Four- and five-body effects in nucleus-nucleus scattering

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Content of the talk

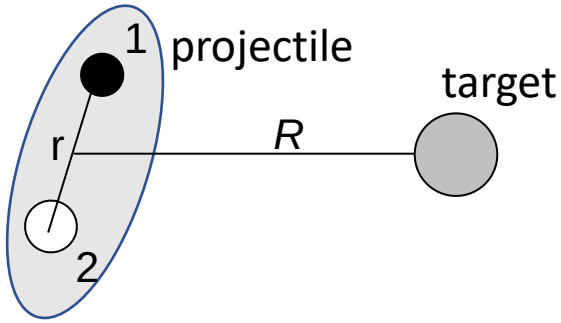
1. Overview of CDCC (Continuum Discretized Coupled Channels)
2. Four- and five-body CDCC
3. Solving the coupled-channel system: Lagrange functions and the R-matrix method
4. Test with 4α bound states
5. Application to «3+1» CDCC: ^{11}Li+p scattering
6. Application to « 2+2 » CDCC: ^{11}Be+d scattering
7. Application to « 3+2 » CDCC: ^{11}Li+d scattering
8. Conclusion

1. Overview of CDCC

- CDCC=Continuum Discretized Coupled Channel
  Introduced in the 70's to deal with deuteron scattering
  Low binding energy of the deuteron → breakup is important
    - G. Rawitscher, Phys. Rev. C 9, 2210 (1974)
    - N. Austern et al., Phys. Rep. 154 (1987) 126

- Two-body projectile, three-body problem

$$H = H_0(\boldsymbol{r}) - \frac{\hbar^2}{2\mu}\Delta_{\boldsymbol{R}} + V_{t1}(\boldsymbol{R},\boldsymbol{r}) + V_{t2}(\boldsymbol{R},\boldsymbol{r})$$



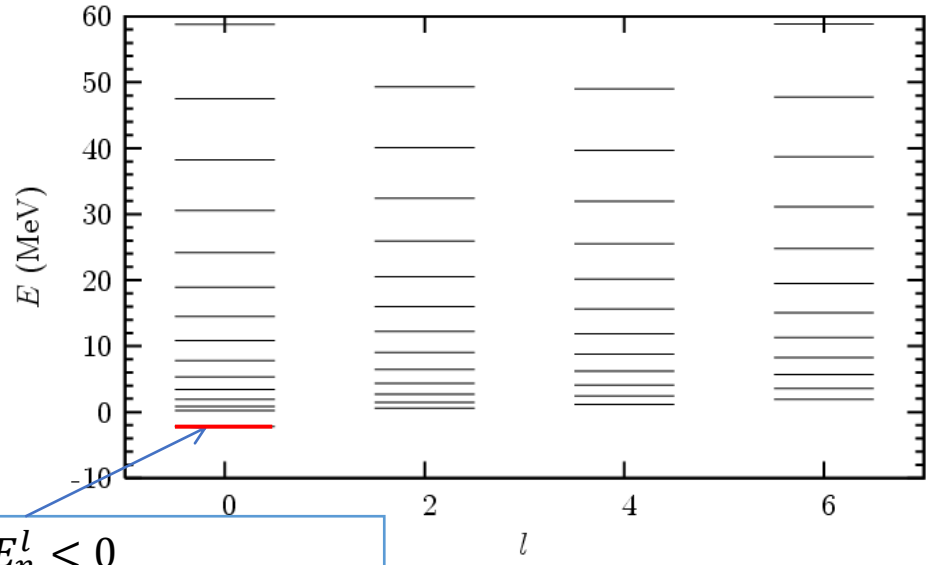
- $H_0(\boldsymbol{r})$ = Hamiltonian associated with the projectile

- V_{t1}, V_{t2}= optical potentials between the target and the fragments
  (high energies: above the resonance region)

1. Overview of CDCC

- Projectile breakup described by approximate (discrete) states: $H_0\Phi_n^{lm}(\boldsymbol{r}) = E_n^l\Phi_n^{lm}(\boldsymbol{r})$

Example: d=p+n



Pseudo state (PS): $E_n^l > 0$
- Simulate breakup effects
- No physical meaning
- Depend on the basis

Ground state: $E_n^l < 0$
- Physical
- Does not depend on the basis

- CDCC well adapted to exotic nuclei (low binding energy)
  Examples: ^{11}Be=^{10}Be+n (0.5 MeV), ^{8}B=^{7}Be+p (0.14 MeV) , ^{7}Li=α+^{3}H (1.5 MeV)

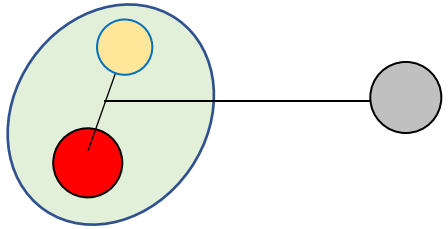
- Low BU energy is not necessary! But BU effects are expected to be more important

- Assuming $E_n^l = E_0$ provides the adiabatic approximation (R. Johnson) → simplifications

1. Overview of CDCC

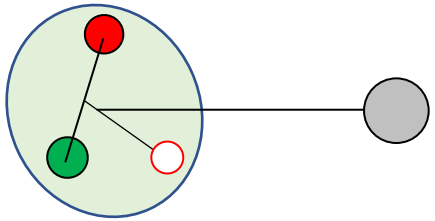
Different variants

- 2-body projectiles: d, ^{11}Be, ^{7}Li, ^{8}Li



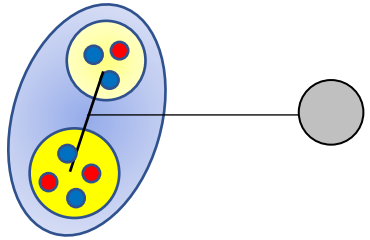
First applications of CDCC
Many references
- G. Rawitscher, PRC 9, 2210 (1974)
- N. Austern et al., Phys. Rep. 154 (1987) 126

- 3-body projectiles: ^{6}He, ^{11}Li, ^{9}Be



More recent
- T. Matsumoto et al., PRC70 (2004) 061601
- M. Rodriguez-Gallardo et al., PRC77 (2008) 064609

- A-body projectiles: ^{7}Li, ^{6}He, ^{8}Li



Based on nucleon-target potentials → no parameter
- Y. Sakuragi et al., PTP Supp. 89 (1986) 136
- P.D., M. Hussein, PRL 111 (2013) 082701

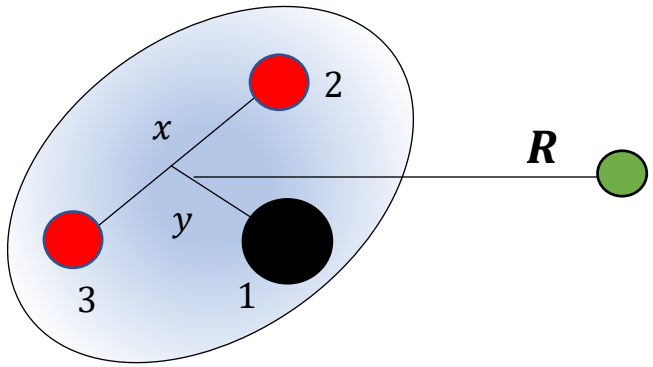
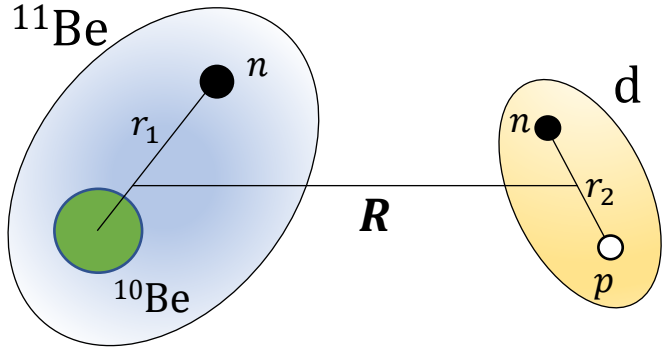
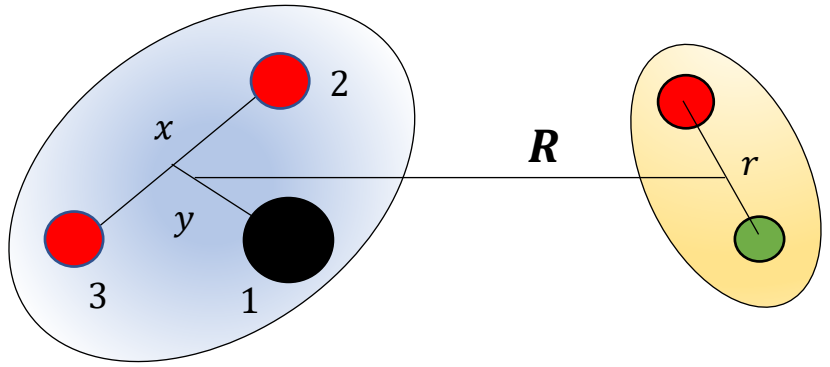
In all cases, the target is considered as inert (no structure)

2. Four- and five body CDCC

2. Four- and five-body CDCC

- For some reactions:
  - the projectile has a 3-body structure- example: ^{11}Li+p
  - both the projectile AND the target have a low breakup threshold - example: ^{11}Be+d

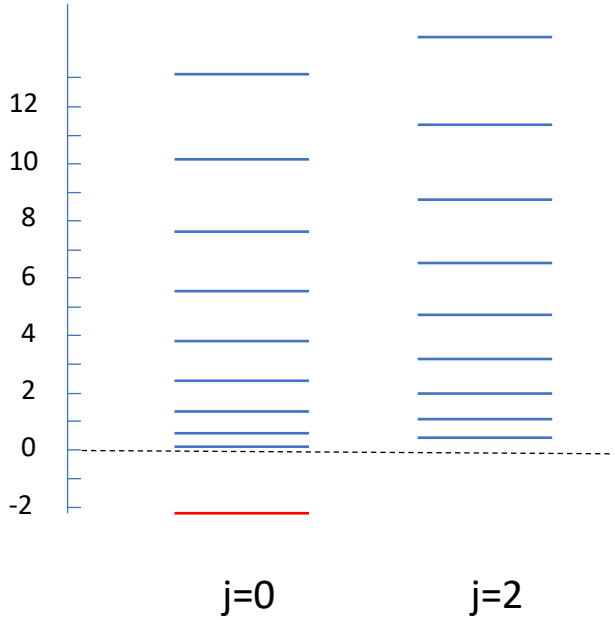
→ Need for extensions of CDCC

4 body « 3+1 »	4-body « 2+2 »	5 body « 3+2 »
		
^{11}Li+p, ^{6}He+^{12}C, ^{6}He+^{208}Pb, etc.	^{11}Be+d, ^{15}C+d, ^{7}Li+d, etc.	^{11}Li+d, ^{16}C+d, etc.

