

# 同济大学物理科学与工程学院学术交流

## 第一性原理研究滴线原子核性质

李健国

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① Gamow 壳模型计算滴线外原子核共振结构

② 核多体方法计算谱因子——核结构与核反应的桥梁

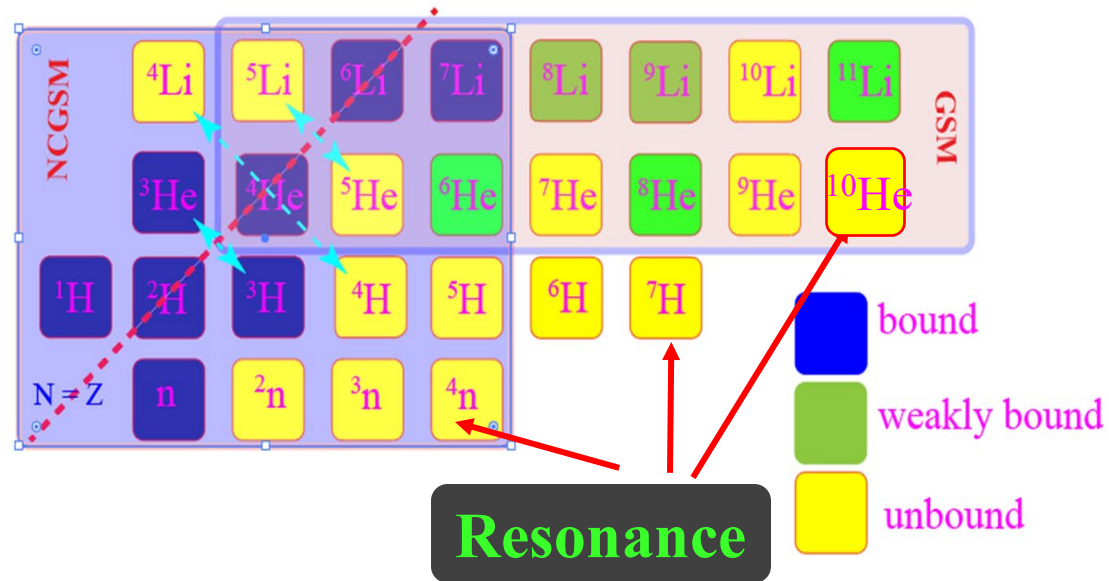
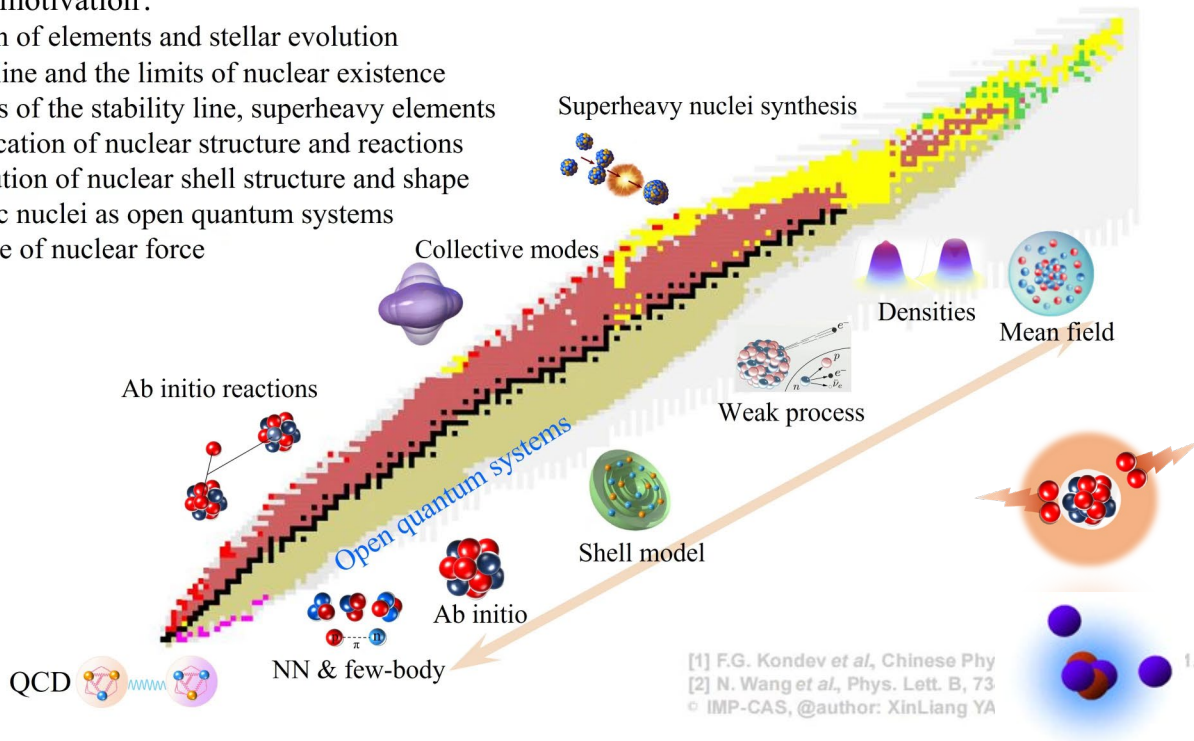
合作者：许甫荣教授, Nicolas Michel, 左维 研究员, 李红蕙, 谢萌冉...

# Gamow 壳模型计算滴线外原子核共振结构

# 滴线原子核

Physics motivation:

- Origin of elements and stellar evolution
- Drip line and the limits of nuclear existence
- Limits of the stability line, superheavy elements
- Unification of nuclear structure and reactions
- Evolution of nuclear shell structure and shape
- Exotic nuclei as open quantum systems
- Nature of nuclear force



HIAF 上重要的科学目标

滴线原子核奇特

- ✓ 晕结构 <sup>6,8</sup>He, <sup>11</sup>Li, <sup>11</sup>Be, <sup>8</sup>B, <sup>17</sup>Ne, <sup>29</sup>F
- ✓ 粒子发射  $p, n, 2p, 2n, 4n, 2n+2n, \dots$
- ✓ Thomas Erhman-shift
- ✓ ...

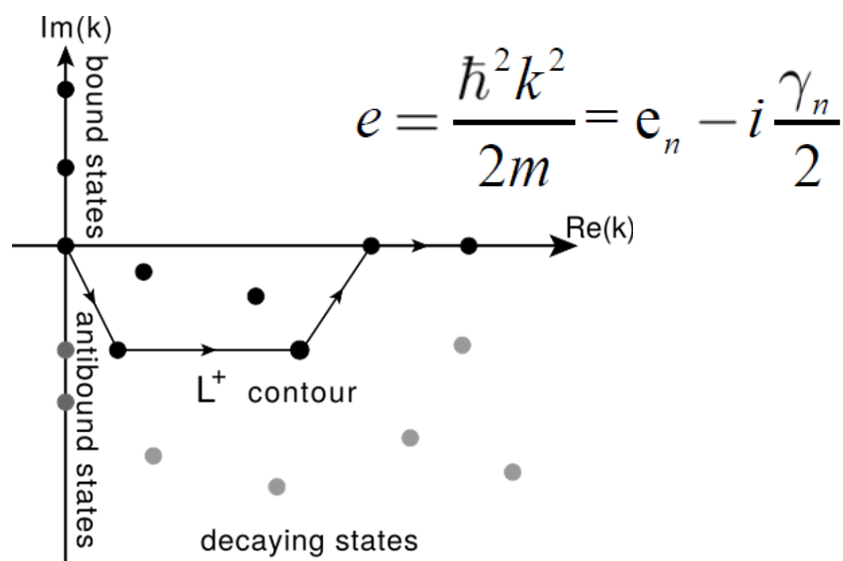
给当前的理论模型提出了巨大的挑战

# 共振态-粒子发射

共振态具有粒子发射特性，是一个时间相关性的量子多体问题

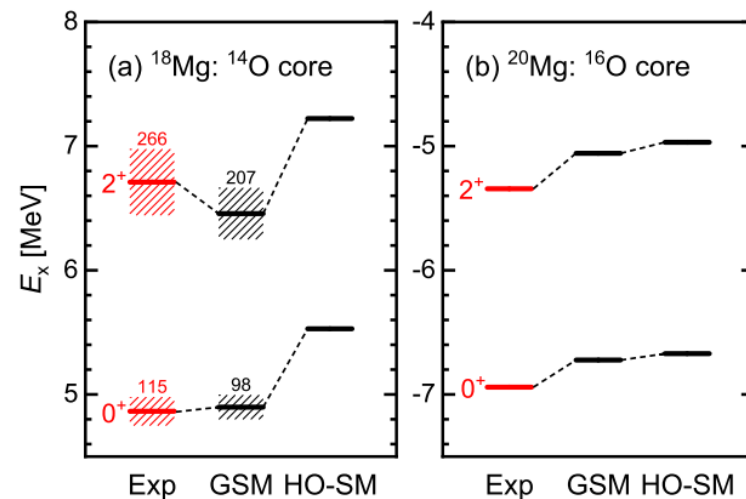
Berggren 复动量空间，将时间相关问题转化为时间无关问题，便于求解薛定谔定态方程

径向波函数  $\psi(\mathbf{r}, t) = e^{-iEt/\hbar} \varphi_E(\mathbf{r}) = e^{-ie_n t/\hbar} \varphi_E(\mathbf{r}) e^{-\gamma_n t/2\hbar}$

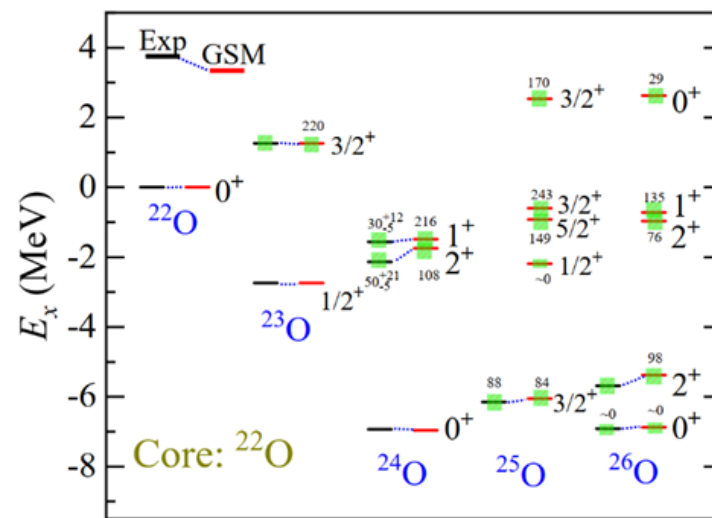


$$\sum_{n \in (b,d)} |u_n\rangle \langle u_n| + \int_{L^+} |u(k)\rangle \langle u(k)| dk = 1$$

T. Berggren, Nucl. Phys. A109 (1968) 265



Y. Jin et al., Phys. Rev. Lett. 127, 262502 (2021).



J. G. Li, et al. Phys. Rev. C 103, 034305 (2021)

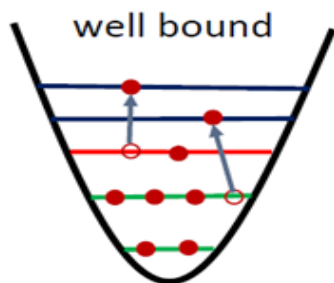
# 量子多体计算- HO /Berggren 完备基矢

$$H = -\sum_i \frac{\hbar^2}{2m} \nabla_i^2 + \sum_{ij} V_{ij} + \sum_{ijk} V_{ijk}$$

NCSM/SM



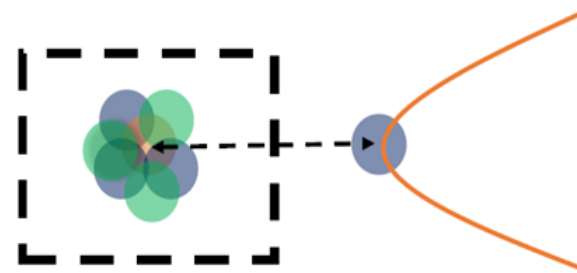
Closed quantum system



HO basis

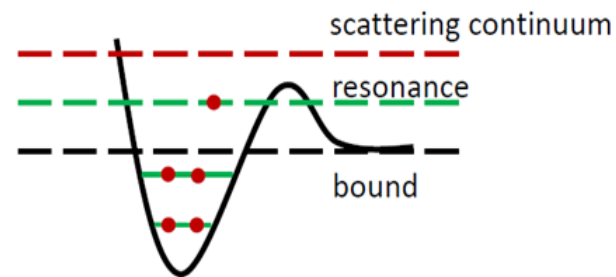
- E. Caurier, et al. RMP 77,427 (2005)*
- T. Otsuka, et al, RMP. 92,015002 (2020)*
- B. R. Barrett, P. Navratil, and J. P. Vary PPNP 69,131(2013)*

NCGSM/GSM



Open quantum system

Berggren 基矢自洽的包含共振与连续谱耦合



Berggren basis

- N. Michel, M. Płoszajczak, The Gamow Shell Model, Springer ;*
- N. Michel, et al., JPG 36,013101 (2009)*
- J. G. Li, et al., Physics 3, 977 (2021)*
- J. G. Li, et al., Phys. Rev. C 100, 054313 (2019)*
- J. G. Li, et al., Phys. Rev. C 104, 024319 (2021)*

# 多中子体系实验研究

- More than 50 years of multi-neutron searches, especially the tetraneutron. See [arXiv:1608.00169](https://arxiv.org/abs/1608.00169)[nucl-th] (2016) Eur. Phys. J. A 57,105(2021),
- Earlier experiments gave negative results, no information about the multi-neutron systems.
- In 2002, Marqués *et al* reported the possible existence of a **bound** tetraneutron observed in a breakup reaction of the  $^{14}\text{Be} \rightarrow ^{10}\text{Be} + 4\text{n}$  channel. Marqués *et al.*, PRC 65, 044006 (2002)
- In 2016, Kisamori *et al* observed of few events of tetraneutron in the doubly charge-exchange reaction  $^4\text{He}(^8\text{He}, ^8\text{Be})$ . Interpreted the tetraneutron as a candidate **resonance** with  $E_r = 0.83 \pm 0.65(\text{stat}) \pm 1.25(\text{syst}) \text{ MeV}$  and width  $\Gamma \leq 2.6 \text{ MeV}$ . Kisamori *et al.*, PRL 116, 044006 (2016)

最新实验工作

reaction	initial state	final state	$\sigma$	results
$^4\text{He}(^8\text{He}, \alpha\alpha)^4\text{n}$ Shimoura, NP1512-SHARAQ10			nb	$N_{\text{evt}} \sim 10\text{s}$ $^4\text{n} : E, \Gamma$
$^8\text{He}(p, p\alpha)^4\text{n}$ Paschalis, NP1406-SAMURAI19			$\mu\text{b}$	$N_{\text{evt}} \sim 1000\text{s}$ $^4\text{n} : E, \Gamma$
$^8\text{He}(p, 2p)\{^3\text{H} + ^4\text{n}\}$ FMM/Yang, NP1512-SAMURAI34			mb	$N_{\text{evt}} \sim 10,000\text{s}$ $^4\text{n} \& ^3\text{H} : E, \Gamma, \Omega$

Nature 2022

杨再宏老师工作

# 多中子体系理论研究-1

Year	Author	Journal	Conclusion
2003	Bertulani et al	J. Phys. G 29, 2431	gave no bound $4n$ combined dineutron-dineutron molecule and a toy NN potential
2003	Steven C. Pieper	PRL. 90, 252501	employed the GFMC calculated the tetraneutron, showed the modern nuclear force can not tolerate a bound tetraneutron and suggested the a tetraneutron resonance near 2MeV
2005	Lazauskas and Carbonell	PRC 72, 034003	used Complex scaling based on Reid 93 NN potential: no low-lying $4n$ resonances : no low-lying tetraneutron resonance.
→ 2016 experiment			
2016	E.Hiyama <i>et al</i>	PRC 93, 044004	employed Complex scaling using AV8'+(toy)NNN, low $4n$ resonance possible only by strongly strongly modify the nuclear force
2016	A.M.Shirokov <i>et al</i>	PRL 117,182502	performed the NCSM with JISP16 interaction conformed a resonant state in tetraneutron around 0.8 MeV, width 1.4 MeV
2017	S. Gandofi <i>et al</i>	PRL 118, 232501(2017)	presented the QMC calculations of multi-neutron systems, suggested the trineutron and tetraneutron were both resonance



