Recent advances in few-body nuclear reactions

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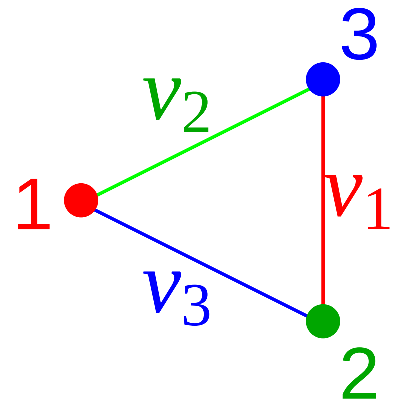
Outline

- core excitation:
  extended Faddeev/AGS formalism
- 3-body nuclear reactions
  — $^{24}\text{Mg}(d,d')$
  — $^{10}\text{Be}(d,p)$, $^{11}\text{Be}(p,d)$, $^{11}\text{Be}(p,pn)$
  — $^{20}\text{O}(d,p)$
- 4-particle scattering
  (p,n), (d,p) and (d,n) reactions in 4N system

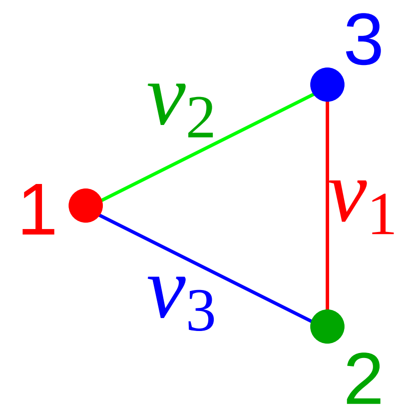
$$\left.\begin{array}{l} p+{}^3\text{H} \\ n+{}^3\text{He} \\ d+d \end{array}\right\} \to \left\{\begin{array}{l} p+n+d \\ p+{}^3\text{H} \\ n+{}^3\text{He} \\ d+d \\ 2p+2n \end{array}\right.$$

Three-particle system



Hamiltonian $H_0 + \sum_{\alpha} v_{\alpha}$

Three-particle system



Hamiltonian $H_0 + \sum_\alpha v_\alpha$

- Faddeev equations

$$(E - H_0 - v_\alpha)|\psi_\alpha\rangle = v_\alpha \sum_\sigma \bar{\delta}_{\alpha\sigma} |\psi_\sigma\rangle$$

$$|\Psi\rangle = \sum_\alpha |\psi_\alpha\rangle$$

difficult to solve
exact solution of 3-body problem:
discrepancy with data
$\rightarrow$ shortcomings of 3-body Hamiltonian

Alt, Grassberger, and Sandhas equations

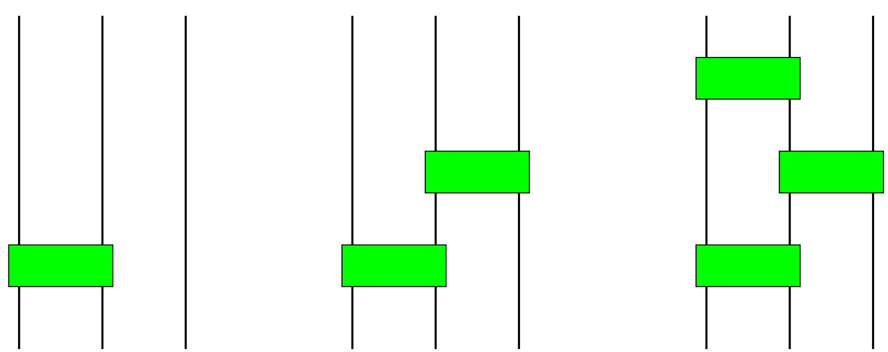
$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\sigma} \bar{\delta}_{\beta\sigma} T_{\sigma} G_0 U_{\sigma\alpha}$$

$$U_{0\alpha} = G_0^{-1} + \sum_{\sigma} T_{\sigma} G_0 U_{\sigma\alpha}$$

$$T_{\sigma} = v_{\sigma} + v_{\sigma} G_0 T_{\sigma}$$

$$G_0 = (E + i0 - H_0)^{-1}$$

channel states $(E - H_0 - v_{\alpha})|\phi_{\alpha}\rangle = 0$



...

AGS equations with 3BF

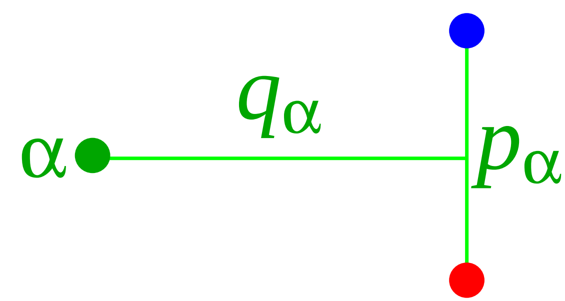
$$V_{3BF} = \sum_{\alpha=1}^{3} w_\alpha$$

$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_\gamma \bar{\delta}_{\beta\gamma} T_\gamma G_0 U_{\gamma\alpha}$$

$$+ w_\alpha + \sum_\gamma w_\gamma G_0 (1 + T_\gamma G_0) U_{\gamma\alpha}$$

AGS equations: numerical solution

$$U_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\sigma} \bar{\delta}_{\beta\sigma} T_{\sigma} G_0 U_{\sigma\alpha}$$

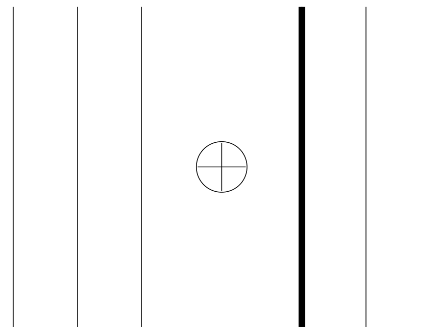


- 3 sets of Jacobi momenta
- momentum-space partial wave basis
- set of coupled 2-variable integral equations
- integrable singularities in kernel
- Coulomb interaction: screening and renormalization