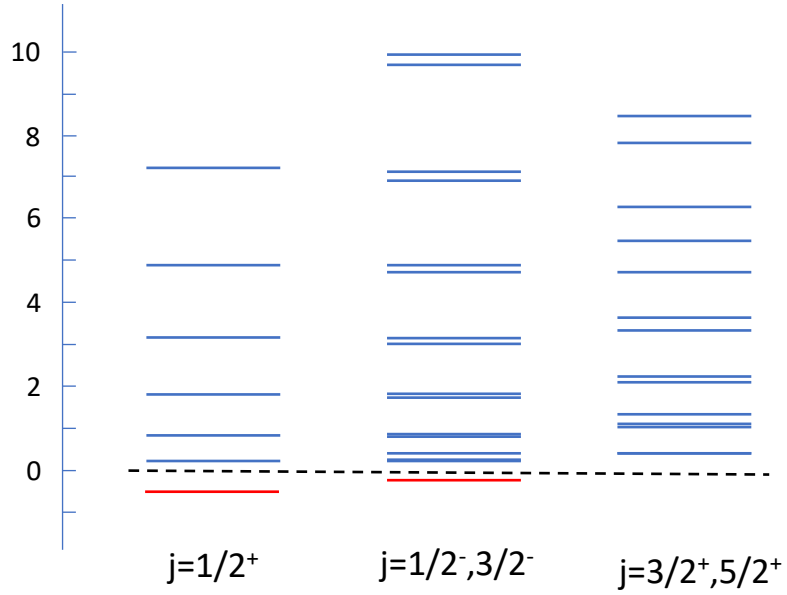
Same principle: determine the pseudostates
compute the coupling potentials
solve the coupled-channel system (scattering states → R-matrix)

2. Four- and five-body CDCC

- Pseudostates in both nuclei: example of ^{11}Be+d



Deuteron p+n : N_d channels

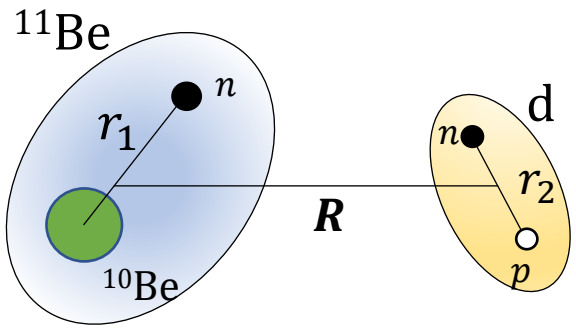


^{11}Be=^{10}Be+n: N_{Be} channels

➔ Total number of channels: N_d x N_{Be} (large!)

2. Four- and five-body CDCC

CDCC equations



Four-body hamiltonian: $H = H_1(\boldsymbol{r_1}) + H_2(\boldsymbol{r_2}) + T_R + \sum_{ij} U_{ij}(\boldsymbol{R}, \boldsymbol{r_1}, \boldsymbol{r_2})$

With $H_i(\boldsymbol{r_i})$=internal hamiltonian of nucleus i (may include core excitation)
T_R =relative kinetic energy
$U_{ij}(s)$ =optical potential between fragments i and j

Then: standard CDCC procedure

Important remark

The potentials $U_{ij}(s)$ should not depend on angular momentum (general assumption of CDCC)

But: presence of forbidden states (spurious bound states)

in spectroscopic calculations: can be removed with a projector → L-dependent

present in CDCC → spurious states at low energies → cannot be applied at low energies

2. Four- and five-body CDCC

Standard CDCC procedure:

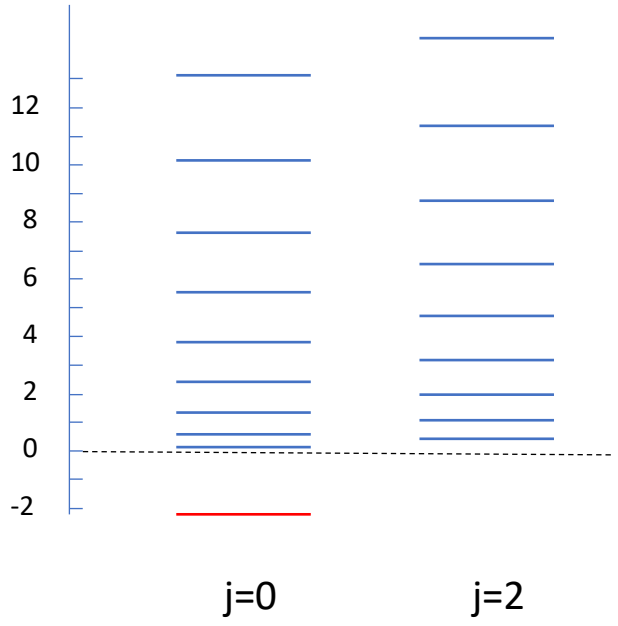
1. Step 1: solve $H_1\Phi_{1k}^{jm} = E_{1k}^{j}\Phi_{1k}^{jm}$ for nucleus 1

   $H_2\Phi_{2k}^{jm} = E_{2k}^{j}\Phi_{2k}^{jm}$ for nucleus 2

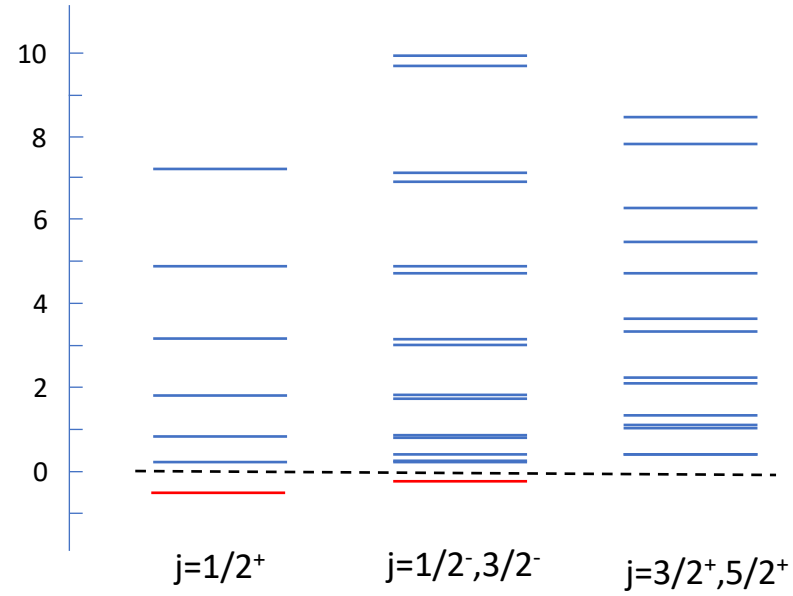
With Φ_{1k}^{jm} expanded on a basis (Lagrange functions: matrix elements are simple)

→ negative energies = physical states

positive energies = pseudostates=(discrete) approximations of the continuum in 1 and 2



Deuteron



^{11}Be

2. Four- and five-body CDCC

Step 2

Define channel functions: $\varphi_c(\boldsymbol{r_1}, \boldsymbol{r_2}, \Omega_R) = \left[\left[\underbrace{\Phi^{j_1}_{1k_1}(\boldsymbol{r_1})}_{\text{d}} \otimes \underbrace{\Phi^{j_2}_{1k_2}(\boldsymbol{r_2})}_{^{11}\text{Be}}\right]^I \otimes Y_L(\Omega_R)\right]^{JM}$

with
I = channel spin
L = angular momentum between d and ^{11}Be
index $c = (j_1, k_1, j_2, k_2, I, L)$

and expand the total wave function as $\Psi^{JM\pi} = \sum_c u_c^{J\pi}(R)\varphi_c(\boldsymbol{r_1}, \boldsymbol{r_2}, \Omega_R)$

$u_c^{J\pi}(R)$ to be determined

Step 3

Compute matrix elements of the potential $\sum_{ij} U_{ij}(\boldsymbol{R}, \boldsymbol{r_1}, \boldsymbol{r_2})$

$$V_{cc'}^{J\pi}(R) = \langle \varphi_c \left| \sum_{ij} U_{ij}(\boldsymbol{R}, \boldsymbol{r_1}, \boldsymbol{r_2}) \right| \varphi_{c'} \rangle$$