# 多中子体系理论研究-2

Year	Author	Journal	Conclusion
2017	K. Fosseze <i>et al.</i>	PRL <b>119</b> , 032501	performed NCGSM gave energy of tetra-neutron may be compatible with experimental value, but the width must be too large
2018	A.Deltuva	PRC <b>97</b> , 034001 (2018), PLB <b>782</b> , 238 (2018)	employed Faddeev method gave the absence of an observable trineutron and tetra-neutron resonance based on modern two-body force
2018	A.M.Shirokov <i>et al.</i>	AIP Conf. proc 020038	Performed NCSM for tetra-neutron with different two-body force, similar results are obtained
2019	A.M.Shirokov <i>et al.</i>	Presentation in Nanjing@China 2019	updated their calculations and gave two resonance states in tetra-neutron
2019	J. G. Li <i>et al.</i>	PRC 100 054313	Performed NCGSM for trineutron and tetra-neutron, predicting that $E(^3n) = 1.29 \text{ MeV}$ $\Gamma(^3n) = 0.91 \text{ MeV}$ $E(^4n) = 2.64 \text{ MeV}$ $\Gamma(^4n) = 2.38 \text{ MeV}$
2020 2021	Michael D. Higgins <i>et al.</i>	PRL 125,052501 PRC 103 024004	Using adiabatic hyperspherical framework, Predicting that that no resonance and no bound state exists for the tetra-neutron system



# 第一性原理无芯Gamow壳模型计算多中子共振态

相互作用 : N3LO

*ab initio* NCGSM

$$E(^3n) = 1.29 \text{ MeV} \quad \Gamma(^3n) = 0.91 \text{ MeV}$$

$$E(^4n) = 2.64 \text{ MeV} \quad \Gamma(^4n) = 2.38 \text{ MeV}$$

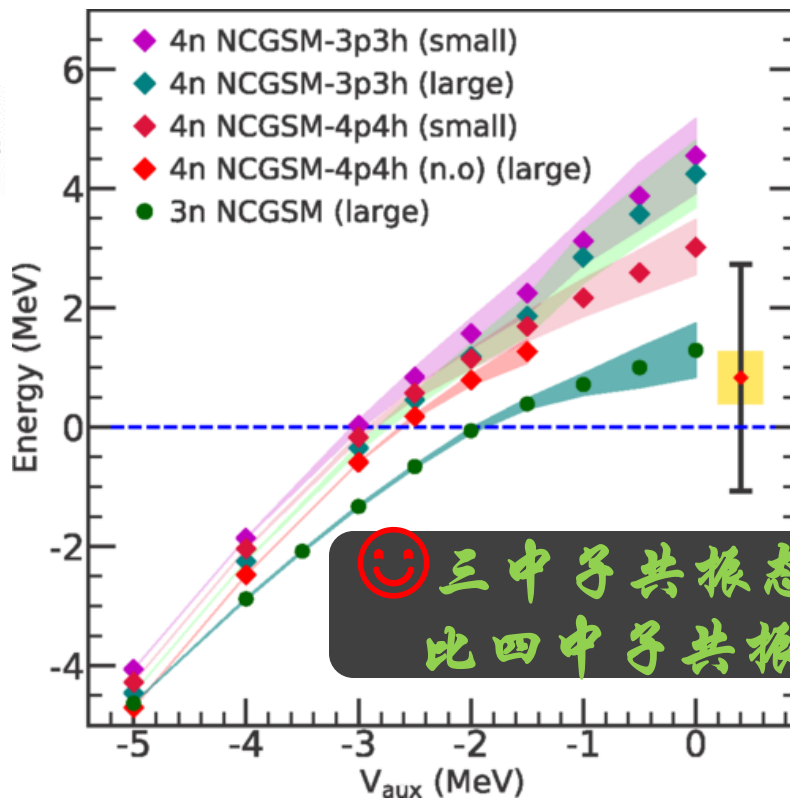
三中子的能量比四中子低，  
与量子蒙卡的外推结果接近

$$H = \frac{1}{A} \sum_i^A \frac{(p_i - p_j)^2}{2m} + \sum_{i < j}^A V_{NN}^{i < j} + \sum_i^A V_{WS}$$

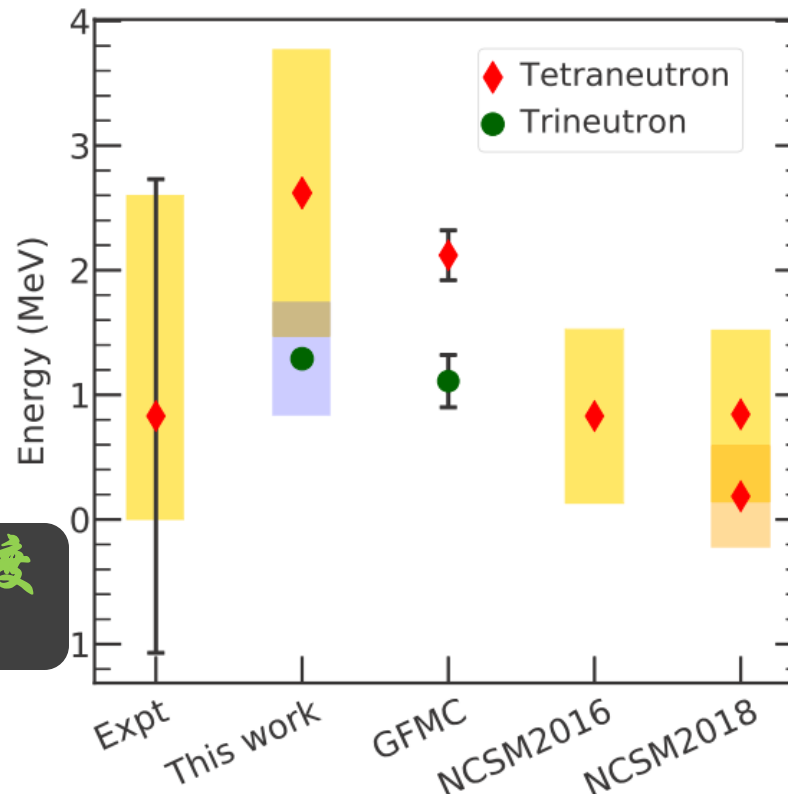
多中子体系

非束缚

连续态耦合强



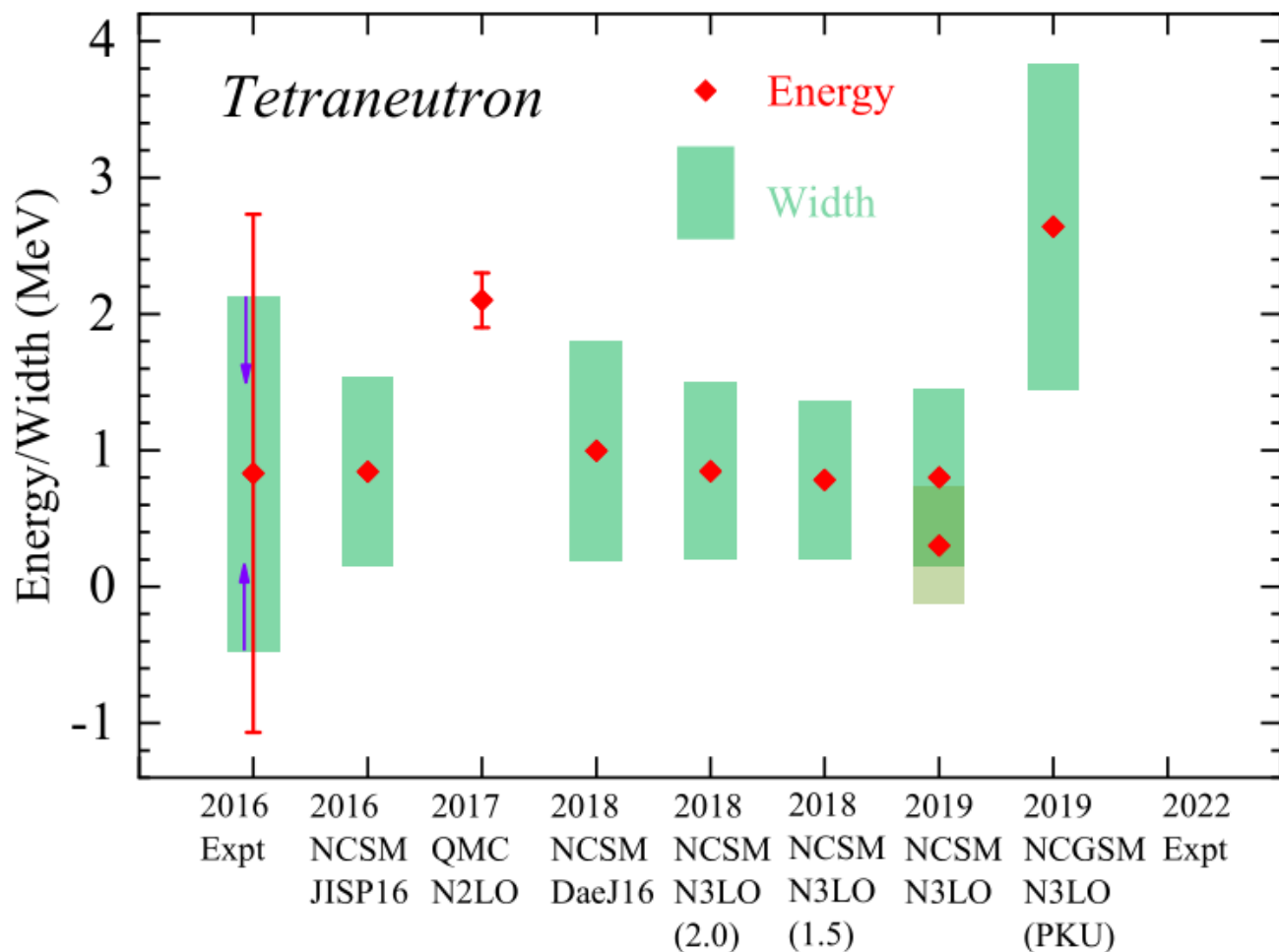
😊 三中子共振态的宽度比四中子共振态小。



😊 三中子共振态在实验上比四中子共振态更容易探测

J. G. Li, N. Michel, B. S. Hu, W. Zuo, and F. R. Xu\*, *Phys. Rev. C* 100, 054313 (2019)

# 四中子研究-理论+实验



- ✓ 2016 Expt : *Kisamori et al., PRL 116, 044006 (2016)*
- ✓ 2016 NCSM: *A. M. Shirokov, et al, PRL 117, 182502 (2016)*
- ✓ 2017 QMC : *S. Gandolfi, et al., PRL 118, 232501(2017)*
- ✓ 2018 NCSM : *A. M. Shirokov, et al AIP Conf. proc 020038 (2018)*
- ✓ 2019 NCSM : *A. M. Shirokov Presentation in Nanjing@China 2019*
- ✓ **2019 NCGSM** : *J. G. Li, N. Michel, B. S. Hu, W. Zuo, and F. R. Xu\*, Phys. Rev. C 100, 054313 (2019)*

# 实验探测到四中子共振态

Article

## Observation of a correlated free four-neutron system


<https://doi.org/10.1038/s41586-022-04827-6>

Received: 4 August 2021

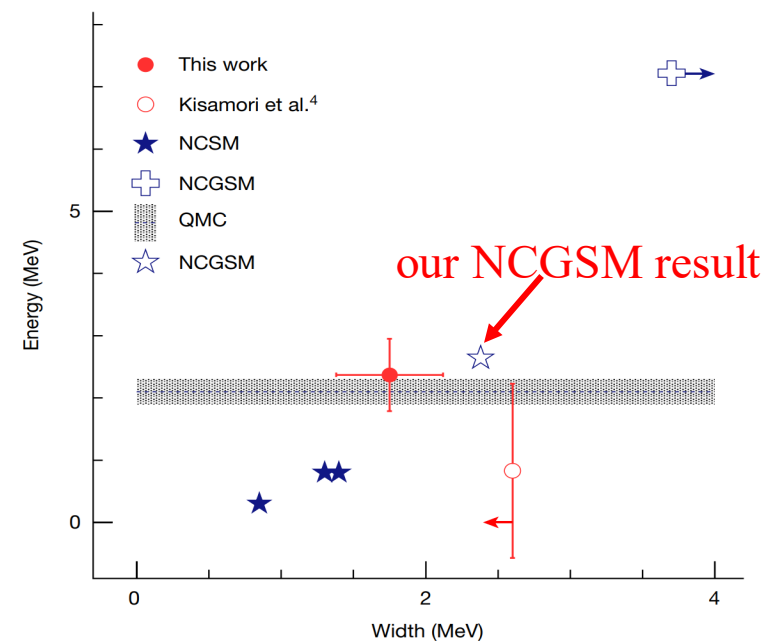
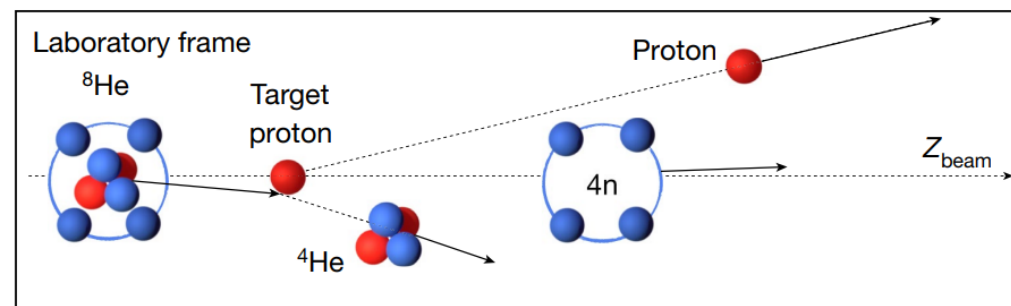
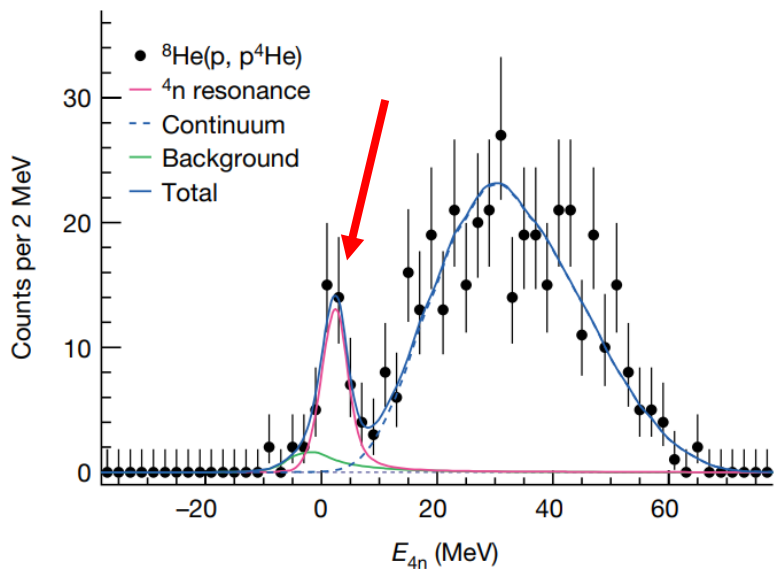
Accepted: 28 April 2022