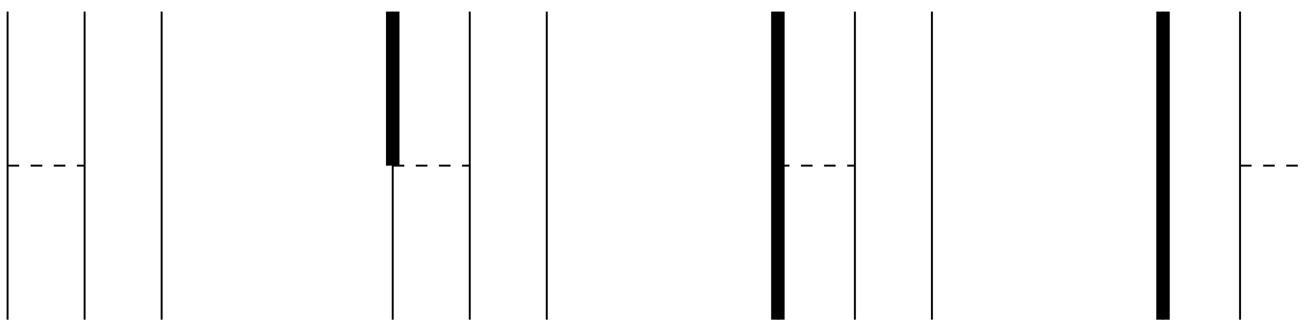
[PRC 71, 054005; PRC 72, 054004; PRC 74, 064001]

Core excitation (CX): extended Hilbert space

$$\mathcal{H} = \mathcal{H}_g \oplus \mathcal{H}_x$$



sector coupling by interaction



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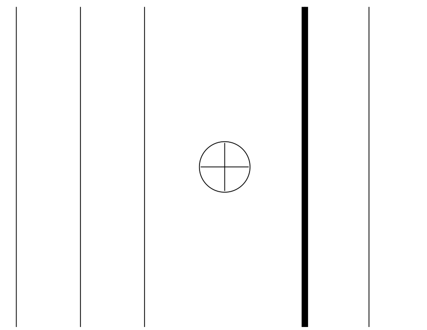

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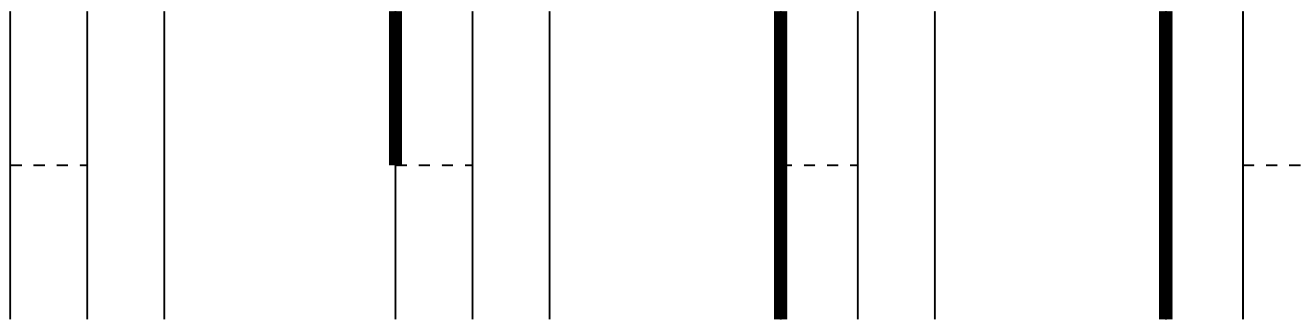


standard operator form of 3-body AGS equations

with $H_0 \to H_0 + h_A^{\text{int}}$

$$h_A^{\text{int}}|\mathcal{H}_a\rangle = (m_{A^*} - m_A)\delta_{ax}|\mathcal{H}_a\rangle$$

3-body AGS equations with core excitation

$$U_{\beta\alpha}^{ba} = \bar{\delta}_{\beta\alpha}\delta_{ba}G_0^{-1} + \sum_{\sigma}\sum_{j}\bar{\delta}_{\beta\sigma}T_{\sigma}^{bj}G_0U_{\sigma\alpha}^{ja}$$

$$U_{0\alpha}^{ba} = \delta_{ba}G_0^{-1} + \sum_{\sigma}\sum_{j}T_{\sigma}^{bj}G_0U_{\sigma\alpha}^{ja}$$

$$T_{\sigma}^{ba} = v_{\sigma}^{ba} + \sum_{j}v_{\sigma}^{bj}G_0T_{\sigma}^{ja}$$