- « 2+2 »: integrals over 8 coordinates (6 angles + 2 radii): 5 analytical + 3 numerical integrals
- « 3+2 »: integrals over 11 coordinates (8 angles + 3 radii): 5 analytical + 6 numerical integrals (use of the Raynal-Revai coefficients)

2. Four- and five-body CDCC

4. Step 4: Solve the coupled-channel system

$$\left[-\frac{\hbar^2}{2\mu}\left(\frac{d^2}{dR^2}-\frac{L(L+1)}{R^2}\right)+E_c-E\right]u_c^{J\pi}(R)+\sum_{c'}V_{cc'}^{J\pi}(R)u_{c'}^{J\pi}(R)=0$$

- Standard coupled-channel system (general form common to most scattering theories)
- At large distances (only Coulomb) : $u_c^{J\pi}(R)\rightarrow I_c(R)\delta_{c\omega}-O_c(R)U_{c\omega}^{J\pi}$ (ω =entrance channel)
  $U_{c\omega}^{J\pi}$ = scattering matrix: provides the cross sections (elastic, inelastic, breakup, etc.)
- Solved with the R-matrix method (space divided in an internal and an external regions)
- The system must be solved for each $J\pi$
- Problems:
  - Many channels c (**up to 9000 for ^{11}Li+d**)
  - Many $J\pi$ values (depends on energy)
  - Long range of the potentials $V_{cc'}^{J\pi}(R)$ (due to Coulomb)

  ➔ Long calculations + many tests

5. Step 5
   Determing the cross sections from the scattering matrices

3. Solving the coupled-channel system: Lagrange functions and the R-matrix method

3. Solving the coupled-channel system: the R-matrix theory

Scattering matrix determined from the R-matrix theory

R-matrix theory: based on 2 regions (channel radius a)

Lane and Thomas, Rev. Mod. Phys. 30 (1958) 257: (ancient) bible of the R-matrix

P.D. and D. Baye, Rep. Prog. Phys. 73 (2010) 036301: modern review of the R-matrix (Bloch operator)

P.D., Computer Physics Communications 200 (2016) 199: general subroutine

Jin Lei, P.D., Phys. Rev. C 102 (2020) 014608 : inhomogeneous equations

Internal region: $R \leq a$ | $R = a$ | External region: $R \geq a$ | R

Full Hamiltonian

$u_c^{J\pi}(R)$ expanded over a basis (N functions $\phi_i(R)$)

$$\boldsymbol{u_{c,int}^{J\pi}(R)} = \sum_{i=1}^{N} c_i \phi_i(R)$$

$\phi_i(R)$=Lagrange functions

typically N ~50

Only Coulomb (monopole)

$$V_{cc'}^{J\pi}(R) = \frac{Z_p Z_t e^2}{R}\delta_{cc'}$$

$u_c^{J\pi}(R)$ has its asymptotic form

$$\boldsymbol{u_{c,ext}^{J\pi}(R)} = I_c(kR)\delta_{c\omega} - O_c(kR)U_{c\omega}^{J\pi}$$

matching at *R=a* provides: scattering matrices $\boldsymbol{U^{J\pi}}$ → cross sections

3. Solving the coupled-channel system: the Lagrange method

Choice of the basis: **the Lagrange-mesh method** (*D. Baye, Phys. Rep. 565 (2015) 1-107)*

- Gauss approximation: $\int_0^a g(x)dx \approx \sum_{k=1}^N \lambda_k g(x_k)$
  - N= order of the Gauss approximation
  - x_k=roots of an orthogonal polynomial $P_N(x)$, λ_k=weights
  - If interval [0,a]: Legendre polynomials
    [0,∞]: Laguerre polynomials

- Lagrange functions for [0,1]: $f_i(x) \sim \frac{P_N(2x-1)}{(x-x_i)}$
  - x_i are roots of $P_N(2x-1)=0$
  - with the Lagrange property: $f_i(x_j) = \lambda_i^{-1/2}$

- Matrix elements with Lagrange functions: Gauss approximation is used

$< f_i|f_j > = \int f_i(x) f_j(x) dx \approx \delta_{ij}$

$< f_i|T|f_j >$ analytical

$< f_i|V|f_j > = \int f_i(x) V(x) f_j(x) dx \approx V(x_i)\delta_{ij} \Rightarrow$ **no integral needed**

Also applicable to non-local potentials

3. Solving the coupled-channel system: the R-matrix with propagation method

Computer time: 2 main parts

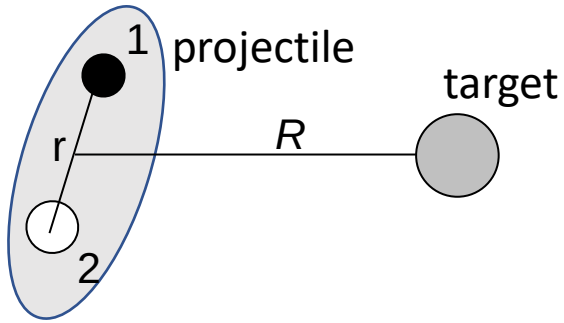
- Matrix elements: very fast with Lagrange functions
- Inversion of a complex matrix C → R-matrix (long times for large matrices)
  size=(number of channels)x(number of radial functions N) →~3000x50 typically

For reactions involving halo nuclei:

- Long range of the potentials (Coulomb)

$$\frac{Z_1 Z_t e^2}{\boldsymbol{R} + \frac{A_2}{A_p}\boldsymbol{r}} + \frac{Z_2 Z_t e^2}{\boldsymbol{R} - \frac{A_1}{A_p}\boldsymbol{r}} = \sum_\lambda V_\lambda(r,R) P_\lambda(\cos\theta_{Rr})$$

$$V_{cc'}(R) \approx \frac{Z_p Z_t e^2}{R} + \frac{Z_t Q_p}{R^3} + \dots$$



Can be large (large quadrupole moments of PS)

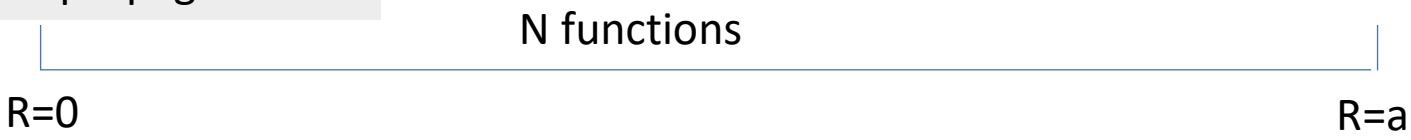
- Radius *a* must be large
- Many basis functions (*N* large)
- Even stronger for dipole terms ($\sim 1/R^2$)

→

- Distorted Coulomb functions (FRESCO)
- Propagation techniques in the R-matrix (well known in atomic physics)
  Ref.: Baluja et al. Comp. Phys. Comm. 27 (1982) 299
  Well adapted to Lagrange-mesh calculations

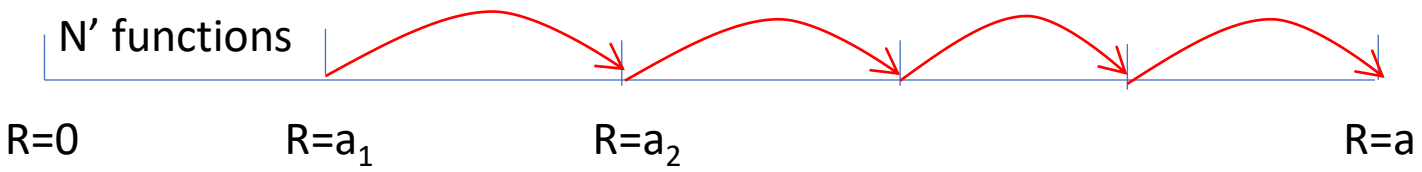
3. Solving the coupled-channel system: the R-matrix with propagation method

Without propagation



- Matrix elements integrated over [0,a]
- Inversion of a matrix of dimension N x N_c

With propagation



- The interval [0,a] is split in N_S subintervals

- In each subinterval N'~N/N_S:
  - Interval 1: determine $R(a_1)$
  - Interval 2 : $R(a_2)$ from $R(a_1)$
  - Interval N_S: $R(a)$ from $R(a_{NS-1})$

  } (inversion of a matrix with size N' x N_c)

➔ N_S smaller calculations:
Lagrange functions well adapted to matrix elements over $[a_i,a_{i+1}]$

4. Test with 4α bound states

References

- P. Descouvemont, *Four-Body Extension of the Continuum-Discretized Coupled-Channels Method*, Phys. Rev. C **97**, 064607 (2018).

- D. C. Cuong, P. Descouvemont, D. T. Khoa, and N. H. Phuc, Coupled-Reaction-Channel Study of the C12(α ,Be8) Reaction and the Be8 + Be8 Optical Potential, Phys. Rev. C 102, 024622 (2020).