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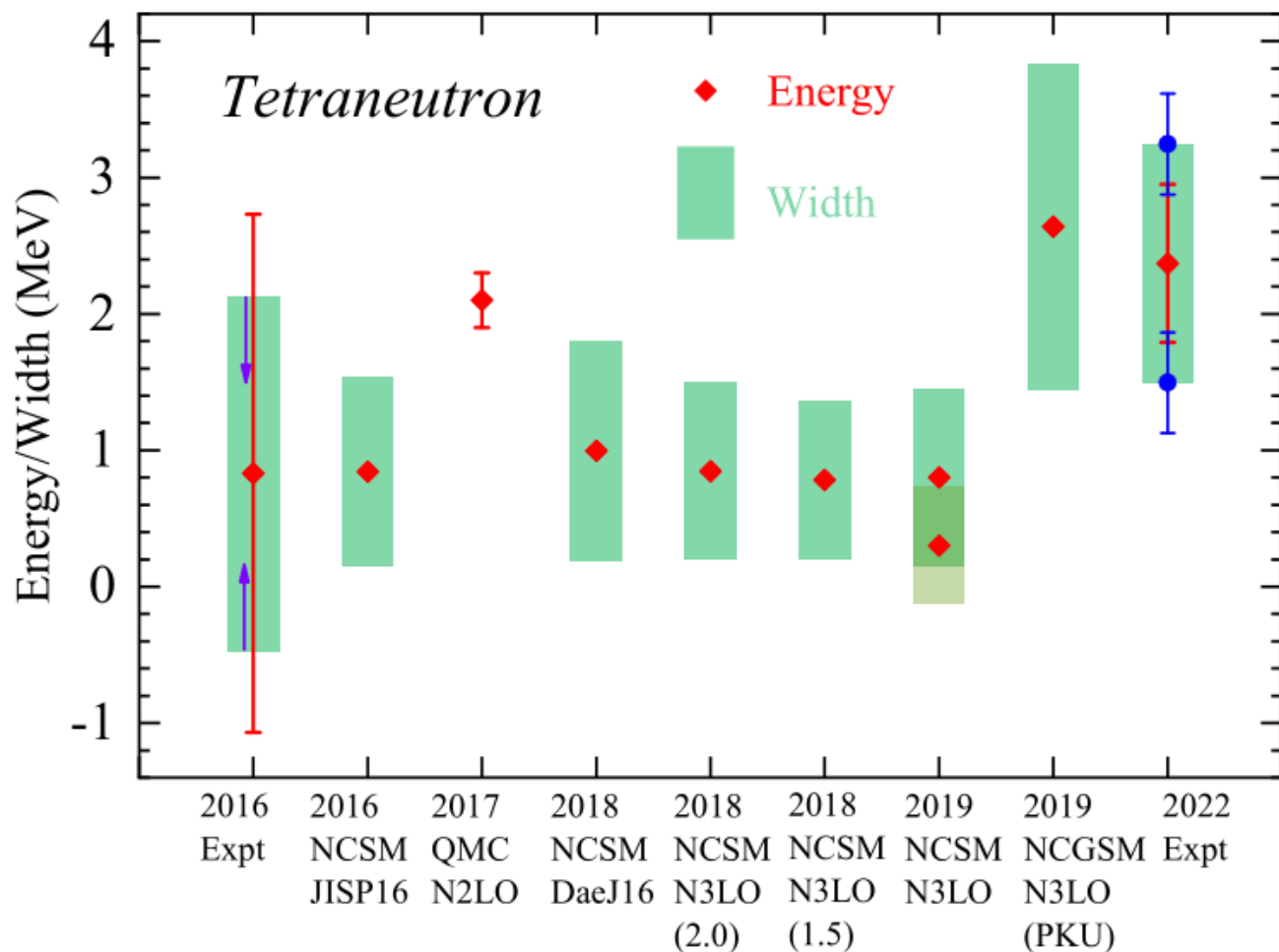
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M. Duer<sup>1,2,3</sup>, T. Aumann<sup>1,2,3</sup>, R. Gernhäuser<sup>4</sup>, V. Panin<sup>2,5</sup>, S. Paschalis<sup>1,6</sup>, D. M. Rossi<sup>1</sup>, N. L. Achouri<sup>7</sup>, D. Ahn<sup>5,16</sup>, H. Baba<sup>8</sup>, C. A. Bertulani<sup>8</sup>, M. Böhmer<sup>4</sup>, K. Boretzky<sup>2</sup>, C. Caesar<sup>1,2,5</sup>, N. Chiga<sup>9</sup>, A. Corsi<sup>9</sup>, D. Cortina-Gil<sup>10</sup>, C. A. Douma<sup>11</sup>, F. Dufter<sup>4</sup>, Z. Elekes<sup>12</sup>, J. Feng<sup>13</sup>, B. Fernández-Domínguez<sup>10</sup>, U. Forsberg<sup>6</sup>, N. Fukuda<sup>5</sup>, I. Gasparic<sup>1,5,14</sup>, Z. Ge<sup>5</sup>, J. M. Gheller<sup>9</sup>, J. Gibelin<sup>7</sup>, A. Gillibert<sup>9</sup>, K. I. Hahn<sup>15,16</sup>, Z. Halász<sup>12</sup>, M. N. Harakeh<sup>11</sup>, A. Hirayama<sup>17</sup>, M. Holl<sup>1</sup>, N. Inabe<sup>5</sup>, T. Isobe<sup>5</sup>, J. Kahlbow<sup>1</sup>, N. Kalantar-Nayestanaki<sup>11</sup>, D. Kim<sup>16</sup>, S. Kim<sup>1,16</sup>, T. Kobayashi<sup>18</sup>, Y. Kondo<sup>17</sup>, D. Körper<sup>2</sup>, P. Koseoglou<sup>1</sup>, Y. Kubota<sup>5</sup>, I. Kuti<sup>12</sup>, P. J. Li<sup>19</sup>, C. Lehr<sup>1</sup>, S. Lindberg<sup>20</sup>, Y. Liu<sup>13</sup>, F. M. Marqués<sup>7</sup>, S. Masuoka<sup>21</sup>, M. Matsumoto<sup>17</sup>, J. Mayer<sup>22</sup>, K. Miki<sup>1,18</sup>, B. Monteagudo<sup>7</sup>, T. Nakamura<sup>17</sup>, T. Nilsson<sup>20</sup>, A. Obertelli<sup>19</sup>, N. A. Orr<sup>7</sup>, H. Otsu<sup>5</sup>, S. Y. Park<sup>15,16</sup>, M. Parlog<sup>7</sup>, P. M. Potlog<sup>23</sup>, S. Reichert<sup>4</sup>, A. Revel<sup>7,9,24</sup>, A. T. Saito<sup>17</sup>, M. Sasano<sup>5</sup>, H. Scheit<sup>1</sup>, F. Schindler<sup>1</sup>, S. Shimoura<sup>21</sup>, H. Simon<sup>2</sup>, L. Stuhl<sup>16,21</sup>, H. Suzuki<sup>5</sup>, D. Symochko<sup>1</sup>, H. Takeda<sup>5</sup>, J. Tanaka<sup>1,5</sup>, Y. Togano<sup>17</sup>, T. Tomai<sup>17</sup>, H. T. Törnqvist<sup>12</sup>, J. Tscheuschner<sup>1</sup>, T. Uesaka<sup>5</sup>, V. Wagner<sup>1</sup>, H. Yamada<sup>17</sup>, B. Yang<sup>15</sup>, L. Yang<sup>21</sup>, Z. H. Yang<sup>5</sup>, M. Yasuda<sup>17</sup>, K. Yoneda<sup>5</sup>, L. Zanetti<sup>1</sup>, J. Zenihiro<sup>5,25</sup> & M. V. Zhukov<sup>20</sup>



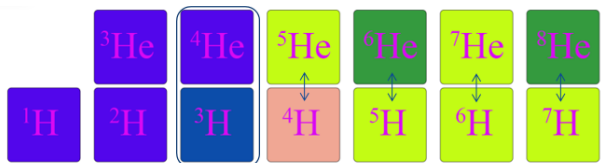
20. Li, J. G., Michel, N., Hu, B. S., Zuo, W. & Xu, F. R. Ab-initio no-core Gamow shell-model calculations of multineutron systems. *Phys. Rev. C* **100**, 054313 (2019).

# 四中子研究-理论+最新实验

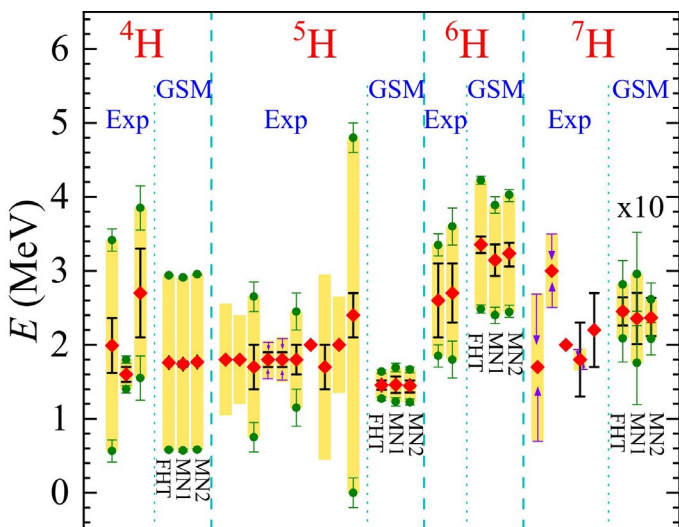


- ✓ 2016 Expt : *Kisamori et al., PRL 116, 044006 (2016)*
- ✓ 2016 NCSM: *A. M. Shirokov, et al, PRL 117, 182502 (2016)*
- ✓ 2017 QMC : *S. Gandolfi, et al., PRL 118, 232501(2017)*
- ✓ 2018 NCSM : *A. M. Shirokov, et al AIP Conf. proc 020038 (2018)*
- ✓ 2019 NCSM : *A. M. Shirokov Presentation in Nanjing@China 2019*
- ✓ 2019 NCGSM : *J. G. Li, N. Michel, B. S. Hu, W. Zuo, and F. R. Xu\*, Phys. Rev. C 100, 054313 (2019)*
- ✓ 2022 Expt : *M. Duer et al. Nature 606, pages 678–682 (2022)*

# Gamow壳模型研究滴线外原子核共振结构

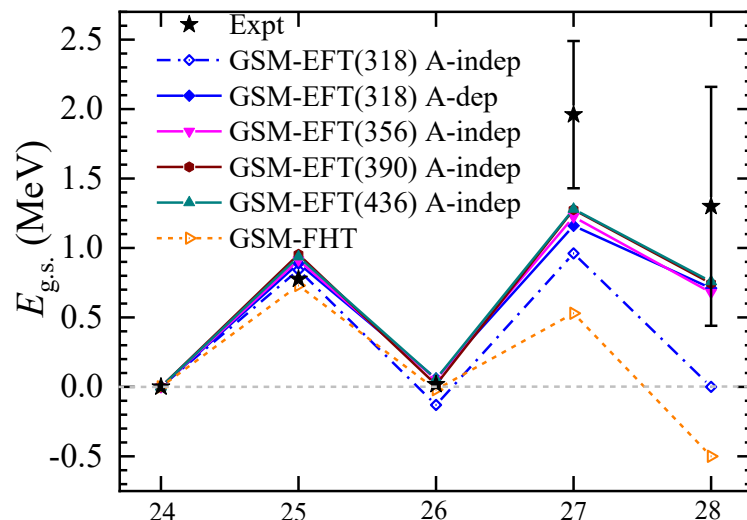
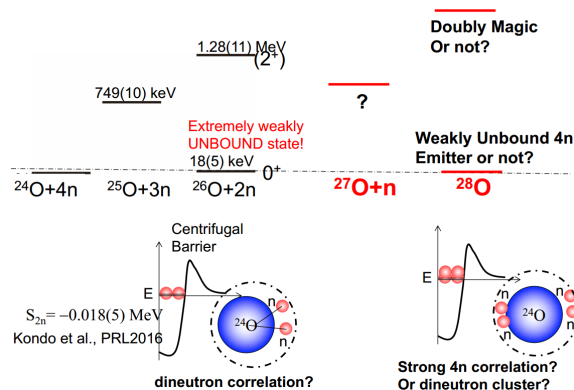


$^7\text{H}$  基态: 窄共振态

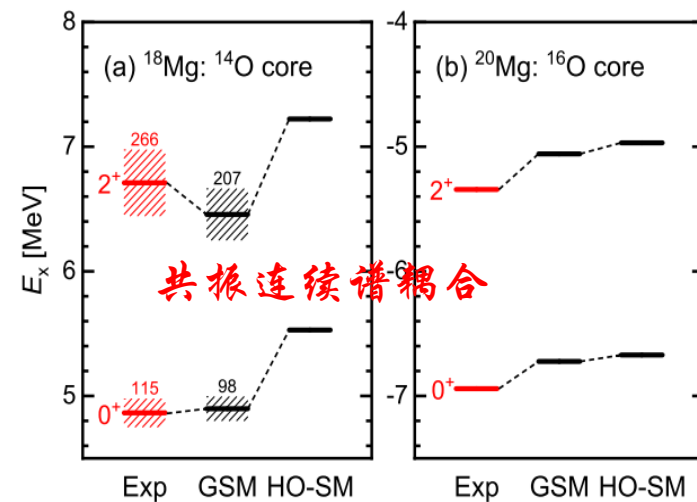
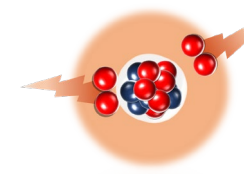
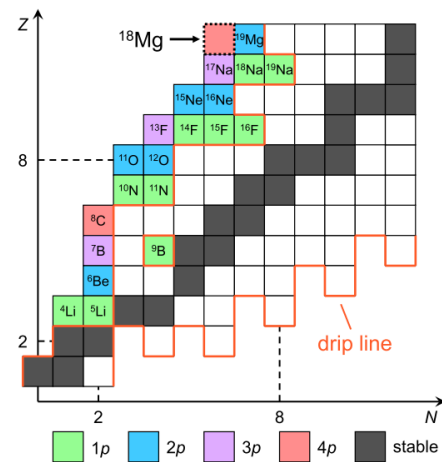


H. H. Li, [J. G. Li](#), N. Michel, and W. Zuo, Phys. Rev. C 104, L061306 (2021)

Dineutron Cluster?



[J.G. Li](#), N. Michel, W. Zuo and F.R. Xu, Phys. Rev. C 103, 034305 (2021)



N. Michel, [J. G. Li](#), F. R. Xu, and W. Zuo, Phys. Rev. C 103, 044319 (2021)

# 小结

## 1. 第一性原理无芯Gamow壳模型计算多中子共振态的能量与宽度 (三中子+四中子)

- ✓ 计算的四中子共振态的能量与宽度被RIKEN实验证实(*Nature* 606, 678 (2022))
- ✓ 三中子在是实验上比四中子更容易被探测

## 2. Gamow壳模型计算滴线外共振原子核:

共振原子核	Gamow shell model	实验
${}^7\text{H}\sim$ ${}^3\text{H} + 4\text{n}$	H. H. Li, J. G. Li, N. Michel, and W. Zuo, Phys. Rev. C 104, L061306 (2021)	实验工作-未发表 RIKEN
${}^{18}\text{Mg}\sim$ ${}^{14}\text{O}+4\text{p}$	N. Michel, J. G. Li, F. R. Xu, and W. Zuo, Phys. Rev. C 103, 044319 (2021)	北京大学华辉老师课题组 PRL 127, 262502 (2021)
${}^{28}\text{O}\sim$ ${}^{24}\text{O}+4\text{n}$	J. G. Li, N. Michel, W. Zuo, and F.R. Xu. Phys, Rev. C 103, 034305 (2021)	实验工作-已投稿 RIKEN

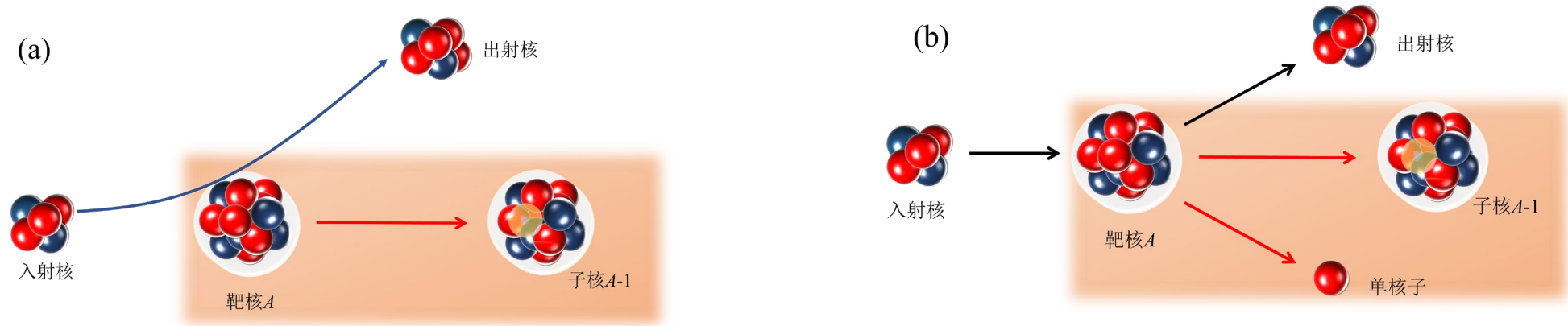
# 核多体方法计算谱因子

核结构与核反应的桥梁



# 谱因子 $\rightarrow$ 核反应/核天体关键核反应

## 当前敲出/转移反应截面的理论框架



单核子敲出反应截面  $\sigma$  与谱因子直接相关  $C^2S$

$$\sigma = \sum_{nlj} \left( \frac{A}{A-1} \right)^N C^2S(\alpha, nlj) \sigma_{sp}(nlj, S_\alpha^*)$$

↑ **Total cross section**                      ↑ **Spectroscopic factor**

$\rightarrow$  **knockout /transfer cross section of nucleon in a single particle state**