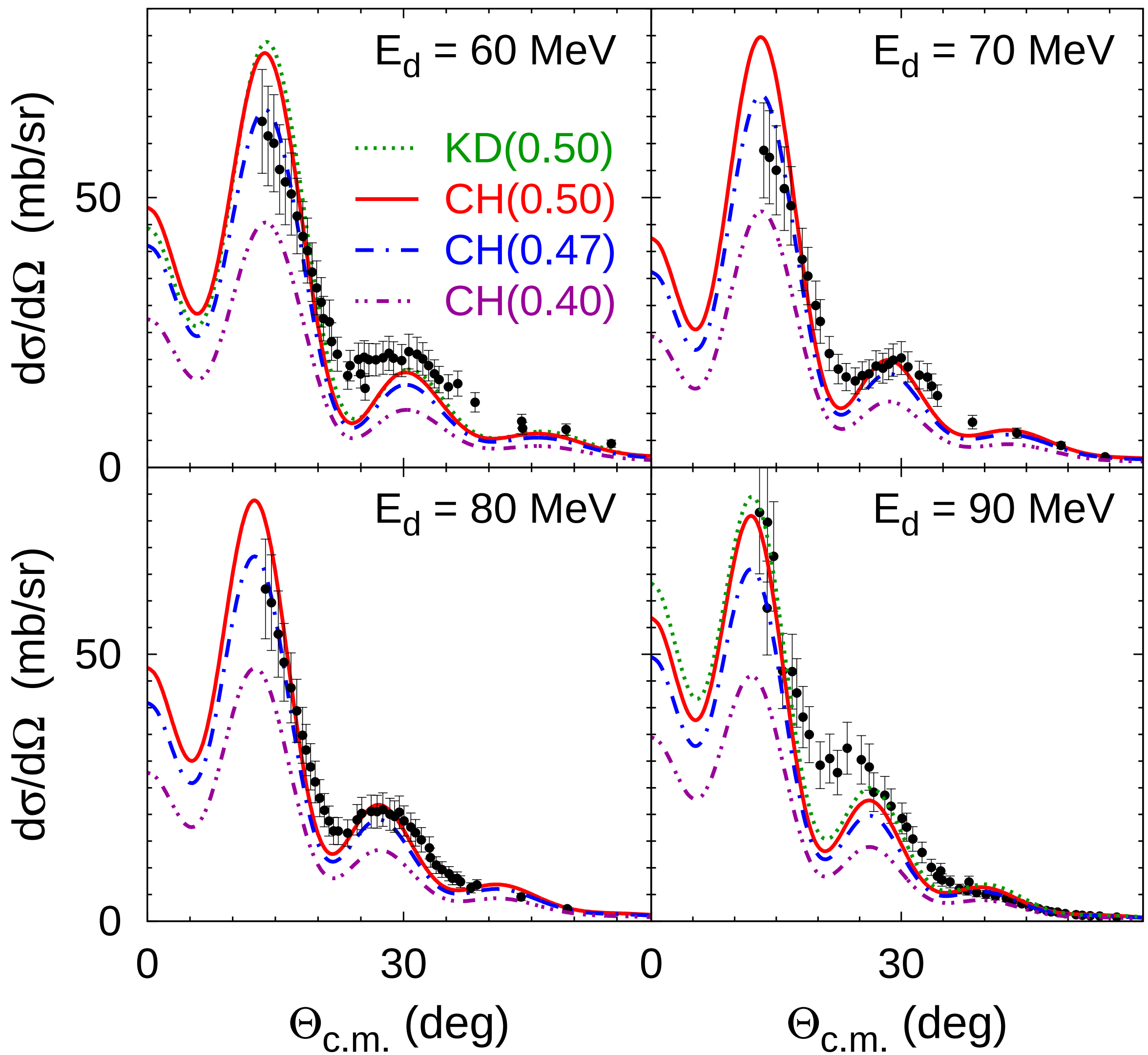
$$G_0 = (E + i0 - H_0)^{-1}$$

channel states $(E - H_0)|\phi_{\alpha}^{a}\rangle = \sum_{j} v_{\alpha}^{aj}|\phi_{\alpha}^{j}\rangle$

$$H_0|\mathbf{p}_{\alpha}\mathbf{q}_{\alpha}\rangle^{a} = [p_{\alpha}^{2}/2\mu_{\alpha} + q_{\alpha}^{2}/2M_{\alpha} + (m_{A^*} - m_{A})\delta_{ax}]|\mathbf{p}_{\alpha}\mathbf{q}_{\alpha}\rangle^{a}$$

[PRC 88, 011601(R)]

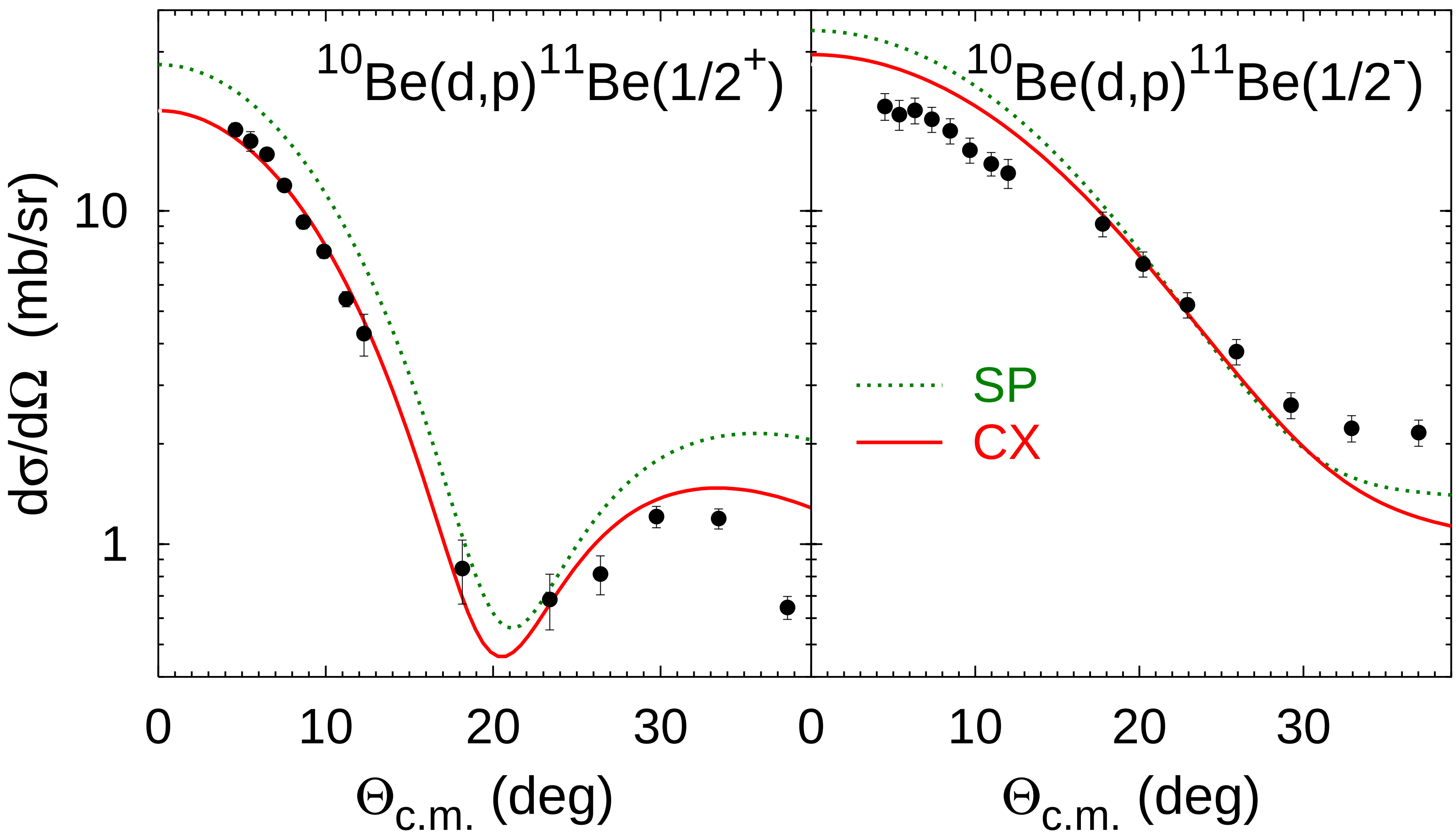
^{24}Mg(d,d')^{24}Mg(2^+) inelastic scattering



Rotational model for V_{NA} with $\beta_2 = 0.4\ldots0.5$ [NPA 947, 173]