4. Test with 4α bound states

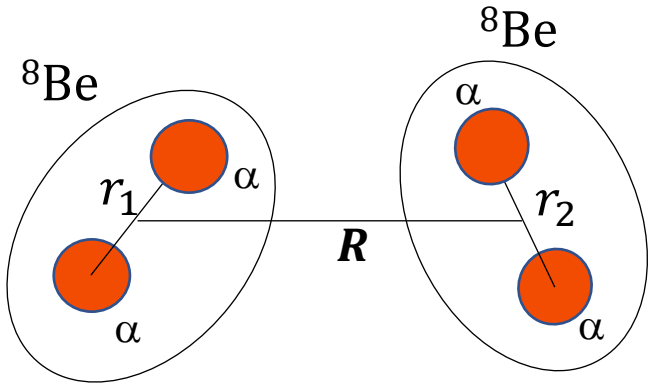
Two main parts in the calculation:

1. Calculation of the 4-body matrix elements
2. Solution of the coupled-channel problem (R-matrix method)

Test of the matrix elements: 4α bound states (**benchmarks exist in the literature**)

- No R-matrix calculation
- Diagonalisation of the Hamiltonian matrix
- Potential: Ali-Bodmer $V_{\alpha-\alpha}(R) = 500\exp(-(R/0.7)^2) - 130\exp(-(R/2.105)^2)$

- Two calculations
  - Without Coulomb potential
  - With Coulomb potential

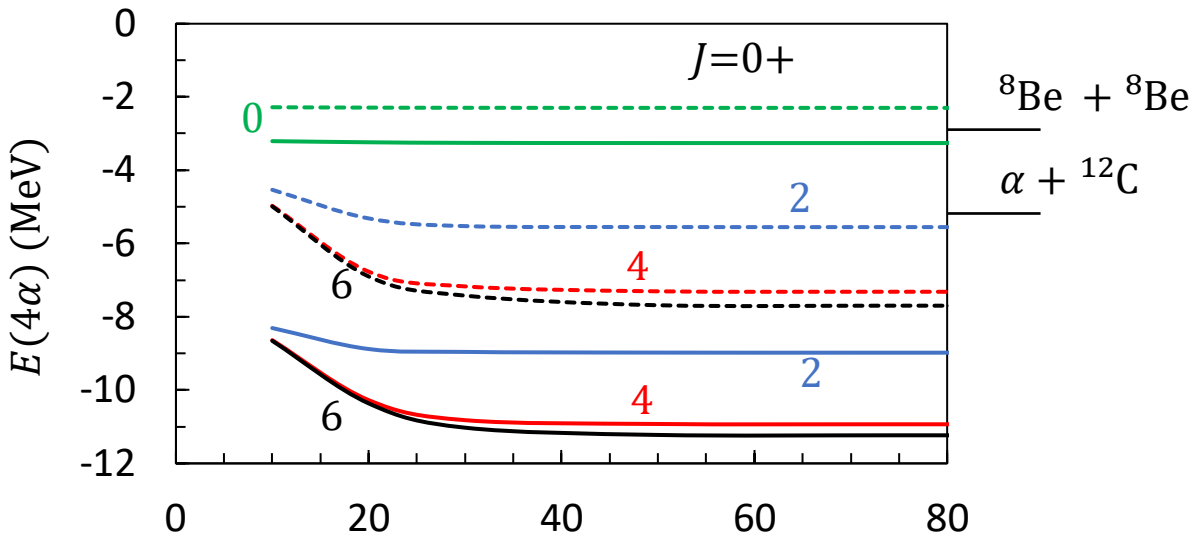


2 parameters

- L_{max} in ^{8}Be
- E_{max} in ^{8}Be

4. Test with 4α bound states

1) $V_C = 0$

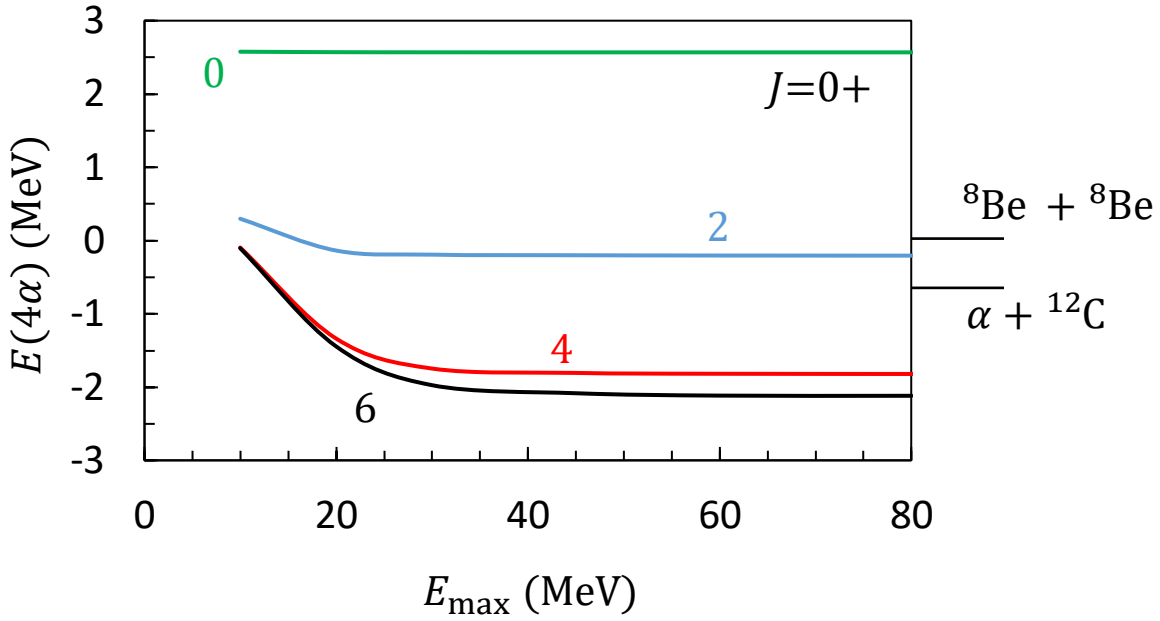


Slow convergence with respect to Lmax and Emax

- Present: -11.23 MeV
- N. K. Timofeyuk, PRC 78 (2008) 054314 : -11.23 MeV
- Y. Suzuki and M. Takahashi, PRC 65 (2002) 064318: -11.17 (revised to -11.23 MeV)

4. Test with 4α bound states

2) $V_C \neq 0$

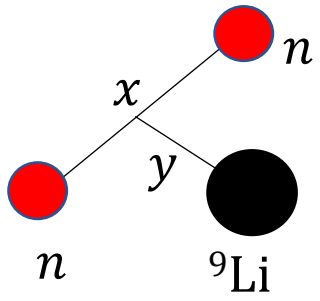


- E(0^+)=-2.12 MeV
- Very different from the ^{16}O experimental energy: -14.44 MeV
- Ali-Bodmer potential excellent for 2α, but poor for 3α and 4α → only for tests

5. Application to « 3+1 »: the ^{11}Li+p scattering

Ref: P. Descouvemont, Phys. Rev. C **101**, 64611 (2020).

5. Application to « 3+1 »: the ^{11}Li+p scattering



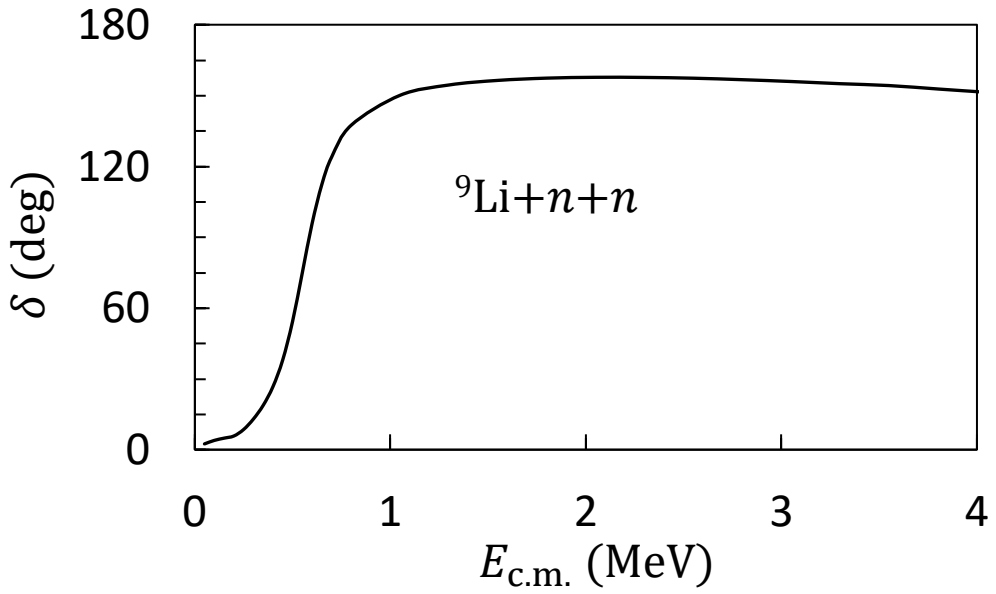
Structure of ^{11}Li: 3-body model ^{9}Li+n+n

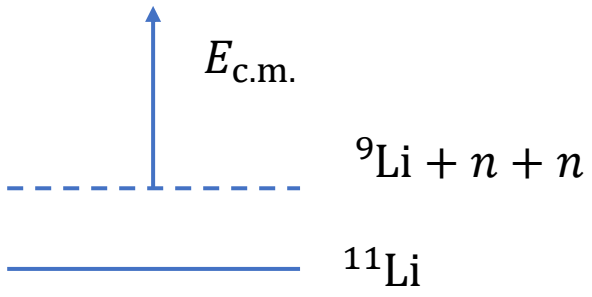
- Hyperspherical coordinates
- Details in E. C. Pinilla et al., Phys. Rev. C 85, 054610 (2012).
- V_{nn}=Minnesota potential
- V_{9Li+n}=Woods-Saxon fitted on the scattering length
- Spin of the ^{9}Li core is neglected
- Forbidden states for $s_{1/2}$ and $p_{3/2}$ → removed by a supersymmetric transformation