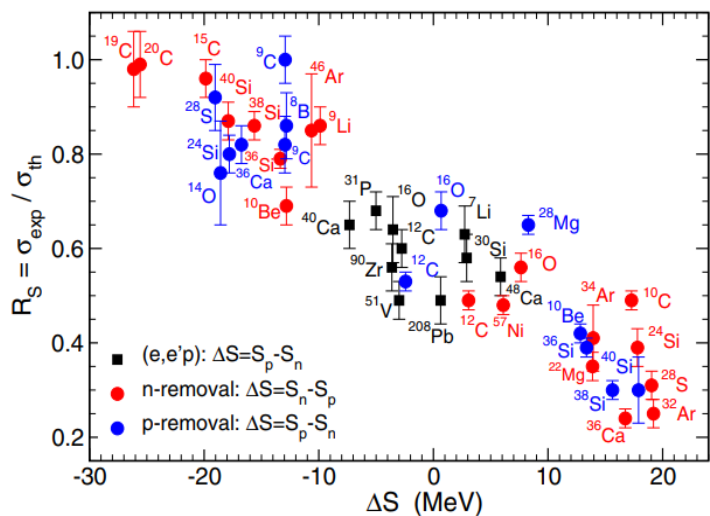
谱因子研究的国内实验方向老师:

核结构与核反应: 楼建玲、陈洁、王惠仁、王世陶、孙志宇、叶沿林、杨再宏、孙叶磊、刘红娜等

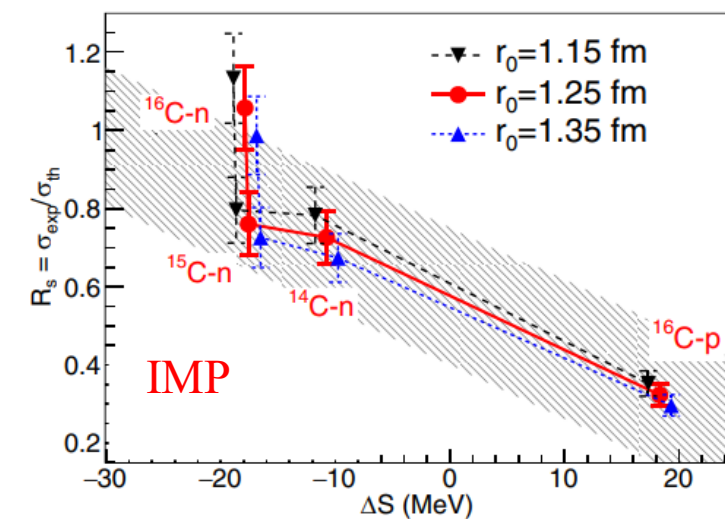
核天体: 何建军、郭冰等

核反应计算: 金磊, 庞丹阳等需要谱因子作为输入量

# 敲出与转移反应中存在的Puzzling



J. A. Tostevin, and A. Gade PRC 90,057602(2014)



Y. Z. Sun, S. T. Wang et al., PRC 104 014310(2021)

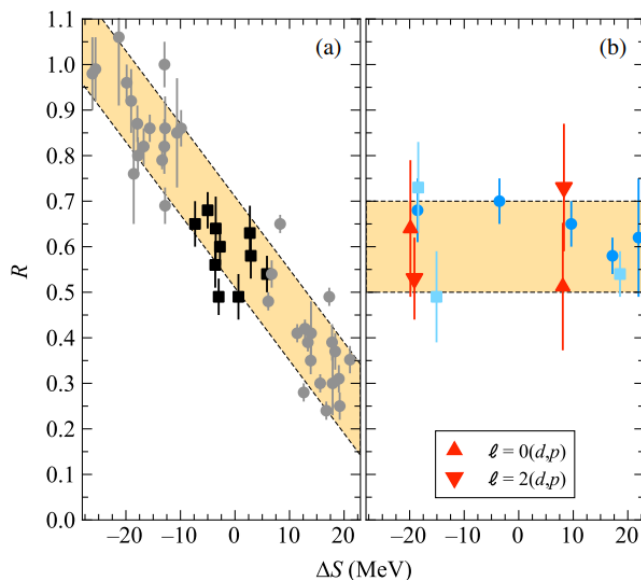
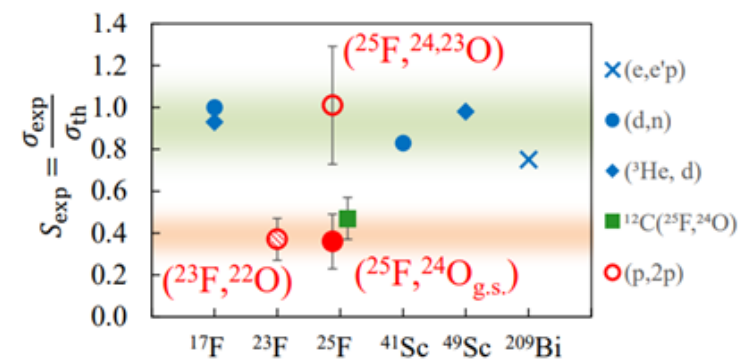


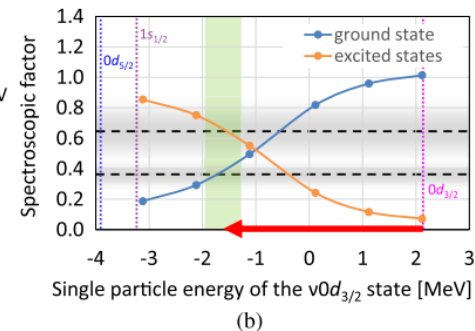
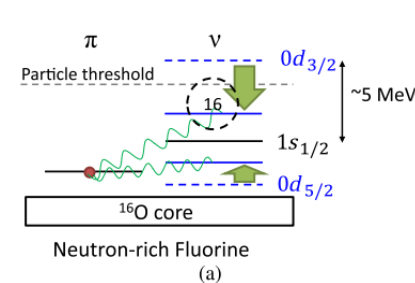
TABLE I. Values of  $\Delta S$ , DWBA (SF), and shell-model ( $SF_{SM}$ ) spectroscopic factors, and  $R$  for the  $1s_{1/2}$  and  $0d_{5/2}$  strength in  $^{15}\text{C}$  and  $^{15}\text{N}$ .

$^A X$	$nlj$	$\Delta S$ (MeV)	SF	$SF_{SM}$	$R$
$^{15}\text{C}$	$1s_{1/2}$	-19.86	0.51(12)	0.80	0.64(15)
	$0d_{5/2}$	-19.12	0.41(7)	0.78	0.53(9)
$^{15}\text{N}$	$1s_{1/2}$	+8.08	0.41(11)	0.80	0.51(14)
	$0d_{5/2}$	+8.29	0.61(12)	0.84	0.73(14)

B. P. Kay, T. L. Tang et al., Phys. Rev. Lett. 129, 152501 (2022)



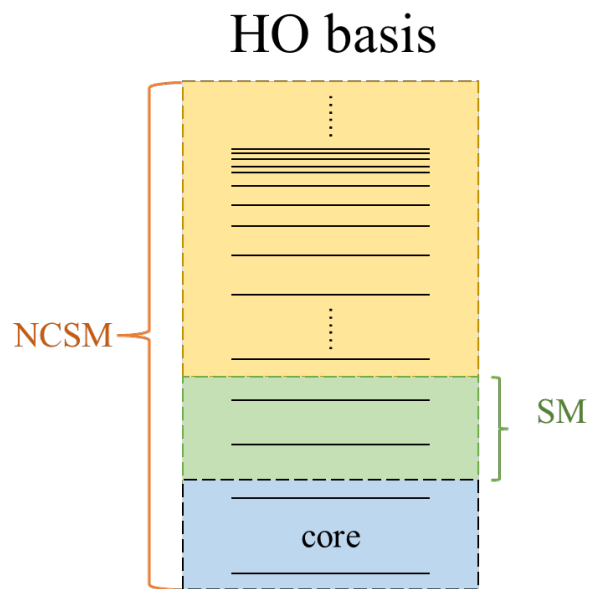
Channel	$J_{th}^{\pi}$	$S_{exp}$	$S_{th}(USDB)$	$S_{th}(SFO)$	$S_{th}(SPDF-MU)$
$(^{25}\text{F}, ^{24}\text{O})$	$5/2^+$	0.36(13)	1.01	0.90	0.95
$(^{25}\text{F}, ^{23}\text{O})$	$5/2^+$	0.65(25)	0.01	0.07	0.05
$(^{25}\text{F}, ^{22}\text{O})$	$1/2^-$	3.43(1.4)		2.19	
$(^{23}\text{F}, ^{22}\text{O})$	$5/2^+$	0.37(10)	1.08	0.92	1.00
$(^{23}\text{F}, ^{21}\text{O})$	$1/2^-, 3/2^-$	4.9(1.5)		5.21	
$(^{23}\text{F}, ^{20}\text{O})$					



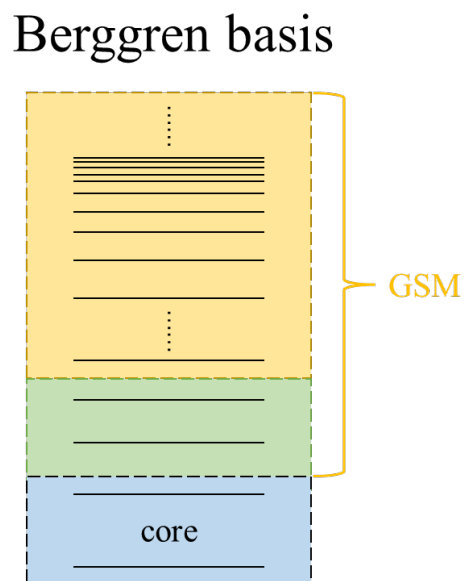
T. L. Tang et al., Phys. Rev. Lett. 124, 212502 (2020)

# 敲出与转移反应中存在的Puzzling

## NCSM/SM



## GSM



$$\hat{H}_{\text{NCSM}} = \sum_{i=1}^A \frac{\mathbf{p}_i^2}{2m} - \frac{\mathbf{P}^2}{2mA} + \sum_{i<j}^A \hat{V}_{ij}^{NN}$$

- ✓ SM/NCSM usually use the HO basis.
- ✓ SM usually choose double magic nuclei as their core.
- ✓ NCSM introduces  $N_{\text{max}}$  as a model space truncation parameter.

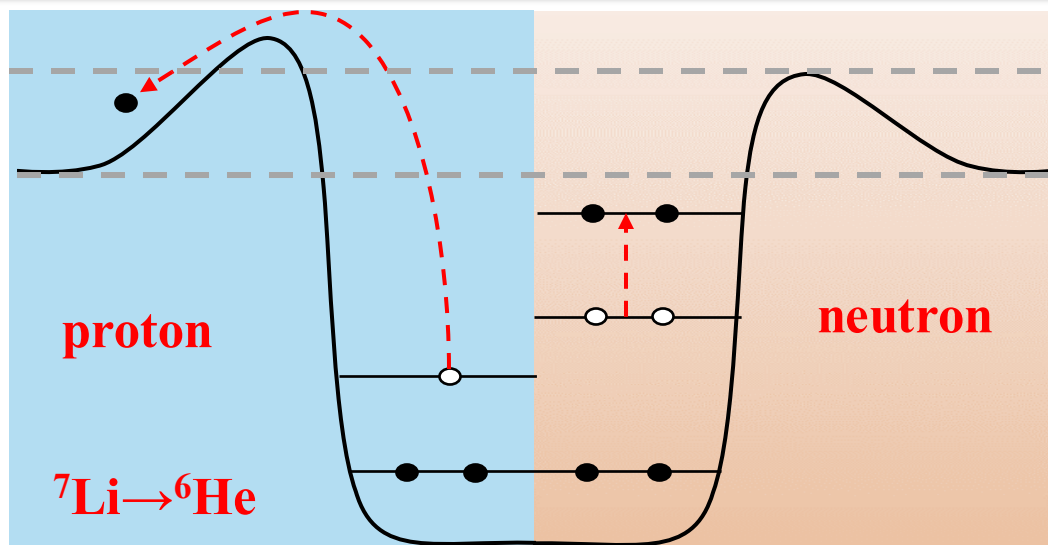
$$\hat{H}_{\text{GSM}} = \sum_{i=1}^{A_{\text{val}}} \left( \frac{\mathbf{p}_i^2}{2\mu_i} + \hat{U}_i^{(c)} \right) + \sum_{i<j}^{A_{\text{val}}} \left( \hat{V}_{ij}^{(\text{res})} + \frac{\mathbf{p}_i \cdot \mathbf{p}_j}{M_c} \right)$$

- ✓ GSM uses the Berggren basis, which contains bound, resonance, and scattering states.

$$O(r) = \frac{1}{\sqrt{2J_A + 1}} \sum_n \langle \Psi_A^{J_A} || a_{n\ell j}^+ || \Psi_{A-1}^{J_{A-1}} \rangle u_n^{(\ell j)}(r), \quad C^2S = \int_0^{+\infty} O(r)^2 dr,$$

量子多体方法计算原子核谱因子  $C^2S$  (GFMC, VMC, SM, NCSM, GSM, CC, SCGF et al.).

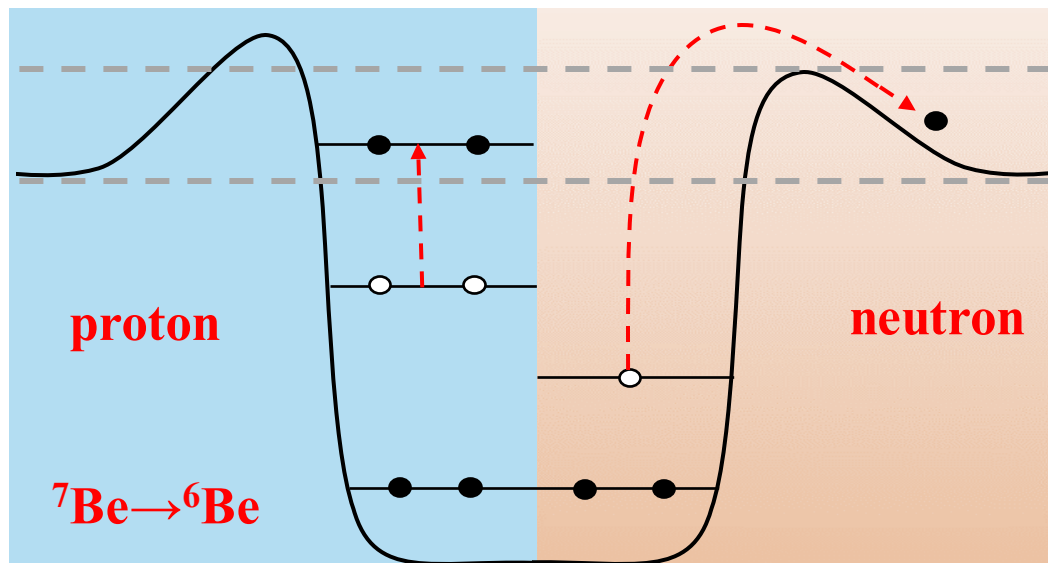
# ① 滴线核深束缚核子谱因子-弱束缚与不束缚原子核



✓  ${}^7\text{Be}$  和  ${}^7\text{Li}$  是深束缚原子核

✓  ${}^6\text{He}$  是弱束缚原子核

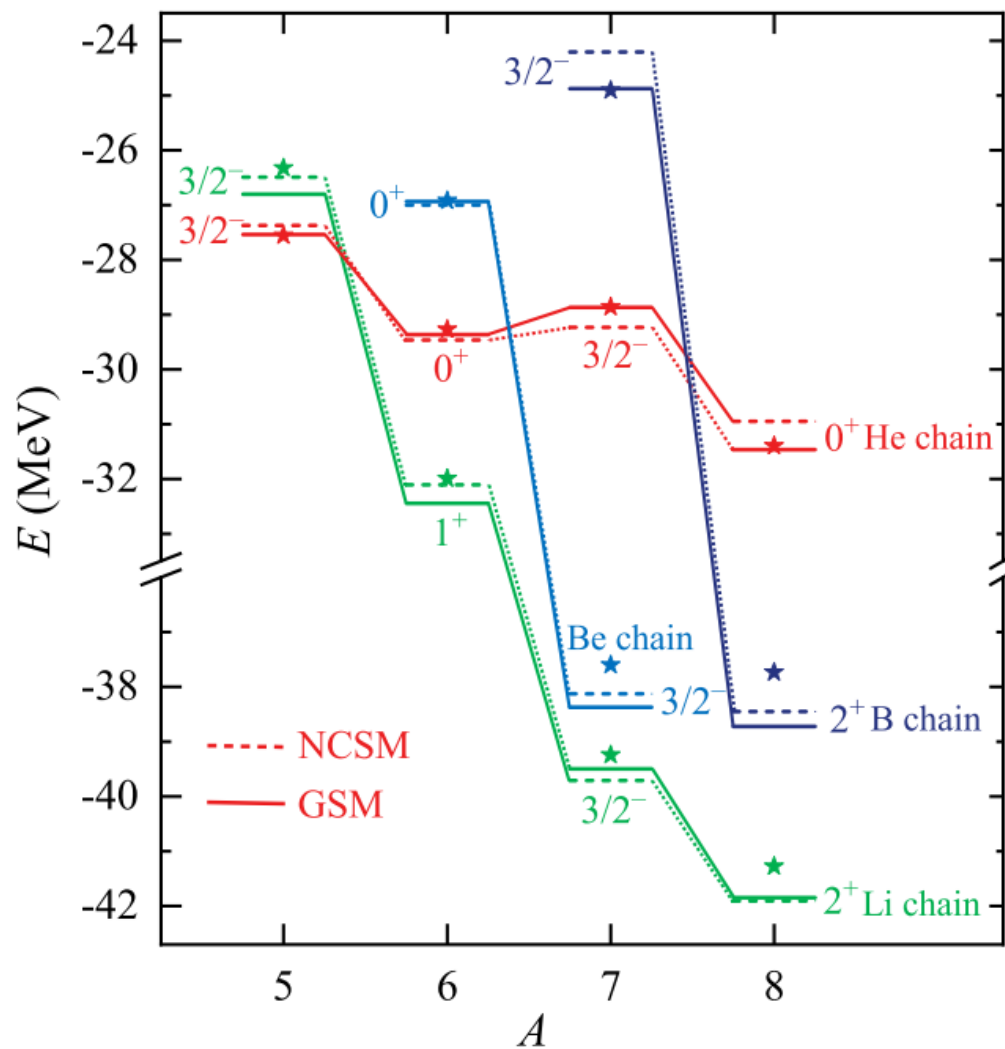
✓  ${}^6\text{Be}$  是不束缚原子核



${}^7\text{Li}$  中价质子与  ${}^7\text{Be}$  中价中子的谱因子是否等于1?