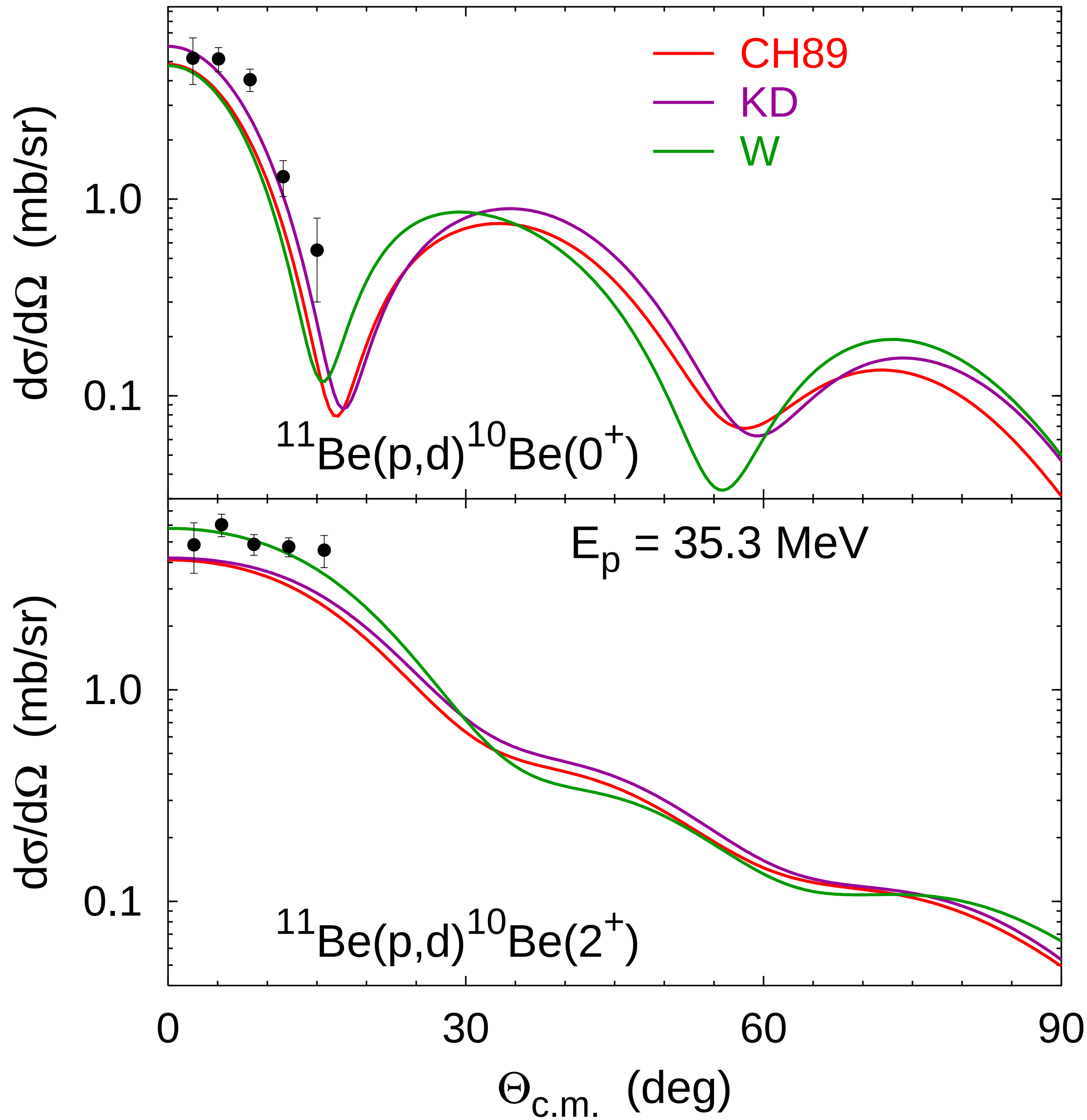
DWBA: $\beta_2 \sim 0.5\ (p,p'),\quad \beta_2 \sim 0.4\ (d,d')$