$J=0^{+}$

- Bound state at E_B=-0.378 MeV
- $\sqrt{<r^2>}$=3.12 fm, exp=3.16±0.11 fm

$J=1^{-}$: ^{9}Li+n+n phase shifts (3-body phase shifts)

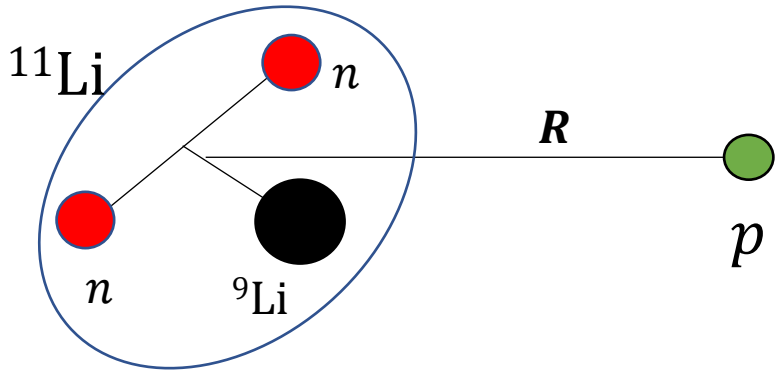




→ Dipole resonance near E_{cm}=0.6 MeV, E_x=1.0 MeV

5. Application to « 3+1 »: the ^{11}Li+p scattering

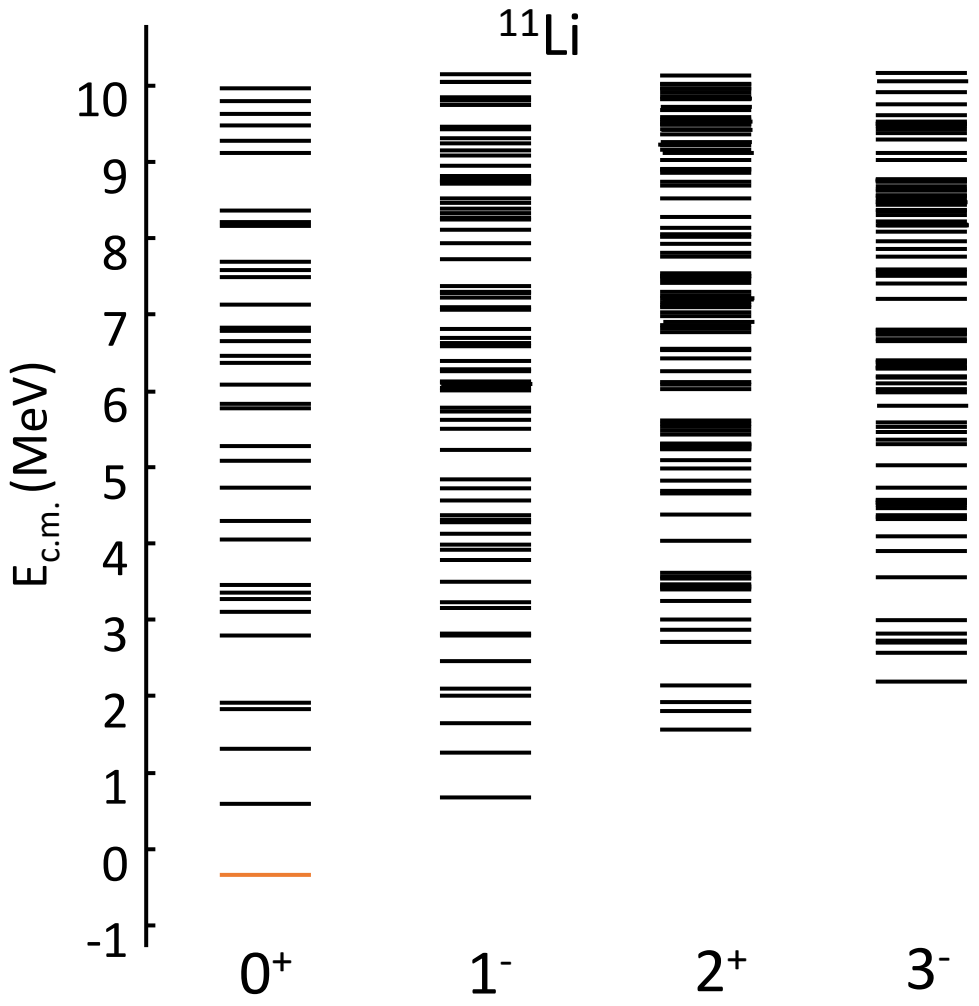
a. Conditions of the calculations



Interactions
- n+p: Minnesota
- ^{9}Li+p: Koning-Delaroche, Chapel Hill

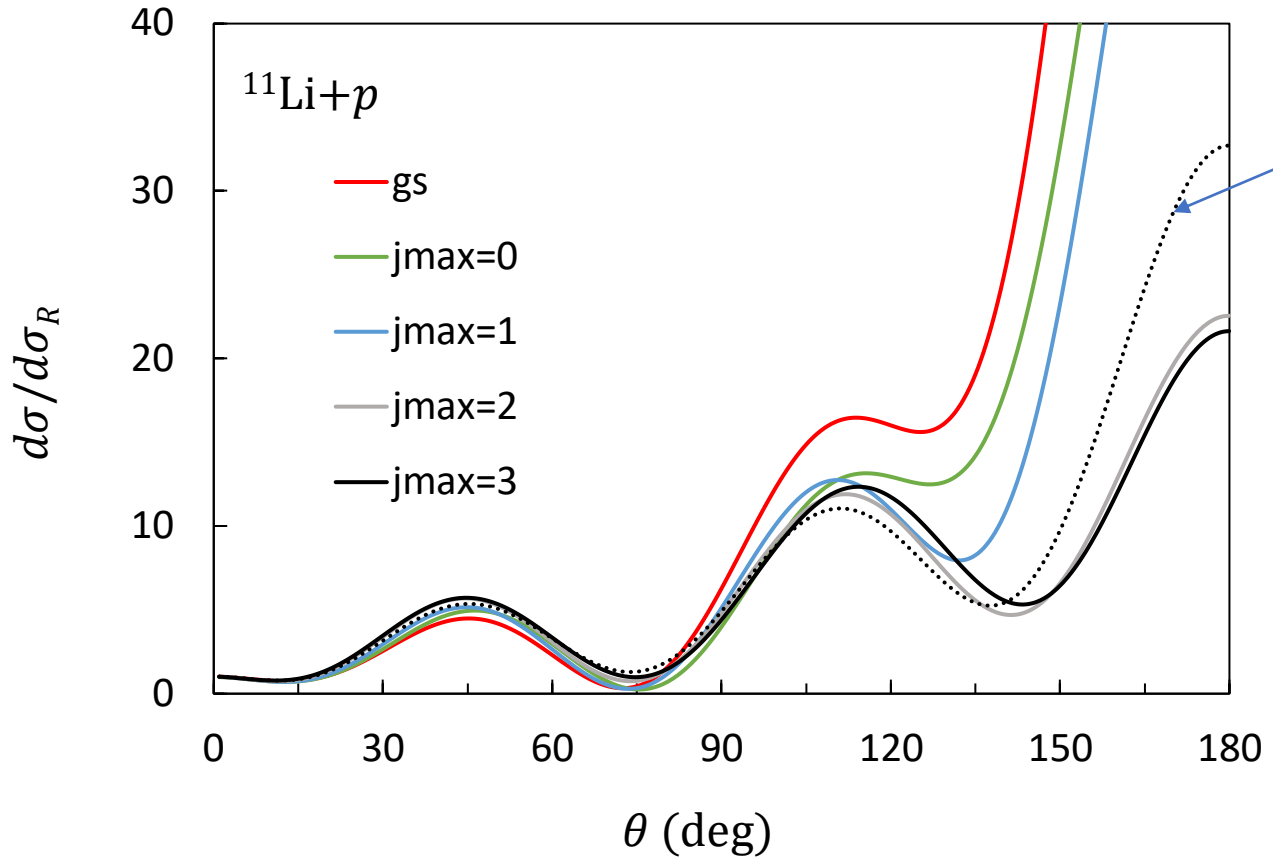
Channel radius
a~25 fm (stability tests)