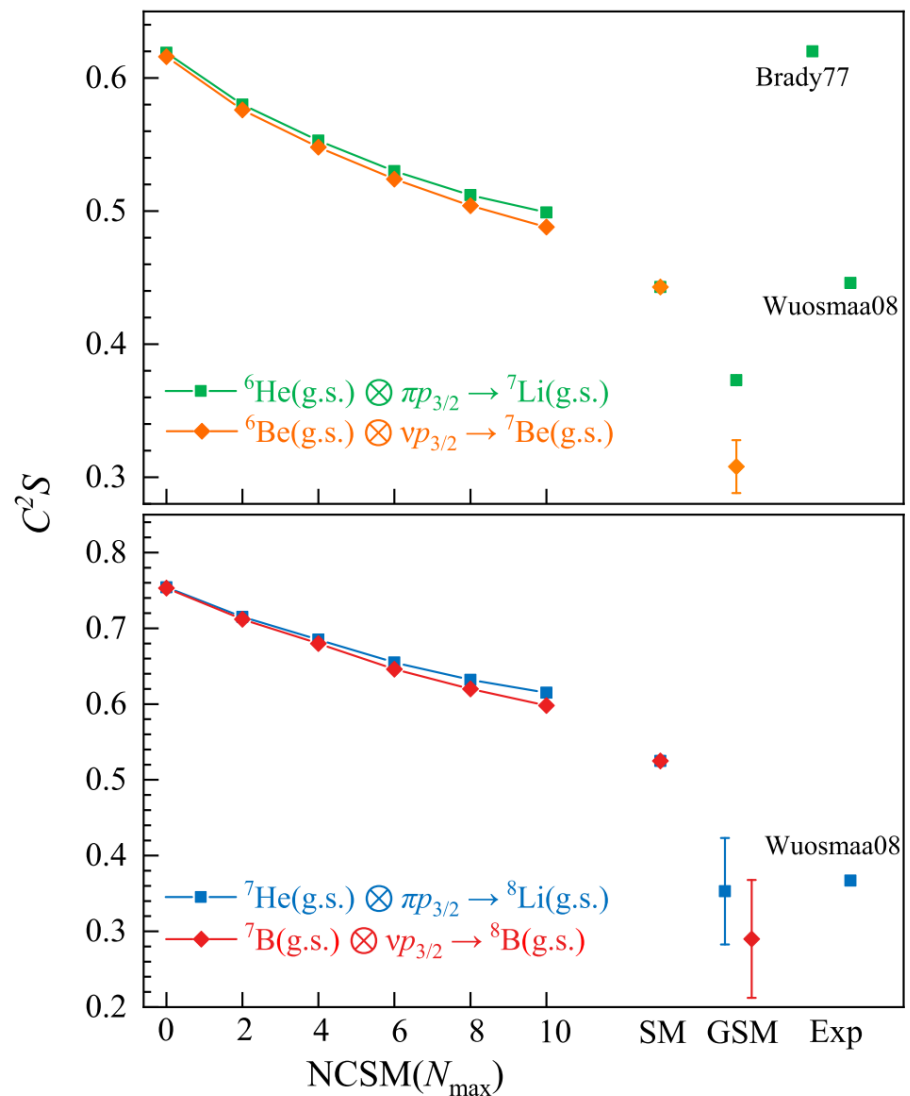
单粒子壳模型图像

# ① 滴线核深束缚核子谱因子



- ✓ NCSM 使用 DJ16 的核子核子相互作用.
- ✓ GSM 计算中使用唯象的 FHT 相互作用, 并选取  $^4\text{He}$  为核芯.
- ✓ GSM 与 NCSM 都  $p$ -壳原子核基态能量提供很好的描述

# ① 滴线核深束缚核子谱因子



- ✓  $C^2S$  NCSM 计算的谱因子随着  $N_{\text{max}}$  增大不收敛;
- ✓ NCSM 计算的谱因子同位旋对称性较小;
- ✓ GSM 计算的  $C^2S$  与实验结果符合较好, 并且其数值小于 NCSM 与 SM 的计算结果.
- ✓ GSM 计算的谱因子  $C^2S$  表示滴线原子核的谱因子中存在明显的同位旋对称性破缺现象  $\rightarrow$  然而, 相关的实验还比较缺乏.

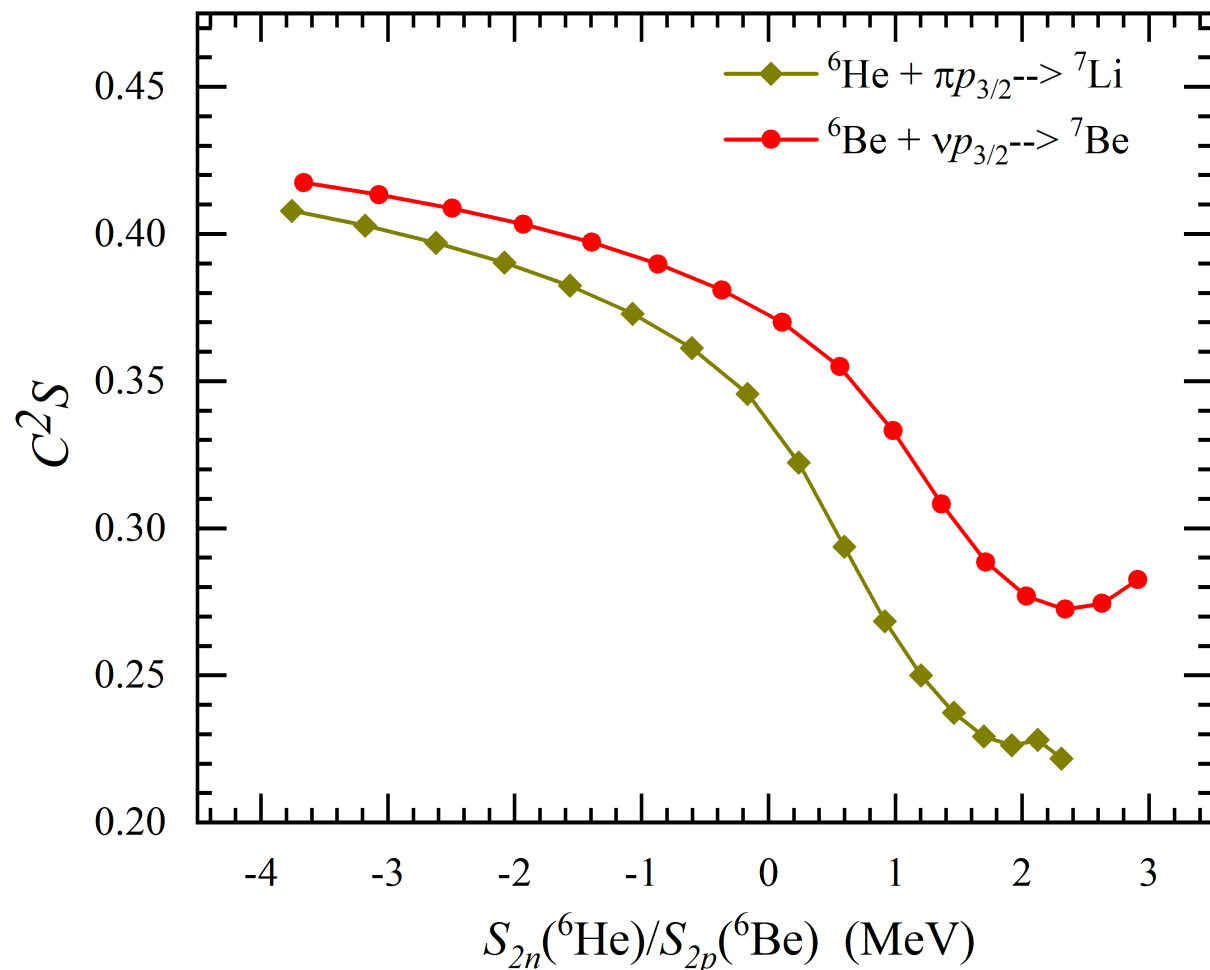
实验物理学家基于国内大科学装置能否开展相关实验研究

M. R. Xie, [J. G. Li](#), N. Michel, H. H. Li, S. T. Wang, H. J. Ong, and W. Zuo, PLB 839 137800 (2023)



## ② 滴线核深束缚核子谱因子—随分离能变化

### Spectroscopic factors of ${}^7\text{Li}$ & ${}^7\text{Be}$



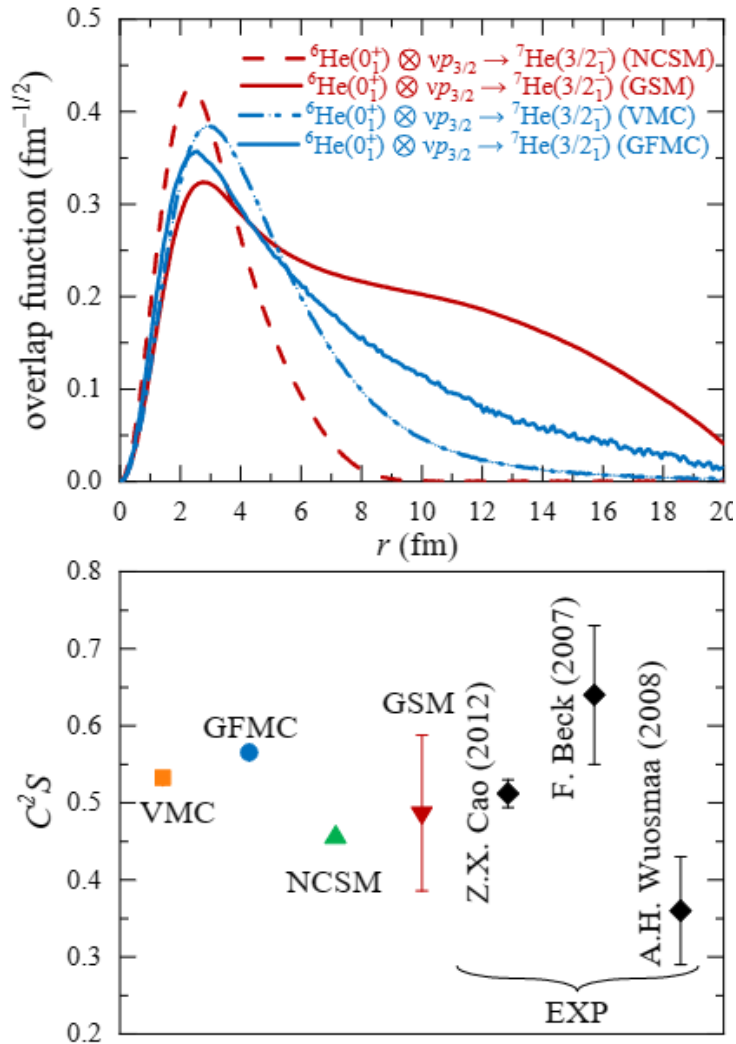
➤ Further study of the effect of **Continuum coupling**

1. Change the depth of WS potential to obtain different  $S_{2n}$ .
2.  $C^2S$  changes significantly near threshold due to continuum coupling
3. There is an inflection point in SF.
4. Further theoretical analysis is needed.

M. R. Xie, J. G. Li\*, N. Michel, W. Zuo, Preliminary results



# ③ 共振态谱因子计算

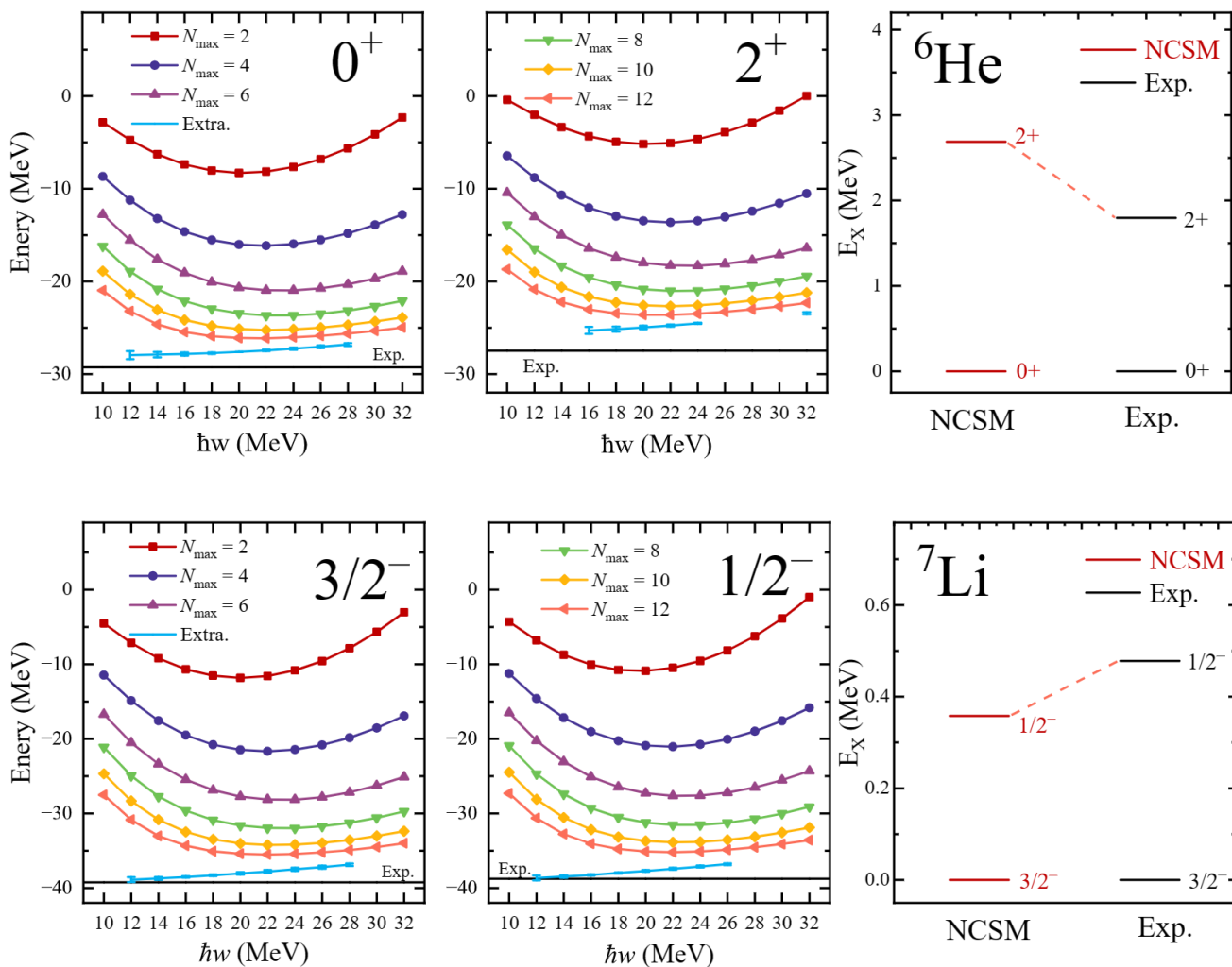


- ✓  ${}^7\text{He}(3/2^-)$  is resonance state, the asymptotic region of the overlap function could only be well described in GSM calculation.
- ✓ The GFMC done in coordinate including continuum effects, but difficult for resonance states.
- ✓ The calculated  $C^2S$  with GSM are close to experimental data.  
*Z. X. Cao, Y. L. Ye, J. Xiao, et al, PLB 707,46 (2012)*
- ✓ Due to the localization of NCSM calculation, the calculated  $C^2S$  of NCSM is smallest.