CX effect in $^{10}Be(d,p)^{11}Be$ at 21.4 MeV

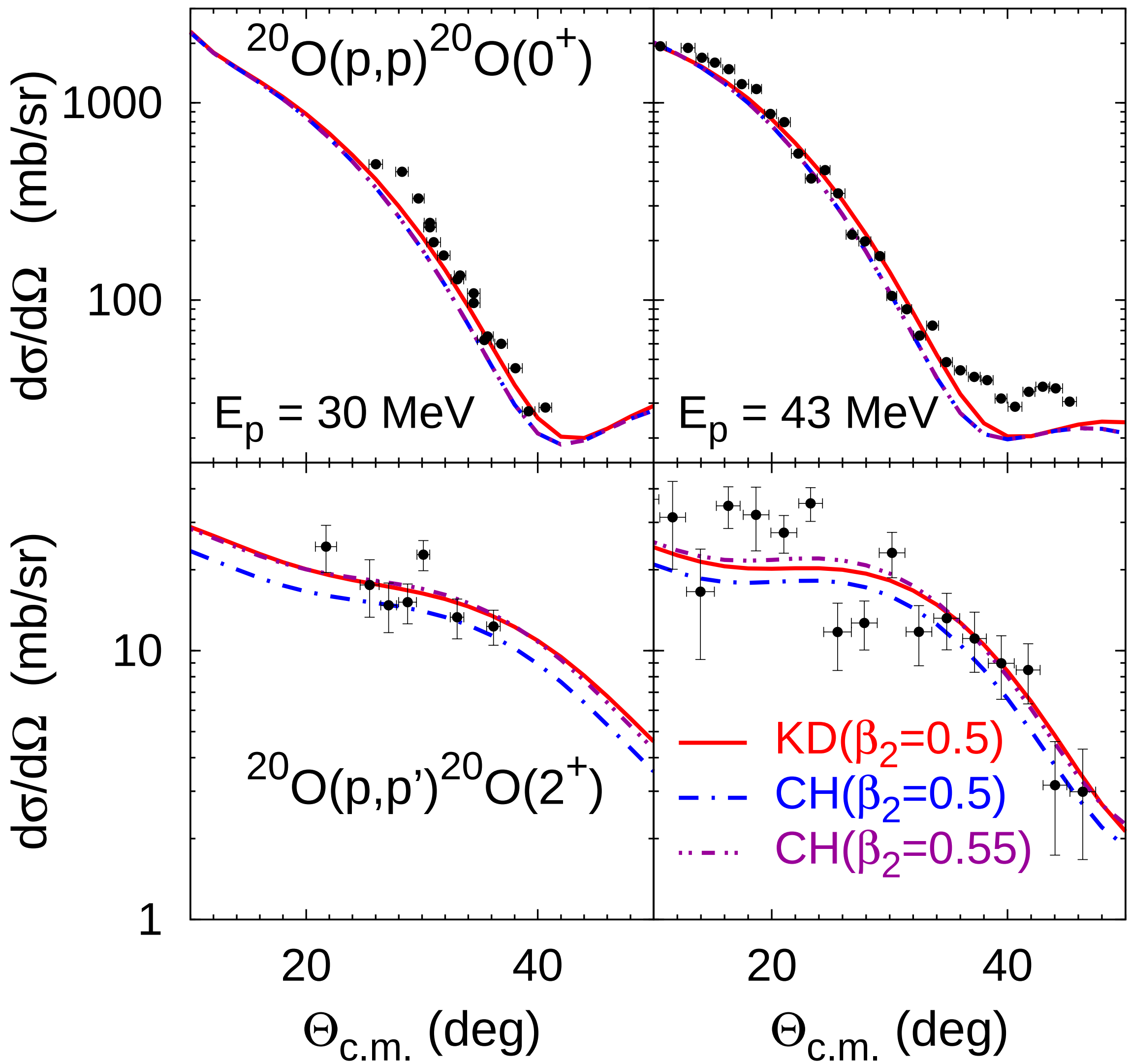


CH89, rotational model for V_{NA} with $\beta_2 = 0.67$
[PRC 91, 024607]