^{11}Li pseudostates
E_{max}=10 MeV, j_{max}=3



5. Application to « 3+1 »: the ^{11}Li+p scattering

b. Convergence of the elastic cross section, E_{lab}=66 MeV, E_{cm}=5.5 MeV

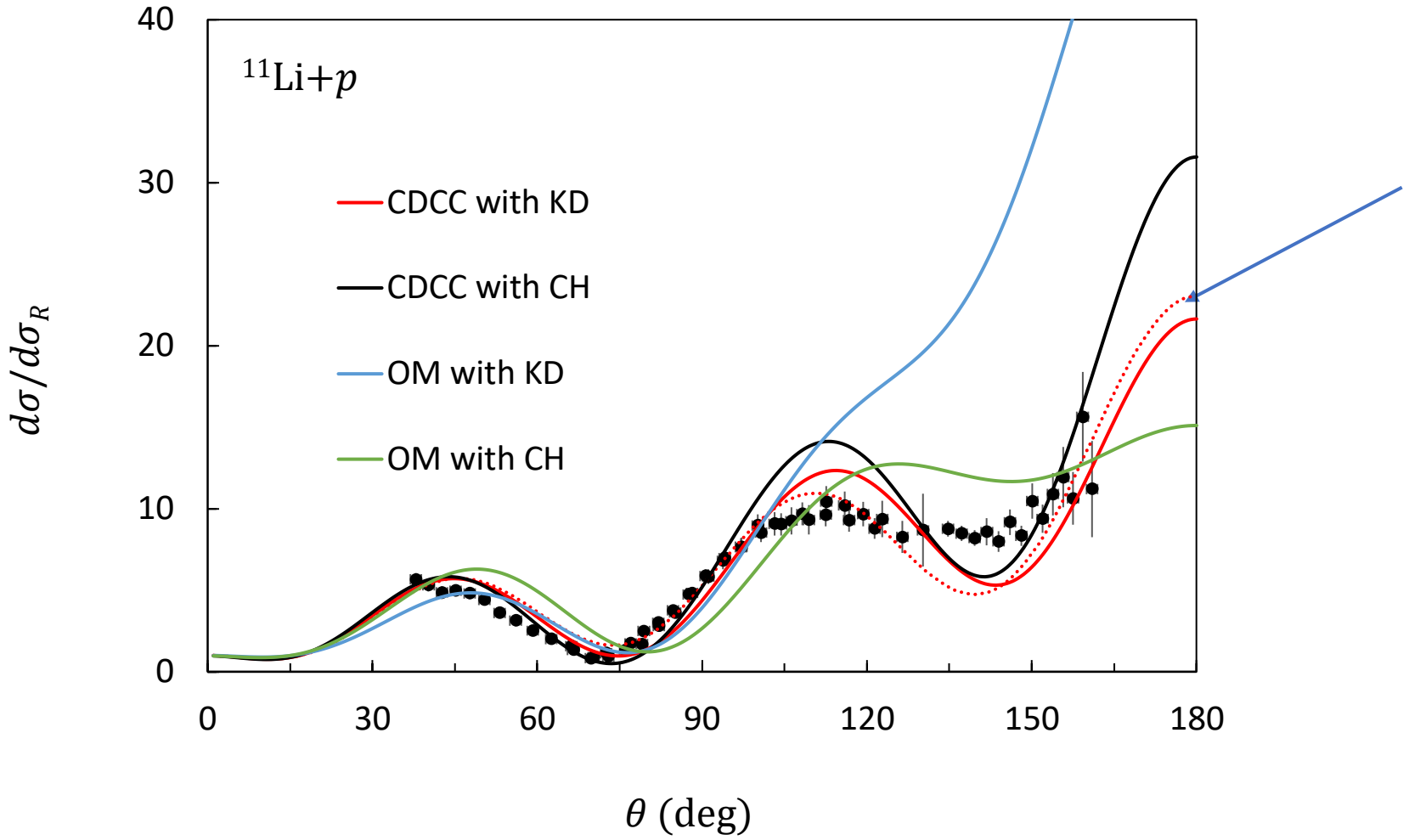


PS up to Emax=5.5 MeV
→ Closed channels are neglected

5. Application to « 3+1 »: the ^{11}Li+p scattering

c. Comparison with experiment

Data from J. Tanaka et al., Phys. Lett. B **774**, 268 (2017).
OM: optical model with global parametrizations (KD03, CH89)



Smaller basis for ^{11}Li
Will be used for ^{11}Li+d

6. Application to « 2+2 »: the ^{11}Be+d scattering

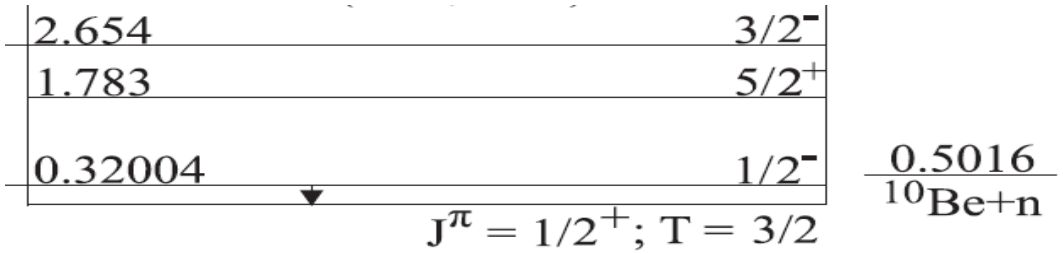
References

- P. Descouvemont, *Four-Body Continuum Effects in ^{11}Be+d Scattering*, Phys. Lett. B **772**, 1 (2017).
- P. Descouvemont, *Four-Body Extension of the Continuum-Discretized Coupled-Channels Method*, Phys. Rev. C **97**, 064607 (2018).

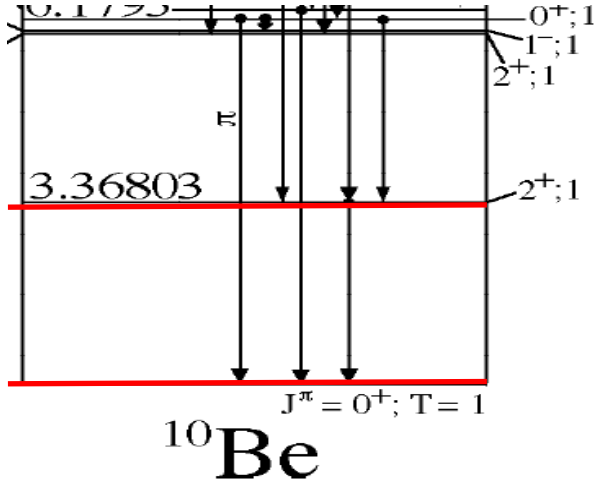
4. Results on ^{11}Be+d

Conditions of the calculation:

- d=p+n: Minnesota potential D. R. Thompson et al., Nucl. Phys. A 286 (1977) 53.
- ^{10}Be+n, ^{10}Be+p optical potentials: Koning-Delaroche 2003, Chapell Hill 89
- ^{11}Be=^{10}Be+n: Two calculations



^{11}Be



^{10}Be

- **^{10}Be(0^+)+n: neglects core excitation**: Potential from Capel et al. PRC C 70 (2004) 064605
  reproduces $1/2^+$, $1/2^-$, $5/2^+$ states
  $B(E1,1/2^- \rightarrow 1/2^+)=0.23\ e^2.fm^2$ (exp: $0.10\ e^2.fm^2$)
- **^{10}Be(0^+,2^+)+n: includes core excitation**
  Potential from N. Summers et al., PLB650 (2007) 124: reproduces $1/2^+$, $1/2^-$, $5/2^+$ states
  $B(E1,1/2^- \rightarrow 1/2^+)= 0.14\ e2.fm^2$ (exp: $0.10\ e2.fm^2$)

6. Application to « 2+2 »: the ^{11}Be+d scattering

Number of channels

$$\Psi^{JM\pi} = \sum_c u_c^{J\pi}(R)\varphi_c(\boldsymbol{r_1}, \boldsymbol{r_2}, \Omega_R)$$

With : $\varphi_c(\boldsymbol{r_1}, \boldsymbol{r_2}, \Omega_R) = \left[\left[\Phi^{j_1}_{1k_1}(\boldsymbol{r_1}) \otimes \Phi^{j_2}_{1k_2}(\boldsymbol{r_2})\right]^I \otimes Y_L(\Omega_R)\right]^{JM}$

with index $c = (j_1, k_1, j_2, k_2, I, L)$

I = channel spin

L =angular momentum between d and ^{11}Be

j_1, k_1: quantum numbers of ^{11}Be

j_2, k_2: quantum numbers of d

Present conditions:

^{11}Be: $j_1 = 1/2^{\pm}, 3/2^{\pm}, 5/2^{+}$: ~ 40 states

d: $j_2 = 0^{+}, 2^{+}$: ~ 15 states

→ ~600 physical channels

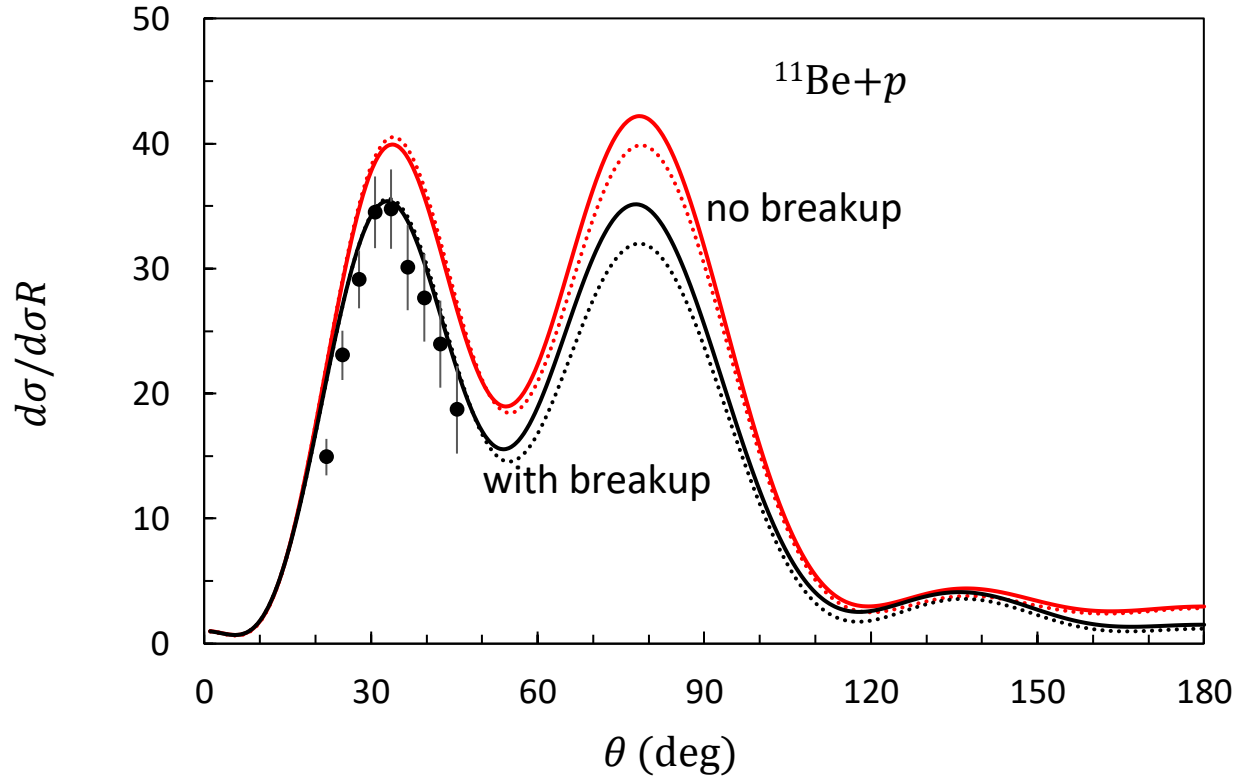
→ ~including (I, L)→ ~3000 values for index c (depends on $J\pi$)

→ Very large systems!