M. R. Xie, [J. G. Li\\*](#), N. Michel, et al., Submitted to PRC

# ④ 第一性原理无芯壳模型计算谱因子

## NCSM calculations with NNLOopt interaction

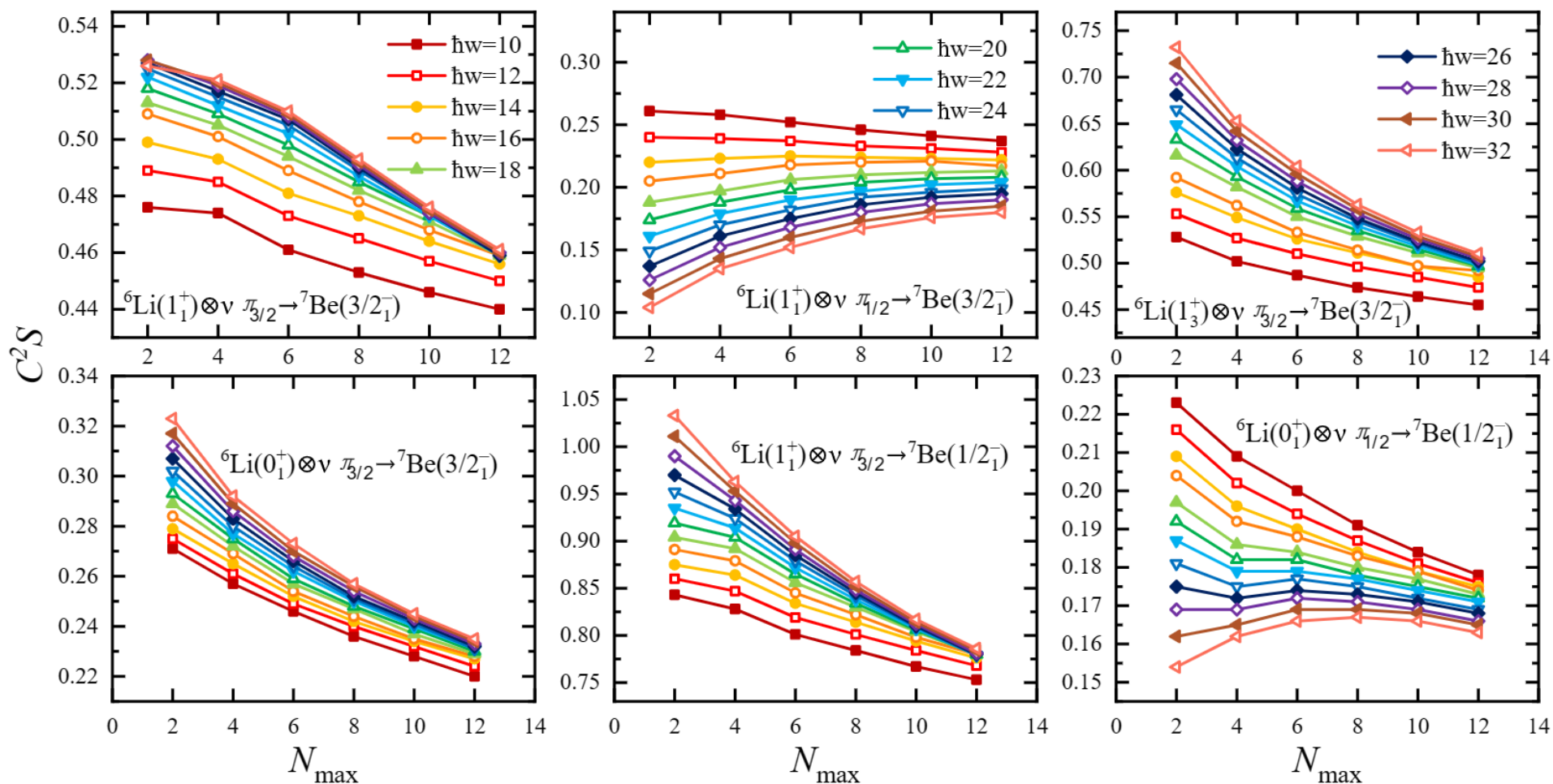


1. The results for energy levels converge quickly.
2. Use the exponential extrapolation formula to obtain spectra convergence results.

$$E(N_{\text{max}}) = A_0 + A_1 \exp(-A_2 * N_{\text{max}})$$

M. R. Xie, [J. G. Li\\*](#), W. Zuo, preliminary results

# ④ 第一性原理无芯壳模型计算谱因子



the exponential extrapolation formula does not apply to SF

M. R. Xie, [J. G. Li\\*](#), W. Zuo, preliminary results

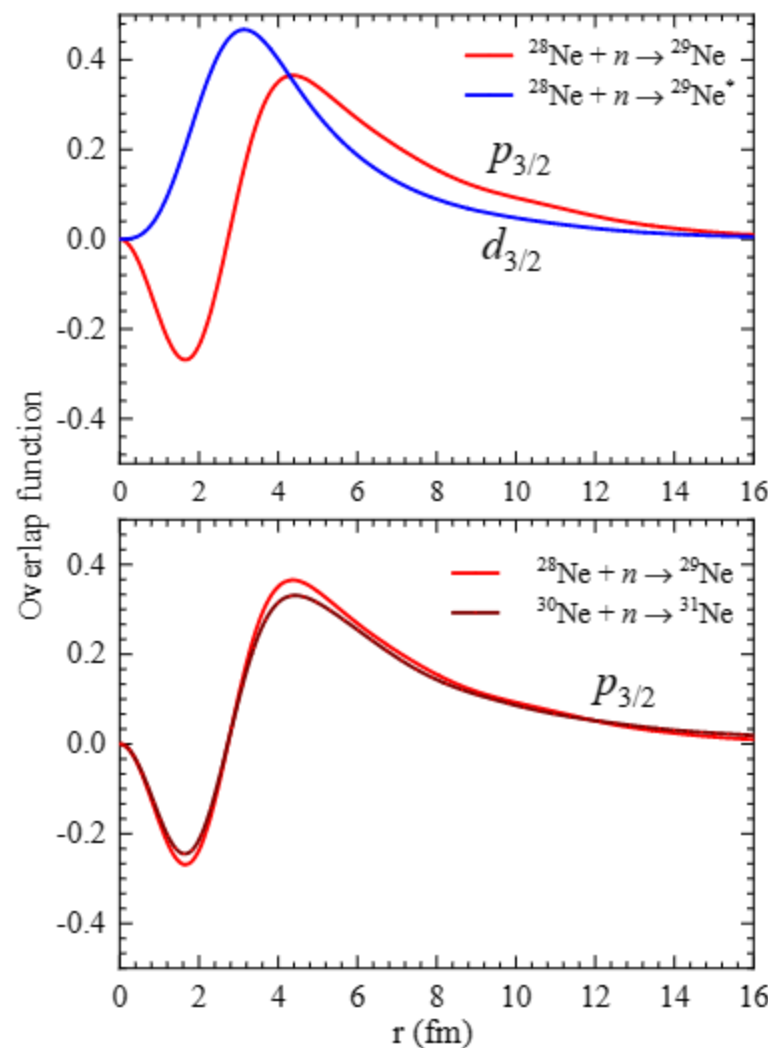
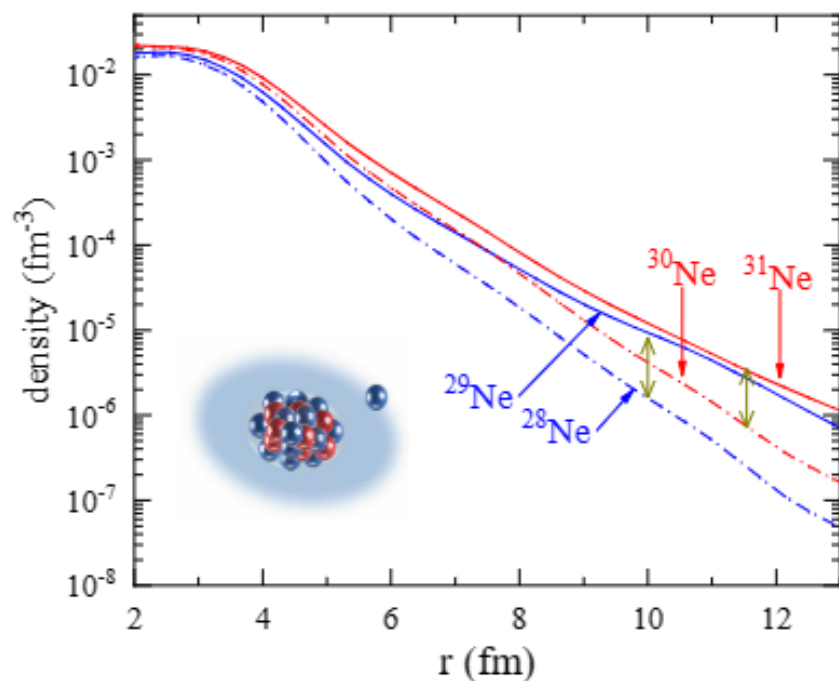
# ⑤ 谱因子/overlap function 研究单核子晕结构性质

## Gamow 壳模型研究 $^{29}\text{Ne}$ 单中子晕结构

✓  $^{31}\text{Ne}$ : 单中子晕

*L. Gaudefroy, et al. PRL. 109, 202503(2012)*

*T. Nakamura et al. PRL. 112, 142501(2014)*



$p_{3/2} C^2S$

$^{29}\text{Ne}$ : 0.540

$^{31}\text{Ne}$ : 0.392

[J. G. Li](#), et al., PLB 832, 137225 (2022)

## □ Gamow壳模型计算原子核共振结构

- ✓ 计算的四中子共振态的能量与宽度被RIKEN实验证实 (Nature 606, 678 (2022))
- ✓ Gamow壳模型预言滴线外原子核共振结构 ( ${}^7\text{H}$ ,  ${}^{28}\text{O}$ ,  ${}^{18}\text{Mg}$ )

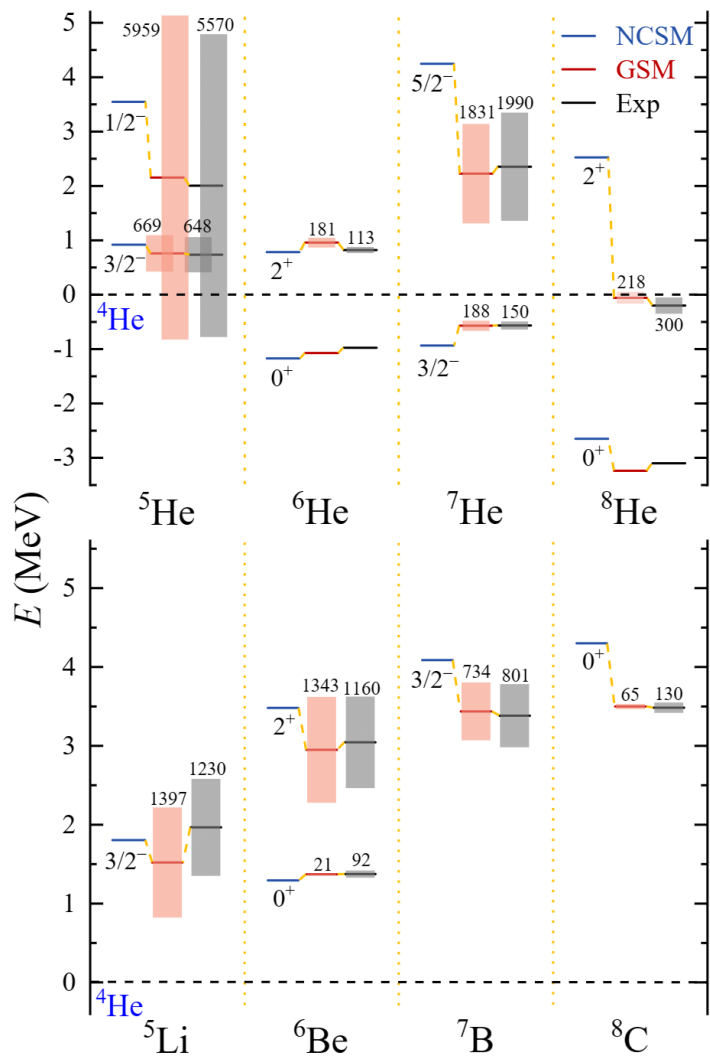
## □ 原子核多体方法计算谱因子

- ✓ 连续态效应对深束缚核子谱因子与共振态的谱因子很关键
- ✓ 第一性原理无芯壳模型对于谱因子计算收敛性差
- ✓ 谱因子/overlap function 研究单核子晕结构

请各位老师批评指正!

# Backslide

# ③ 共振态谱因子计算



Resonance states lie above decay thresholds, so that they can decay via the emission of particle.

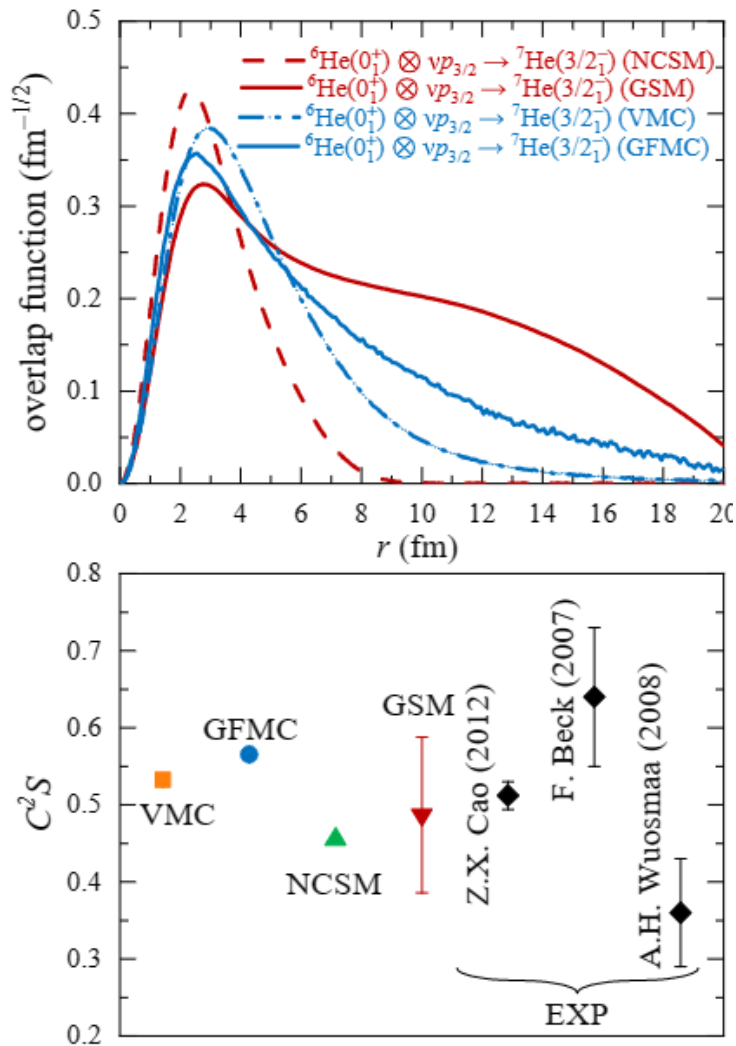
$$E = E_0 - i\Gamma/2$$

- ✓ The resonance can only be expanded with Berggren basis, but can not with HO basis.
- ✓ The resonance states can be well described with GSM, particularly the calculated width is close to experimental data.
- ✓ NCSM lacks continuum coupling
- ✓ Broad resonance state:  ${}^5\text{He}(1/2^-)$ ,  ${}^7\text{He}(5/2^-)$ ,  ${}^5\text{Li}(3/2^-)$ ,  ${}^6\text{Be}(2^+)$ ,  ${}^7\text{B}(3/2^-)$ , et al.
- ✓ Weakly-bound and loose-unbound states :  ${}^6, {}^8\text{He}(0^+)$ ,  ${}^7\text{He}(3/2^-)$ ,  ${}^6\text{Be}(0^+)$ ,  ${}^8\text{C}(0^+)$ , et al.