^{11}Be(p,d)^{10}Be: sensitivity to V_{NA}



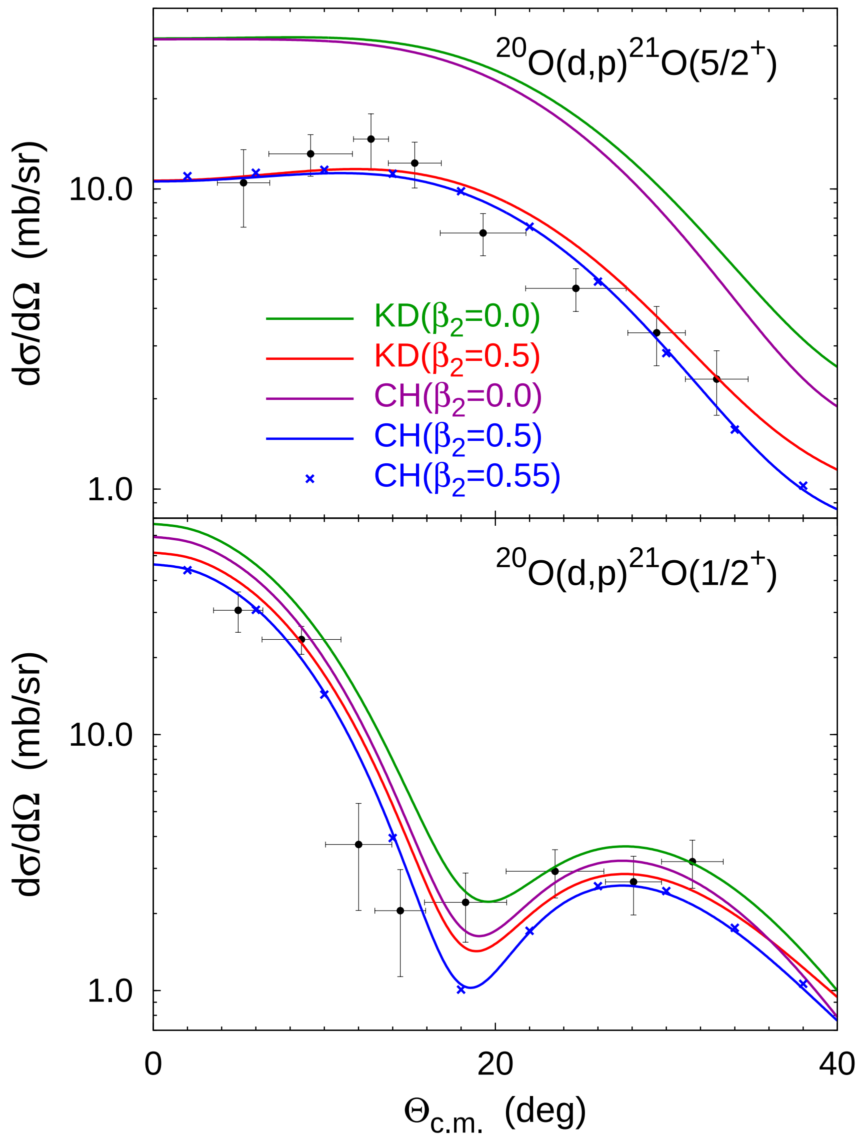
Potential test: N + ^{20}O



Vibrational model for V_{NA}

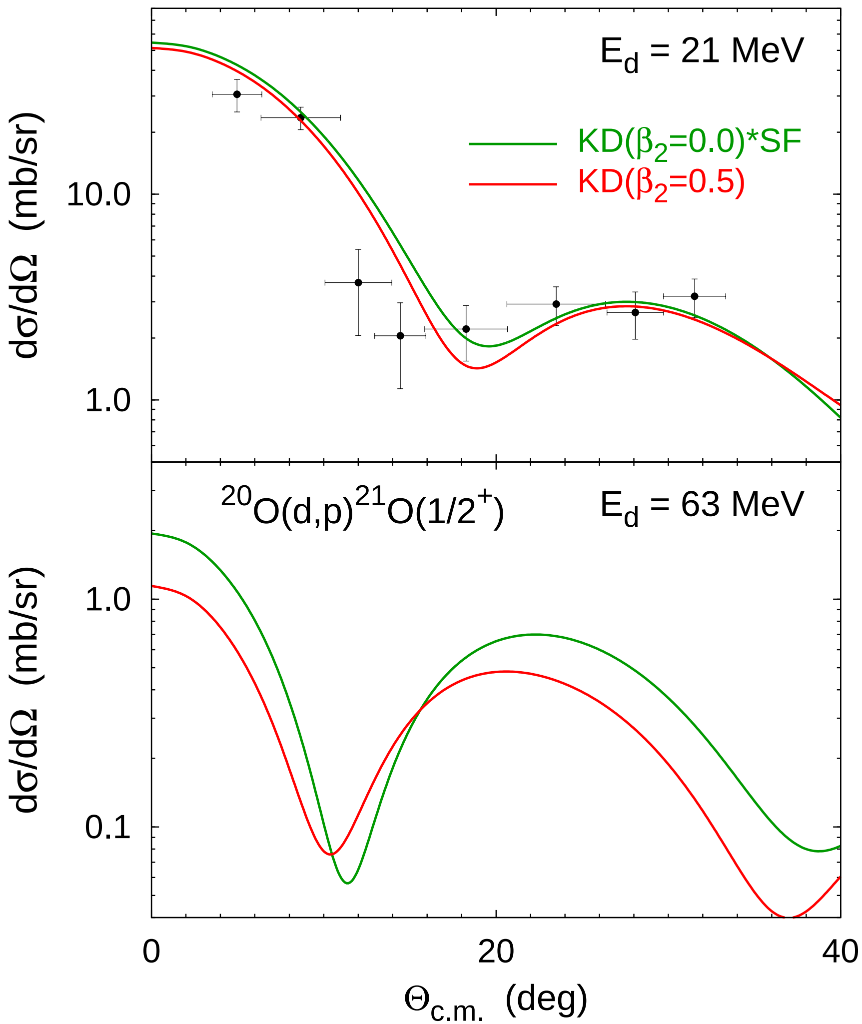
Shell-model SF for ^{21}O: $0.34(\frac{5}{2}^+)$, $0.82(\frac{1}{2}^+)$

$^{20}O(d,p)^{21}O$ at 21 MeV



[PLB 769, 202]

$^{20}O(d,p)^{21}O$: extracting SF?



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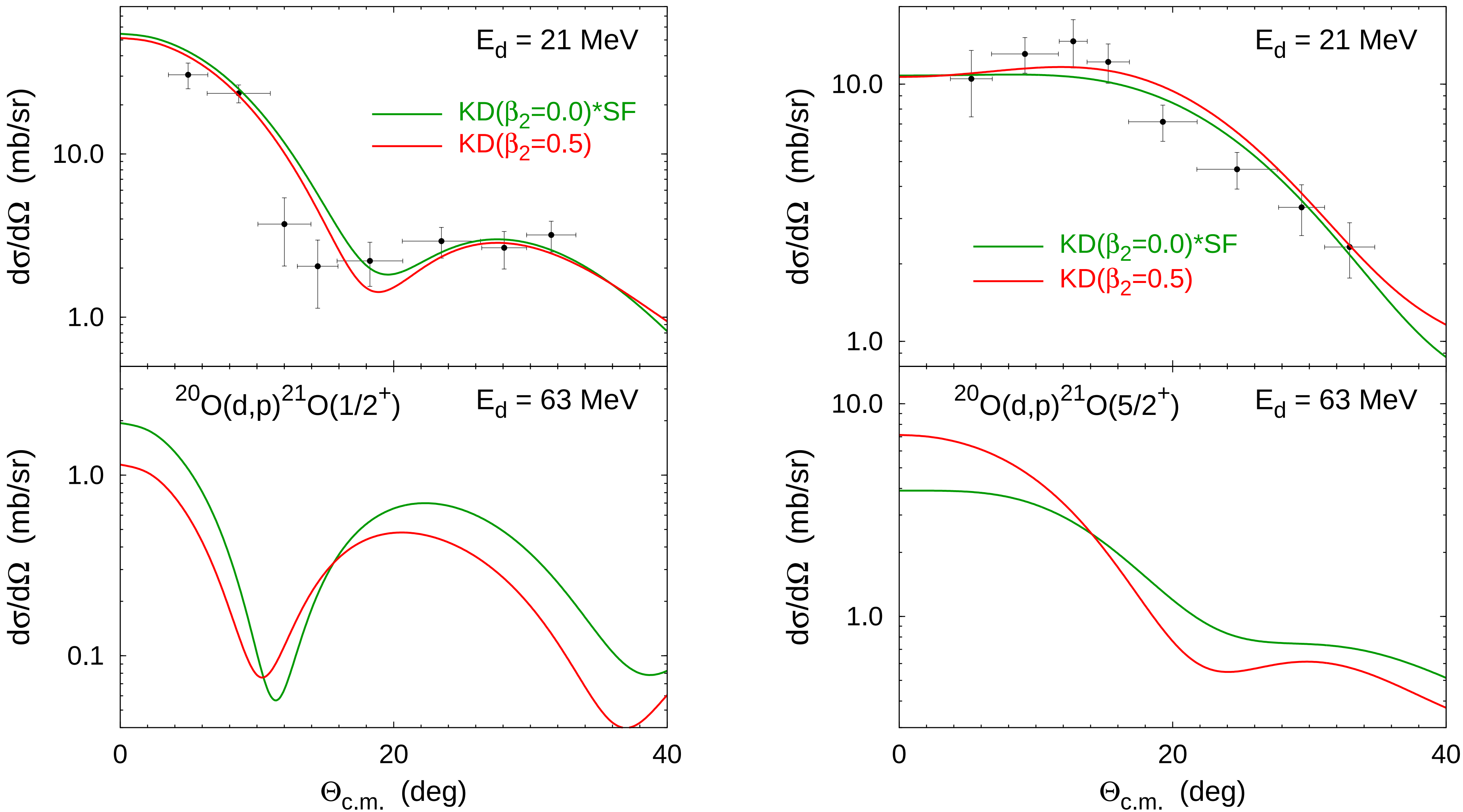
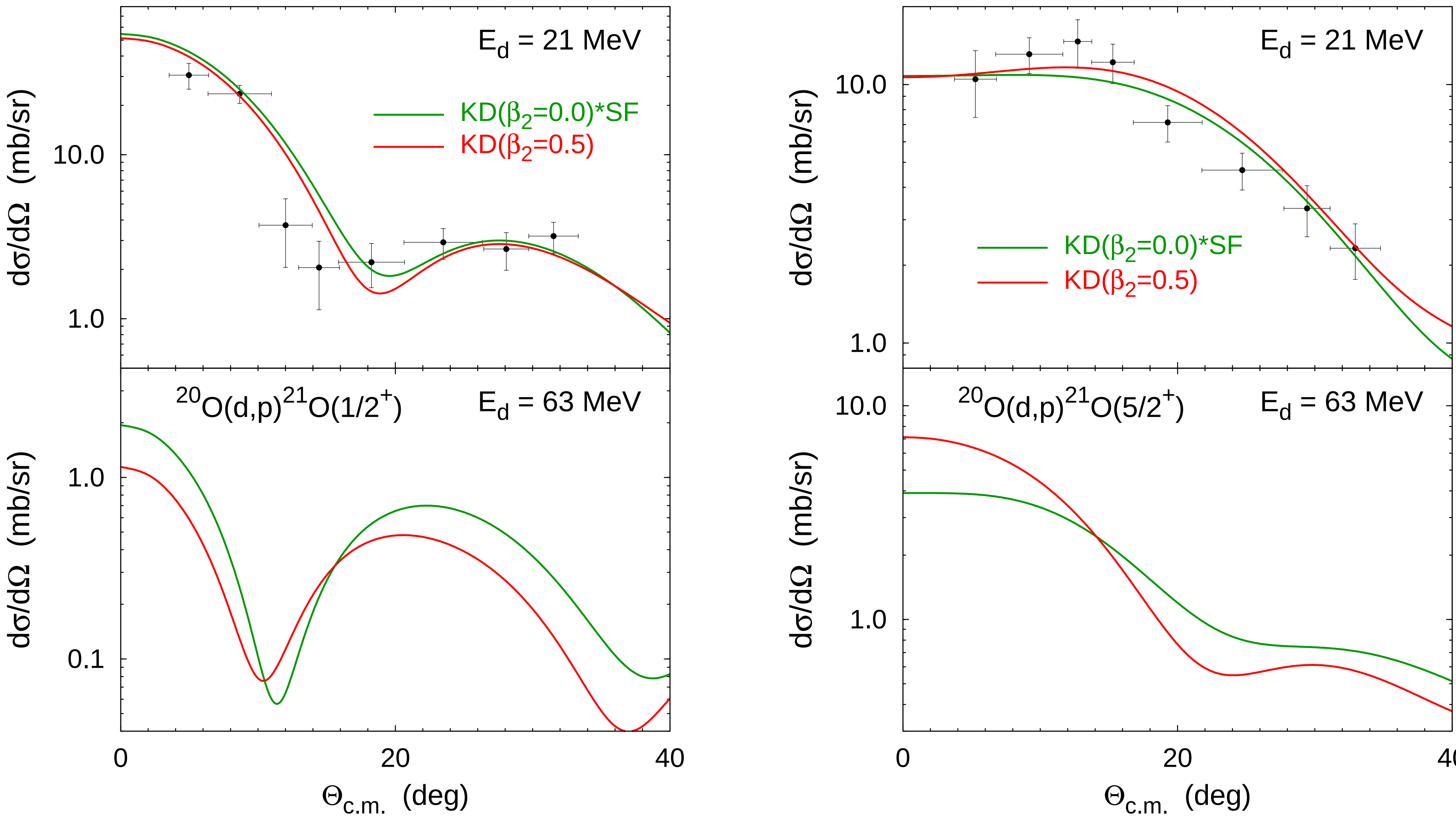