6. Application to « 2+2 »: the ^{11}Be+d scattering

Intermediate calculation: ^{11}Be+p

- Data by Chen et al., PRC93 (2016) 034623 at E(^{11}Be)=26.9A MeV
- 3-body CDCC calculation: same potentials, same ^{11}Be wave functions



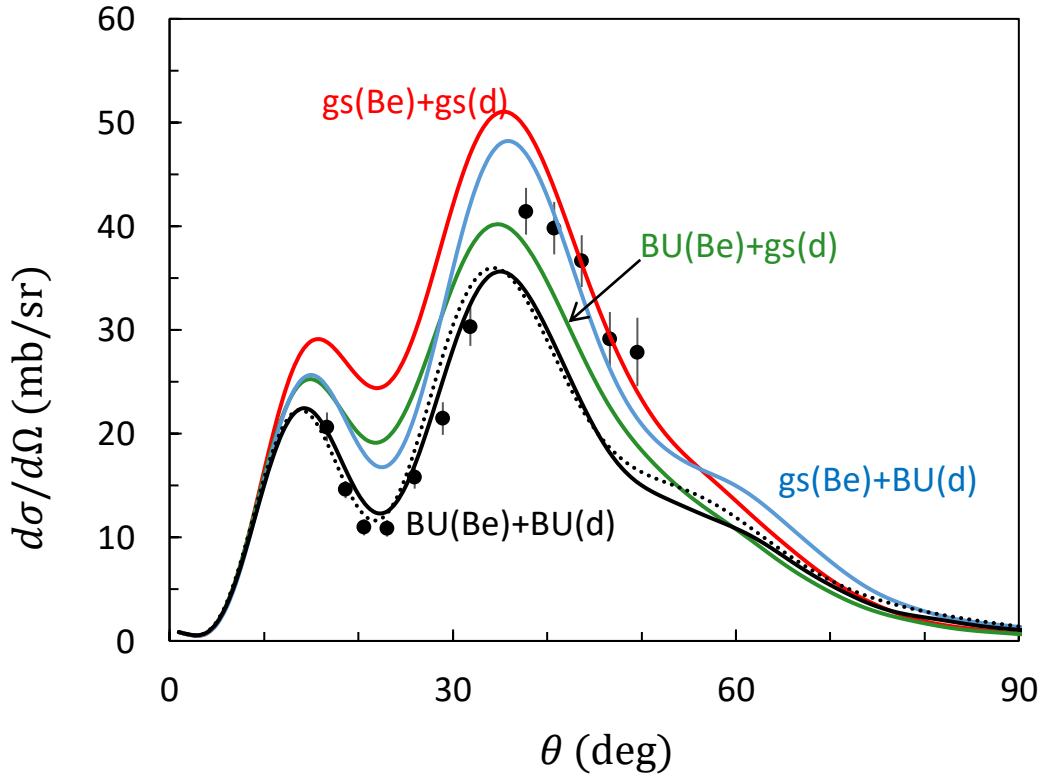
Solid: ^{10}Be(0^+,2^+)+n
Dotted:^{10}Be(0^+)+n

→ Good agreement with experiment
→ Breakup effects non negligible
→ Role of core excitation is minor

6. Application to « 2+2 »: the ^{11}Be+d scattering

^{11}Be+d Elastic scattering: $E_{lab}(^{11}Be)$=26.9A MeV (E_{cm}=45.5 MeV)
data from J. Chen et al., PRC 94 (2016) 064620

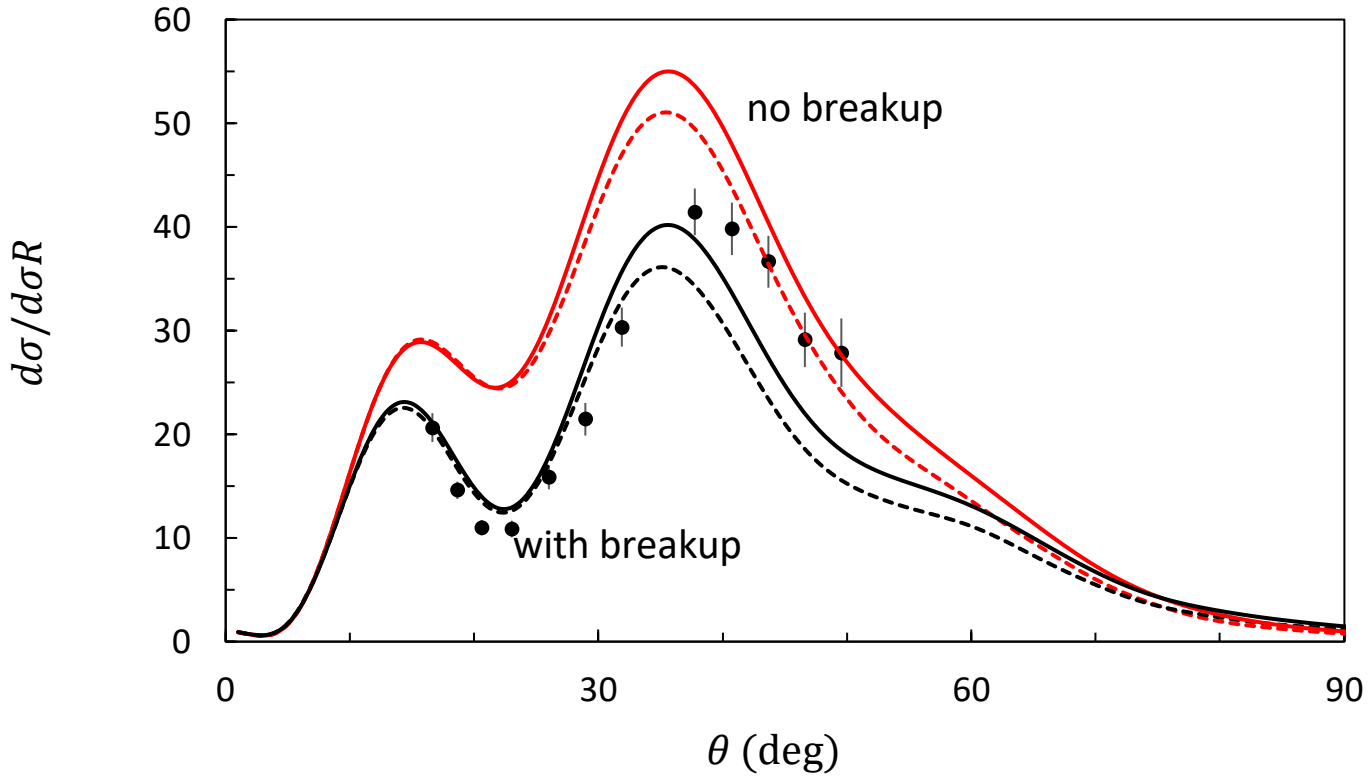
Without core excitation



- Excellent agreement for $\theta \leq 30°$ when ^{11}Be and d BU are introduced
- Weak sensitivity to the ^{10}Be+nucleon optical potential
- Underestimation for $\theta > 30°$
  - Need for ^{12}Be+p channel?
  - ^{10}Be core excitation?
  - Experimental normalization?