M. R. Xie, J. G. Li\*, N. Michel, H. H. Li, and W. Zuo, Submitted to PRC



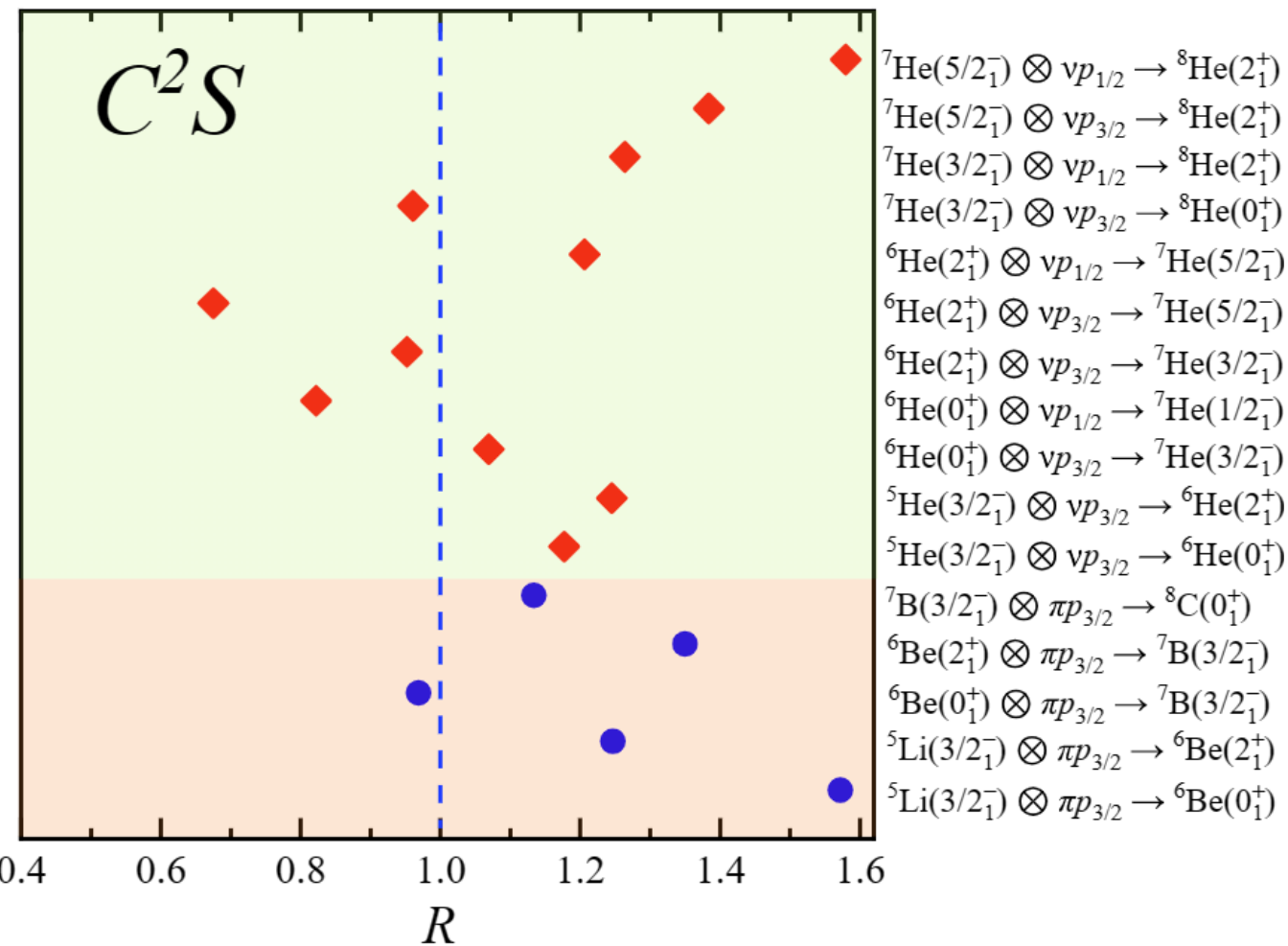
# ③ 共振态谱因子计算



- ✓  ${}^7\text{He}(3/2^-)$  is resonance state, the asymptotic region of the overlap function could only be well described in GSM calculation.
- ✓ The GFMC done in coordinate including continuum effects, but difficult for resonance states.
- ✓ The calculated  $C^2S$  with GSM are close to experimental data.  
*Z. X. Cao, Y. L. Ye, J. Xiao, et al, PLB 707,46 (2012)*
- ✓ Due to the localization of NCSM calculation, the calculated  $C^2S$  of NCSM is smallest.

M. R. Xie, J. G. Li\*, N. Michel, et al., Submitted to PRC

# ③ 共振态谱因子计算



$$R = \frac{C^2 S_{GSM}}{C^2 S_{NCSM}}$$

M. R. Xie, J. G. Li\*, N. Michel, H. H. Li, and W. Zuo, Submitted to PRC

- ✓  $|R - 1| \sim 0.2 - 0.6$  : strong coupling to continuum  $\rightarrow$  related to broad resonance states. For example  ${}^7\text{He}(5/2^-) + p_{3/2} \rightarrow {}^8\text{He}(2^+)$
- ✓ Broad resonance state:  ${}^5\text{He}(1/2^-)$ ,  ${}^7\text{He}(5/2^-)$ ,  ${}^5\text{Li}(3/2^-)$ ,  ${}^6\text{Be}(2^+)$ ,  ${}^7\text{B}(3/2^-)$ , et al.
- ✓  $|R - 1| \sim 0$  : related to narrow resonance states or weakly-bound state.
- ✓ Weakly-bound and loose-unbound states :  ${}^6, {}^8\text{He}(0^+)$ ,  ${}^7\text{He}(3/2^-)$ ,  ${}^6\text{Be}(0^+)$ ,  ${}^8\text{C}(0^+)$ , et al.

# NCGSM for multi-neutron systems

Interaction : N3LO  $V_{\text{low-}k}$   $2.1 \text{ fm}^{-1}$

Model space :  $s_{1/2}, p_{3/2}$  in Berggren basis (45 points)

$p_{1/2}, d_{5/2,3/2}, f_{5/2,7/2}, g_{9/2}$  in HO basis. ( $N_{\text{max}} \leq 20$ )

Method : Davison + overlap method

N. Michel, W. Nazarewicz, M. Ploszajczak, and T. Vertse, JPG 36, 013101 (2009)

Multineutron systems

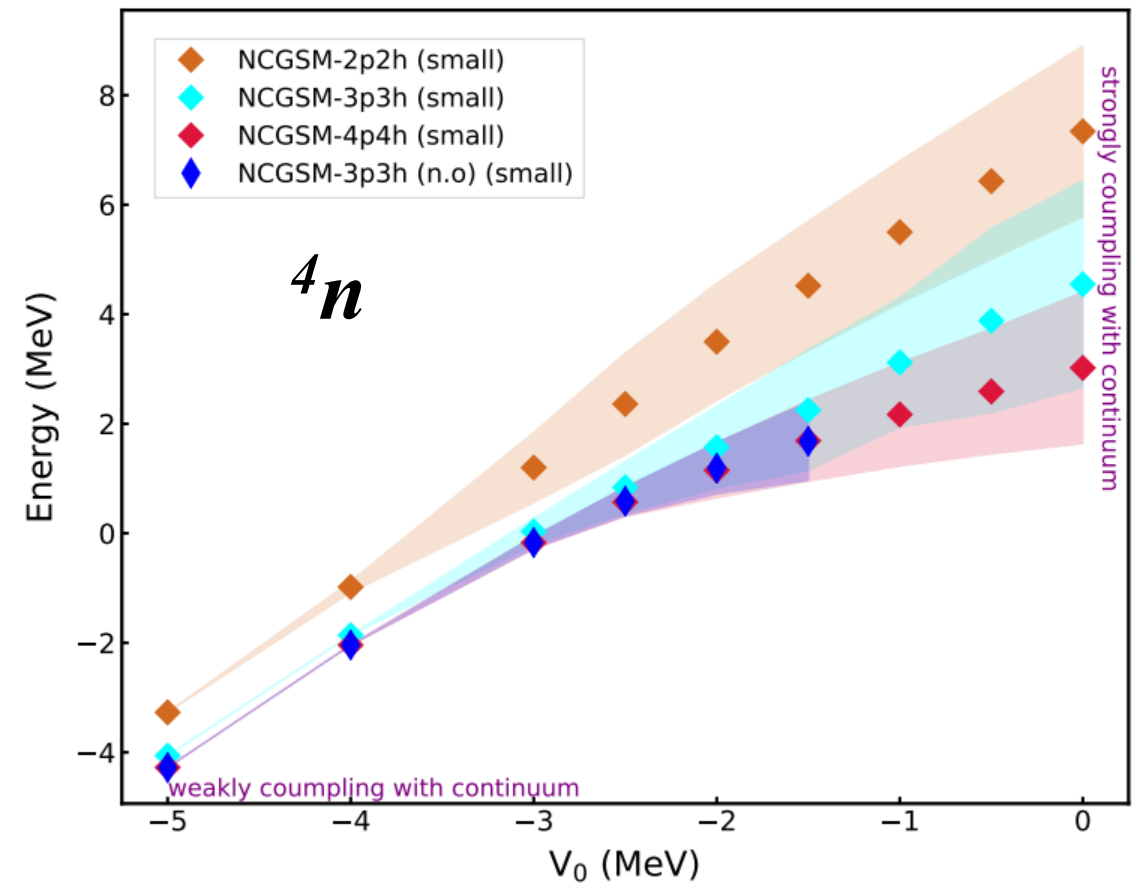
Unbound

Strongly coupling with continuum

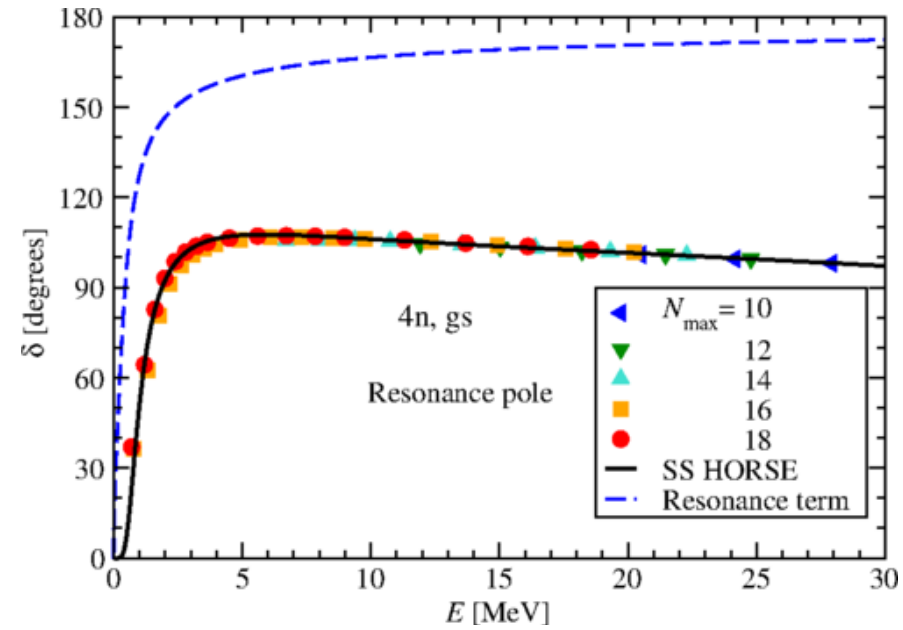
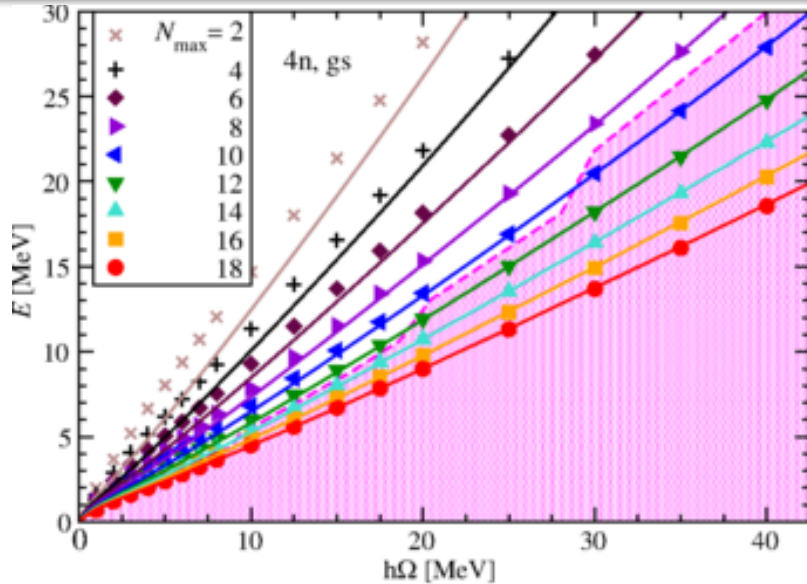
$$H = \frac{1}{A} \sum_i^A \frac{(p_i - p_j)^2}{2m} + \sum_{i < j}^A V_{NN}^{i < j} + \sum_i^A V_{ws}$$

small model space :  $s_{1/2}, p_{3/2}$  in Berggren basis (45 points)

$p_{1/2}, d_{5/2,3/2}, f_{5/2,7/2}, g_{9/2}$  in HO basis. ( $N_{\text{max}} \leq 4$ )



# 第一性原理无芯壳模型计算四中子共振态



$E(\text{tetraneutron}) = 0.8 \text{ MeV}$     $\Gamma(\text{tetraneutron}) = 1.4 \text{ MeV}$

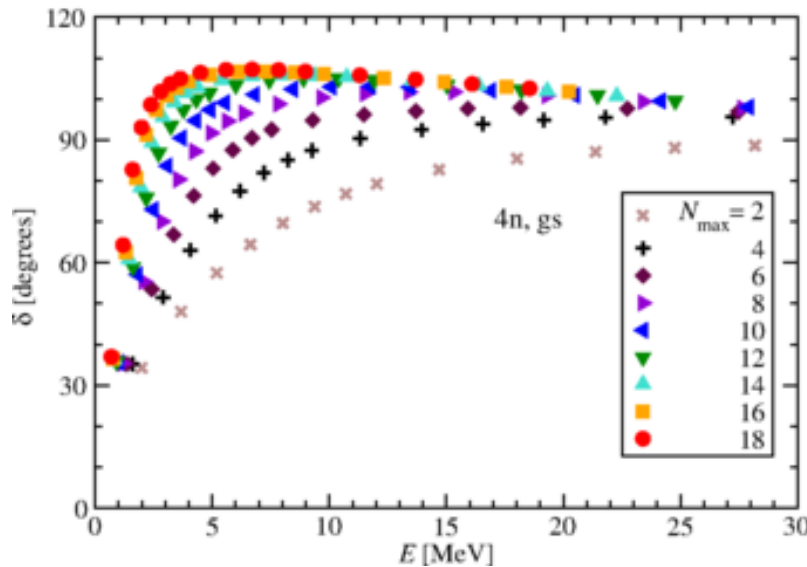
*A. M. Shirokov, et al, PRL 117,182502 (2016)*

2019南京会议上, 预言四中子具有两个共振态

1<sup>st</sup>:  $E(4n) \sim 0.3 \text{ MeV}$     $\Gamma(4n) \sim 0.8 \text{ MeV}$