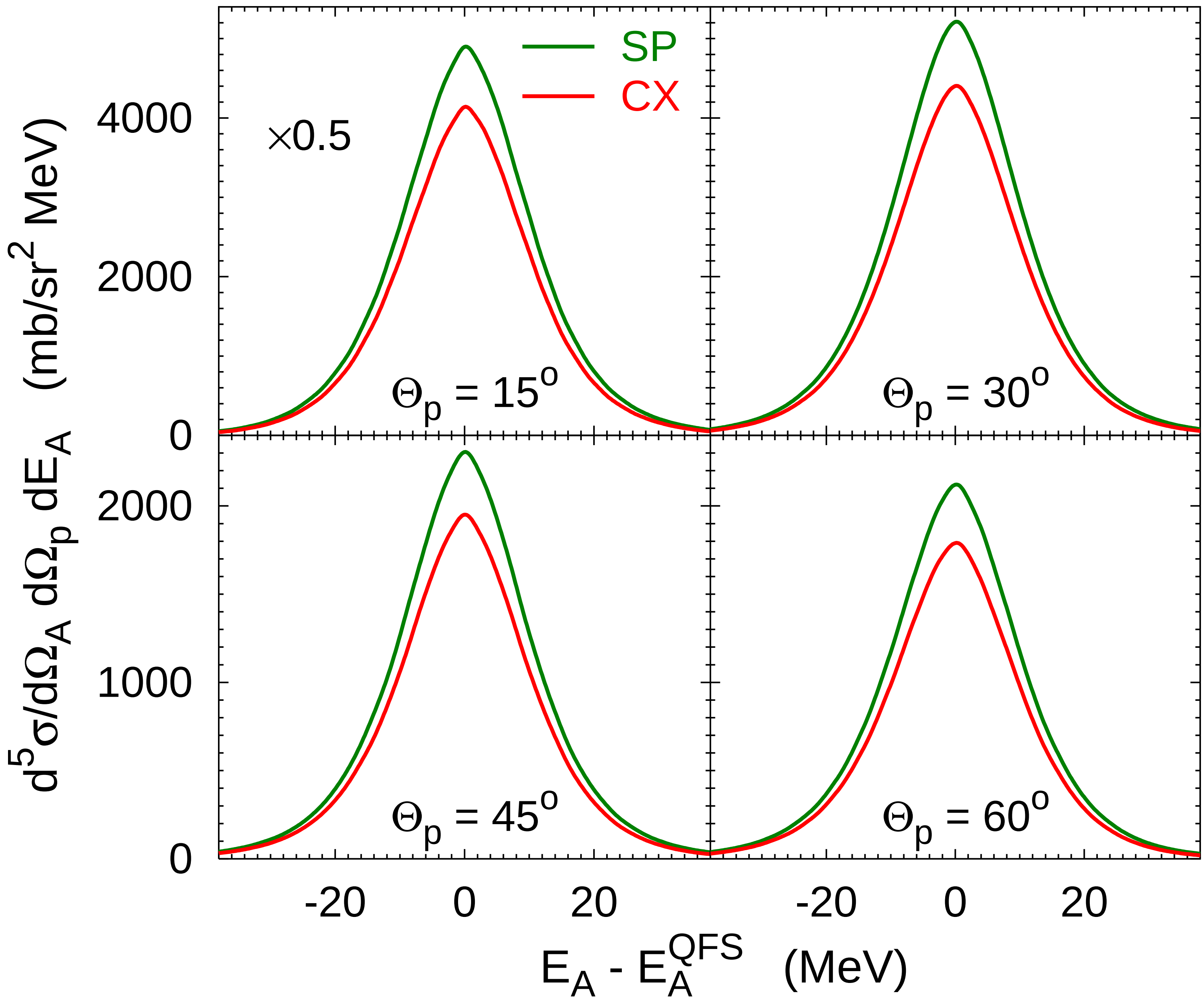
$^{20}O(d,p)^{21}O$: extracting SF?