6. Application to « 2+2 »: the ^{11}Be+d scattering

With ^{10}Be core excitation



Solid: ^{10}Be(0^+,2^+)+n
Dashed:^{10}Be(0^+)+n

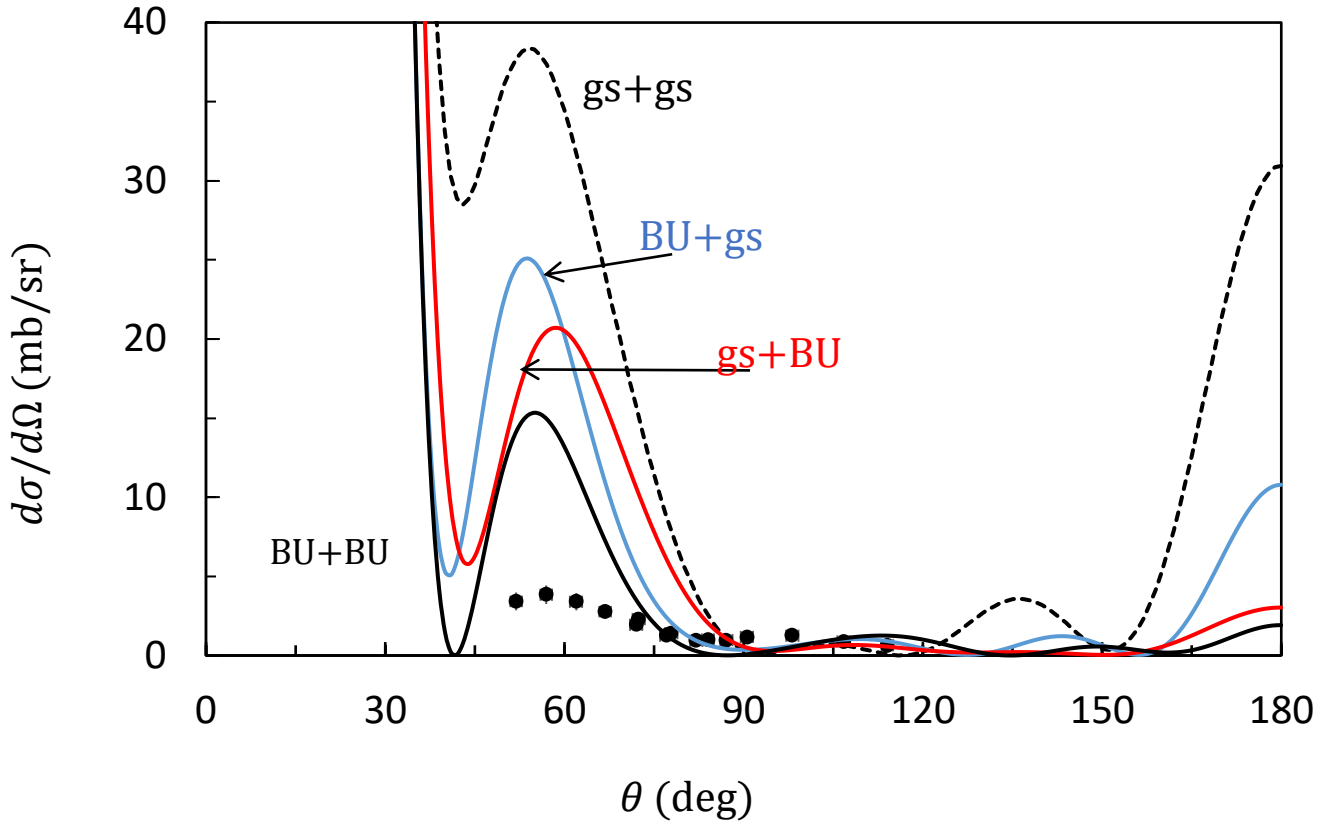
→ Weak role of core excitation
→ Breakup important

7. Application to « 3+2 »: the ^{11}Li+d scattering

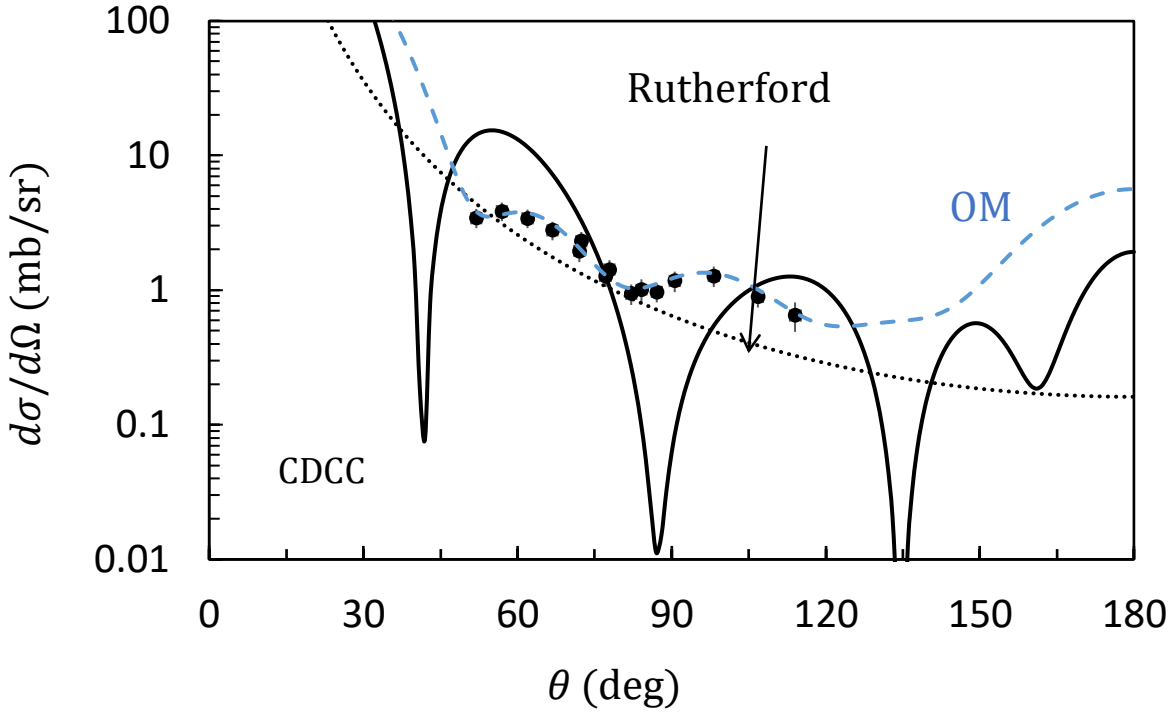
Ref: P. Descouvemont, Phys. Rev. C **101**, 64611 (2020).

7. Application to « 3+2 »: the ^{11}Li+d scattering

- Main goal: simultaneous study of ^{11}Li+p and ^{11}Li+d (same conditions)
- Data from R. Kanungo et al., Phys. Rev. Lett. **114**, 1 (2015), Elab=55.3 MeV, Ecm=8.5 MeV
- Much more difficult: many channels, coupling potentials require long computer times, etc
  → no full convergence

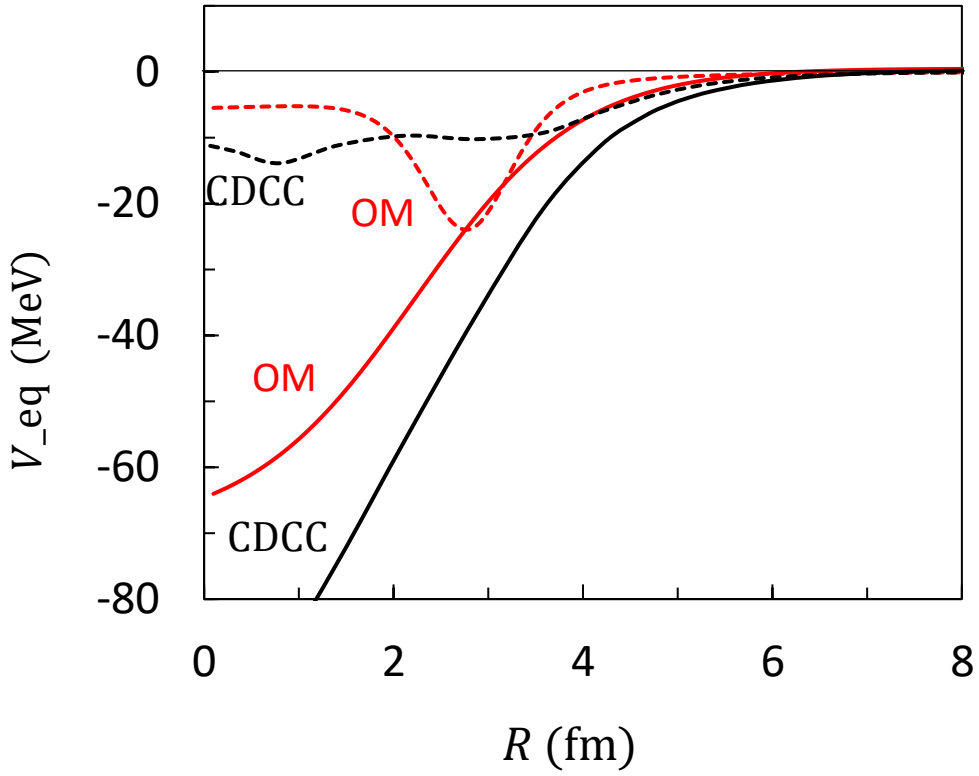


7. Application to « 3+2 »: the ^{11}Li+d scattering



- OM: optical potential fitted by Kanungo et al.
- Data are close to Rutherford scattering

Equivalent potential



A short range is necessary
(surprising…)

8. Conclusions

8. Conclusions

- No fitting parameter: the model is based on optical potentials known from the literature
- 2+2 and 3+2: heavy numerical calculations (many channels)
  - $J\pi$ scattering matrices computed in parallel
  - Potential matrix elements saved on files
  - Optimization of the R-matrix paramaters (number of basis functions)
- Extension to 3+3 (^{6}He+^{9}Be) : feasible (but complicated!)
- Limitations of CDCC at low energies
  - presence of resonances?
  - treatment of Pauli forbidden states?
- Future (and current) work: transfer reactions
  - Use of CDCC scattering wave functions
    example: ^{11}Li(p,t)^{9}Li: P. Descouvemont, Phys. Rev. C **104**, 024613 (2021).
  - Use of microscopic overlap integrals
    ^{6}He(d,n)^{7}Li (microscopic description of ^{7}Li): P. Descouvemont, Eur. Phys. J. A 2022 5810 **58**, 1 (2022).
    ^{16}C(d,p)^{17}C (microscopic description of ^{17}C): Le Hoang Chien and P. Descouvemont, Phys. Rev. C **108**, 044605 (2023).
    ^{11}Li(p,t)^{9}Li(gs, exc) (microscopic description of ^{11}Li): in progress

Thank you for your attention!