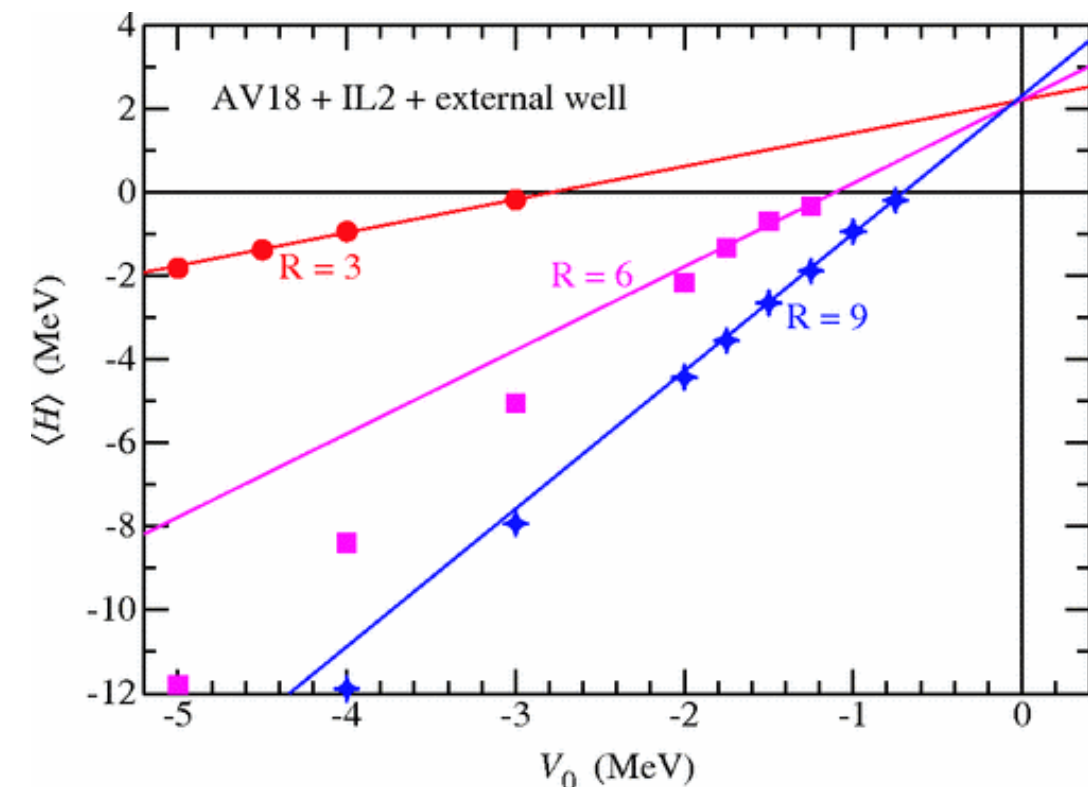
2<sup>nd</sup>:  $E(4n) \sim 0.8 \text{ MeV}$     $\Gamma(4n) \sim 1.3 \text{ MeV}$

*A. M. Shirokov Presentation in Nanjing@China 2019*

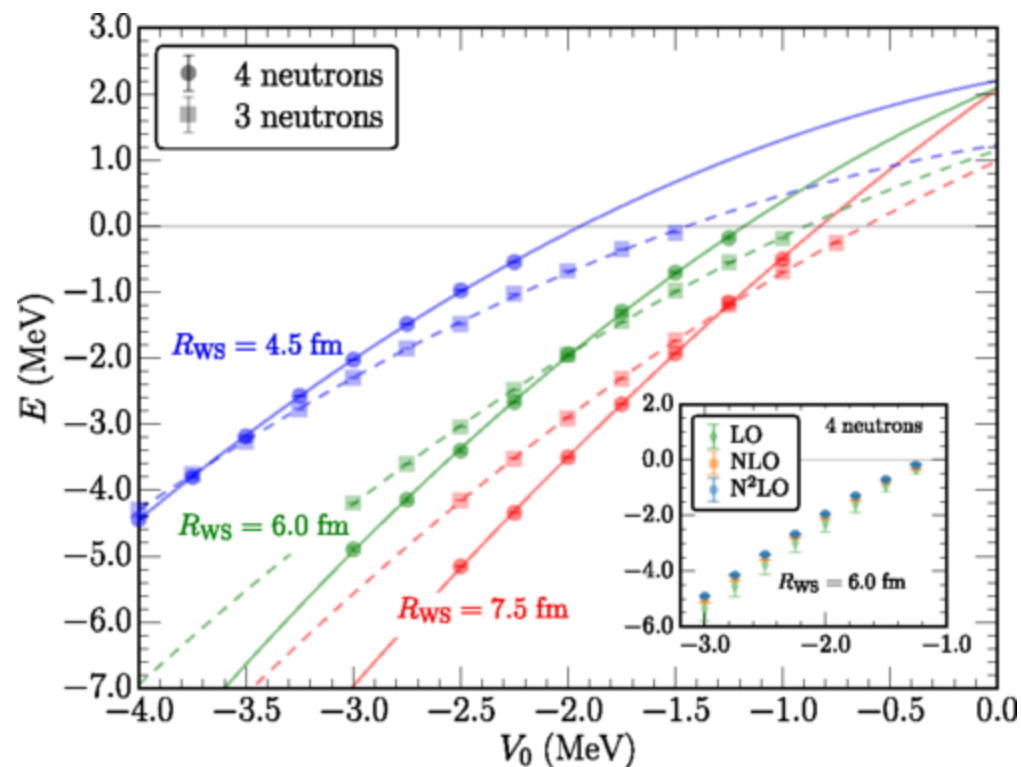


# 量子蒙特卡洛方法计算多中子体系

$$H = -\sum_i \frac{\hbar^2}{2m} \nabla_i^2 + \sum_i V_{\text{ws}}(r_i) + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk}$$



Steven C. Pieper PRL 90, 252501(2002)

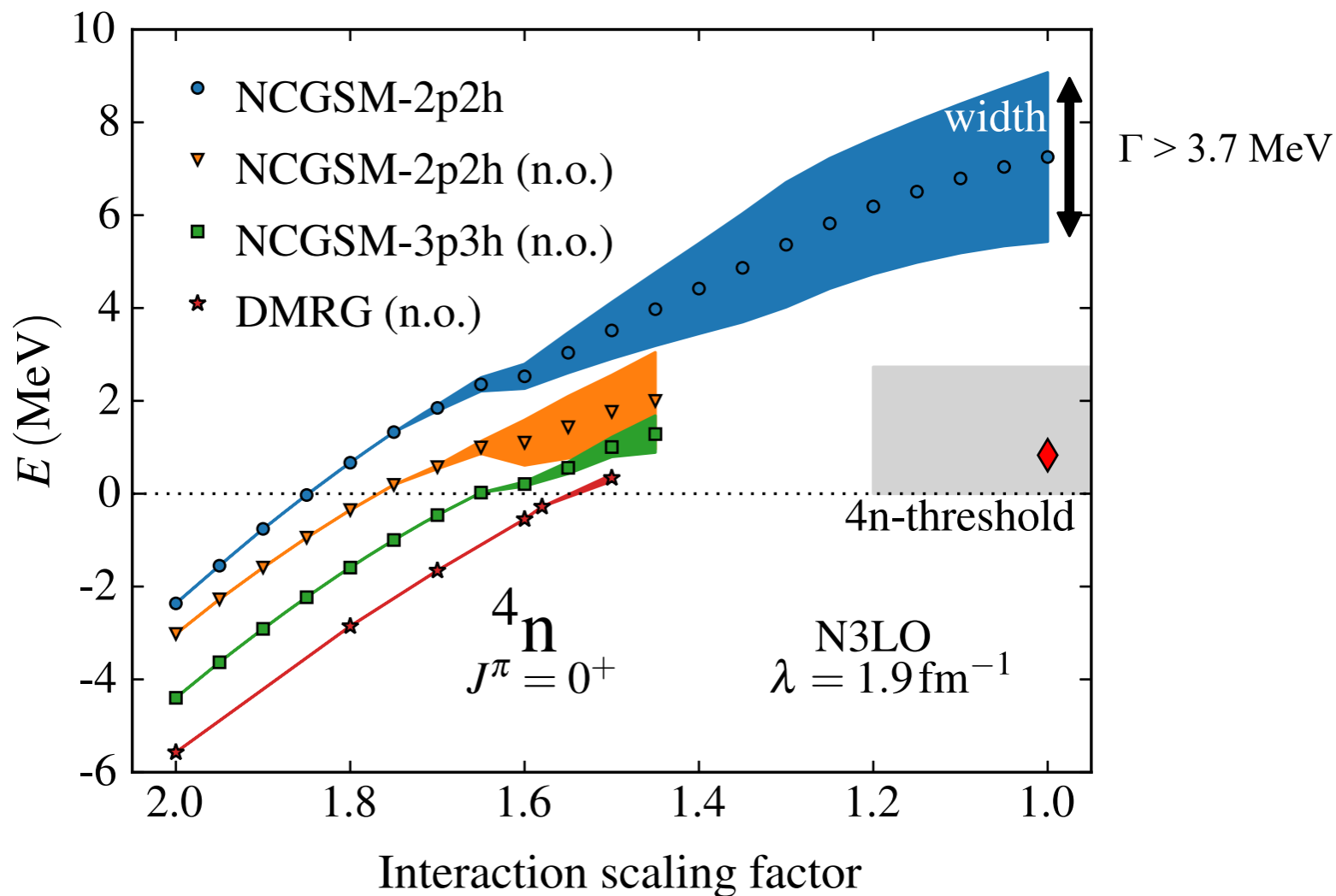


S. Gandolfi, et al., PRL 118, 232501(2017)

只能计算能量，无法计算宽度

# 无芯Gamow壳模型计算四中子

	$\lambda = 1.7 \text{ fm}^{-1}$	$\lambda = 1.9 \text{ fm}^{-1}$	$\lambda = 2.1 \text{ fm}^{-1}$
N3LO	7.27 (3.69)	7.28 (3.67)	7.28 (3.69)
N2LO <sub>opt</sub> *	7.32 (3.74)	7.33 (3.78)	7.34 (3.95)
N2LO <sub>sat</sub>	7.24 (3.48)	7.22 (3.58)	7.27 (3.55)
JISP16		7.00 (3.72)	



*K. Fosse, J. Rotureau, N. Michel, and M. Ploszajczak, PRL 119, 032501 (2017)*



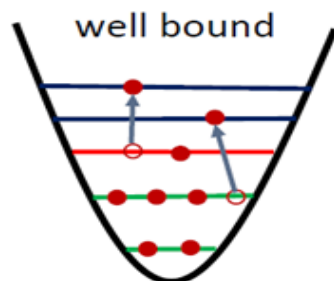
# 量子多体计算- HO /Berggren 完备基矢

$$H = -\sum_i \frac{\hbar^2}{2m} \nabla_i^2 + \sum_{ij} V_{ij} + \sum_{ijk} V_{ijk}$$

NCSM/SM



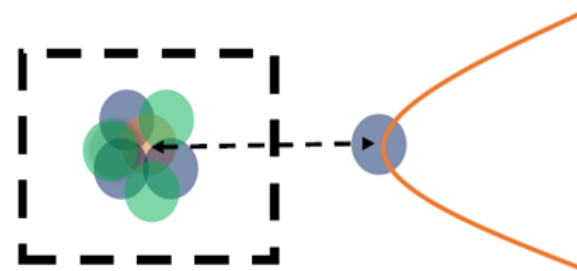
Closed quantum system



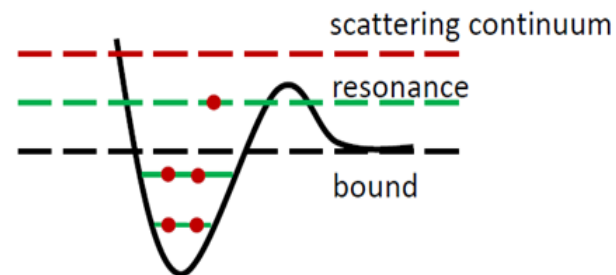
HO basis

- E. Caurier, et al. RMP 77,427 (2005)*
- T. Otsuka, et al, RMP. 92,015002 (2020)*
- B. R. Barrett, P. Navratil, and J. P. Vary PPNP 69,131(2013)*

NCGSM/GSM



Open quantum system



Berggren basis

- N. Michel, M. Płoszajczak, The Gamow Shell Model, Springer ;*
- N. Michel, et al., JPG 36,013101 (2009)*
- J. G. Li, et al., Physics 3, 977 (2021)*
- J. G. Li, et al., Phys. Rev. C 100, 054313 (2019)*
- J. G. Li, et al., Phys. Rev. C 104, 024319 (2021)*

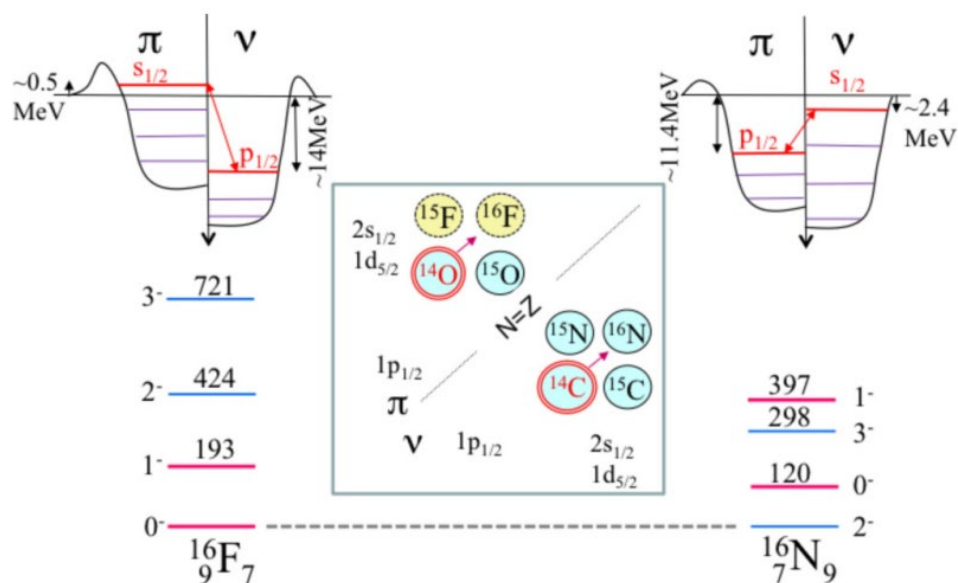
Berggren 基矢自洽的包含共振与连续谱耦合





# 滴线原子核的共振与连续谱耦合: 举例

## $^{16}\text{F}/^{16}\text{N}$ 中的基态反转

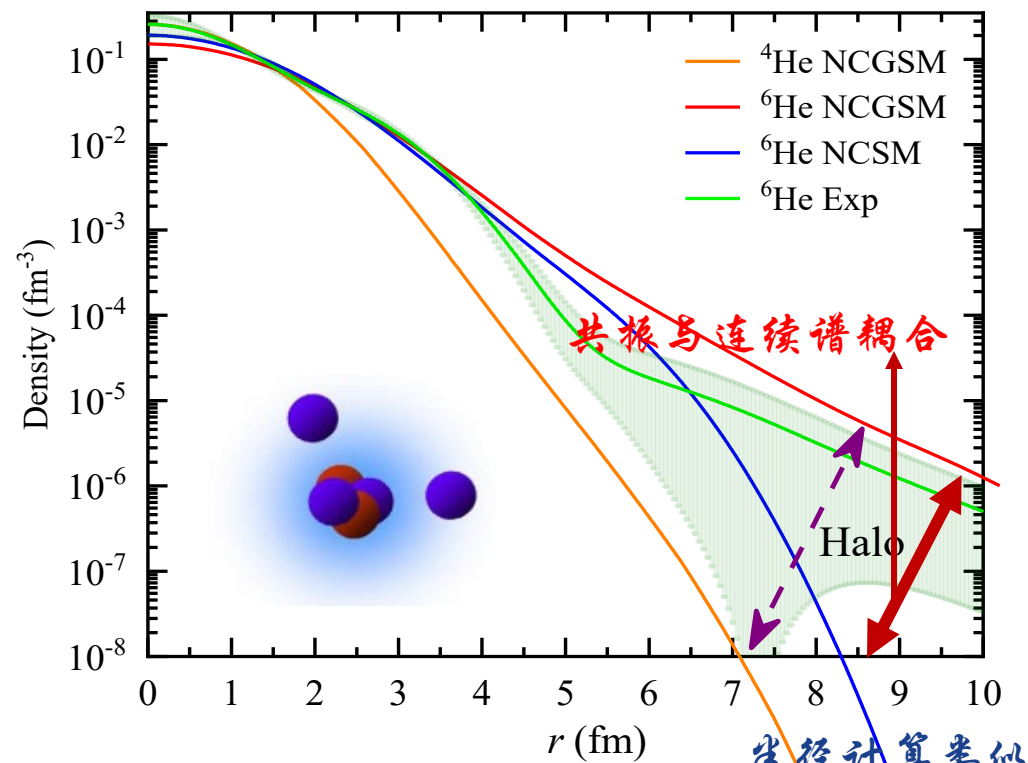


I. Stefan et al., PRC 90, 014307(2014)

N. Michel, J. G. Li, et al PRC 106, L011301(2022)

S. Zhang, F. R. Xu et al, submitted to PRC

## $^6\text{He}$ : 双中子晕结构



J. G. Li, et al., 文章准备中

共振与连续谱耦合非常关键，理论模型计算中应该考虑