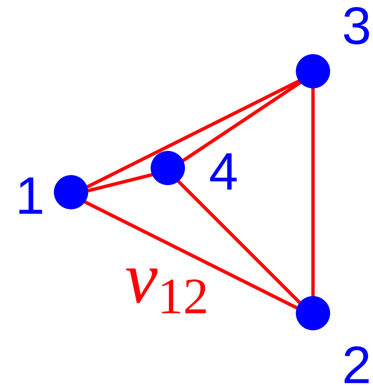
SF$= \sigma_{\text{exp}}/\sigma_{\text{SP}}$ in general unreliable !

Faddeev/AGS: (V_{NA} - SF - data) compatibility check

^{11}Be(p,pn)^{10}Be at 200 MeV/u near np QFS ($\Theta_A = 0^\circ$)



4N scattering



Hamiltonian $$H_0 + \sum_{i>j} v_{ij}$$

- Wave function:
  Schrödinger equation (HH + Kohn VP, r-space)
  [M. Viviani, A. Kievsky, L. E. Marcucci, S. Rosati, L. Girlanda]
- Wave function components:
  Faddeev-Yakubovsky equations (r-space)
  [R. Lazauskas, J. Carbonell]
- Transition operators:
  Alt-Grassberger-Sandhas equations (p-space)
  [AD, A. C. Fonseca]

4-body scattering: AGS equations

4-body transition operators

$$t_i = v_i + v_i G_0 t_i$$

$$G_0 = (E + i0 - H_0)^{-1}$$

$$U_\gamma^{jk} = G_0^{-1} \bar{\delta}_{jk} + \sum_i \bar{\delta}_{ji} t_i G_0 U_\gamma^{ik}$$

$$\mathcal{U}_{\beta\alpha}^{ji} = (G_0 t_i G_0)^{-1} \bar{\delta}_{\beta\alpha} \delta_{ji} + \sum_{\gamma k} \bar{\delta}_{\beta\gamma} U_\gamma^{jk} G_0 t_k G_0 \, \mathcal{U}_{\gamma\alpha}^{ki}$$

i, j, k: pairs ($\equiv$ three-cluster (2+1+1) partitions)
α, β, γ: two-cluster (1+3 or 2+2) partitions

4-body scattering: AGS equations

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i, j, k: pairs ($\equiv$ three-cluster (2+1+1) partitions)
α, β, γ: two-cluster (1+3 or 2+2) partitions

wave function

$$|\Psi_\alpha\rangle = |\Phi_\alpha\rangle + \sum_{\gamma jki} G_0 \, t_j \, G_0 \, U_\gamma^{jk} G_0 \, t_k \, G_0 \, \mathcal{U}_{\gamma\alpha}^{ki} |\phi_\alpha^i\rangle$$

$$|\Phi_\alpha\rangle = \sum_i |\phi_\alpha^i\rangle, \qquad |\phi_\alpha^i\rangle = G_0 \sum_j \bar{\delta}_{ij} t_j |\phi_\alpha^j\rangle$$

4-body scattering amplitudes

two-cluster reactions:

$$\langle \Phi_\beta | T_{\beta\alpha} | \Phi_\alpha \rangle = \sum_{ji} \langle \phi_\beta^j | \mathcal{U}_{\beta\alpha}^{ji} | \phi_\alpha^i \rangle$$

three-cluster breakup:

$$\langle \Phi^j | T_\alpha^j | \Phi_\alpha \rangle = \sum_{\beta k i} \langle \Phi^j | U_\beta^{jk} G_0 \, t_k \, G_0 \, \mathcal{U}_{\beta\alpha}^{ki} | \phi_\alpha^i \rangle$$

four-cluster breakup:

$$\langle \Phi_0 | T_{0\alpha} | \Phi_\alpha \rangle = \sum_{\beta j k i} \langle \Phi_0 | t_j \, G_0 \, U_\beta^{jk} G_0 \, t_k \, G_0 \, \mathcal{U}_{\beta\alpha}^{ki} | \phi_\alpha^i \rangle$$

[PRC 75, 014005; PRA 85, 012708]

Symmetrized AGS equations

$$t = v + vG_0 t$$

$$G_0 = (E + i\varepsilon - H_0)^{-1}$$

$$U_j = P_j G_0^{-1} + P_j t G_0 U_j$$

$$3+1: \quad P_1 = P_{12} P_{23} + P_{13} P_{23}$$

$$2+2: \quad P_2 = P_{13} P_{24}$$

$$\mathcal{U}_{11} = (G_0 t G_0)^{-1} \zeta P_{34} + \zeta P_{34} U_1 G_0 t G_0 \, \mathcal{U}_{11} + U_2 G_0 t G_0 \, \mathcal{U}_{21}$$

$$\mathcal{U}_{21} = (G_0 t G_0)^{-1} (1 + \zeta P_{34}) + (1 + \zeta P_{34}) U_1 G_0 t G_0 \, \mathcal{U}_{11}$$

$$\mathcal{U}_{12} = (G_0 t G_0)^{-1} + \zeta P_{34} U_1 G_0 t G_0 \, \mathcal{U}_{12} + U_2 G_0 t G_0 \, \mathcal{U}_{22}$$

$$\mathcal{U}_{22} = (1 + \zeta P_{34}) U_1 G_0 t G_0 \, \mathcal{U}_{12}$$

$\zeta = -1 \; (+1)$ for fermions (bosons)

basis states partially symmetrized

Scattering amplitudes: $E + i\varepsilon \to E + i0$

2-cluster reactions:

$$T_{fi} = s_{fi} \langle \phi_f | \mathcal{U}_{fi} | \phi_i \rangle$$
$$|\phi_j\rangle = G_0 t P_j |\phi_j\rangle$$
$$|\Phi_j\rangle = (1 + P_j) |\phi_j\rangle$$

3-cluster breakup/recombination:

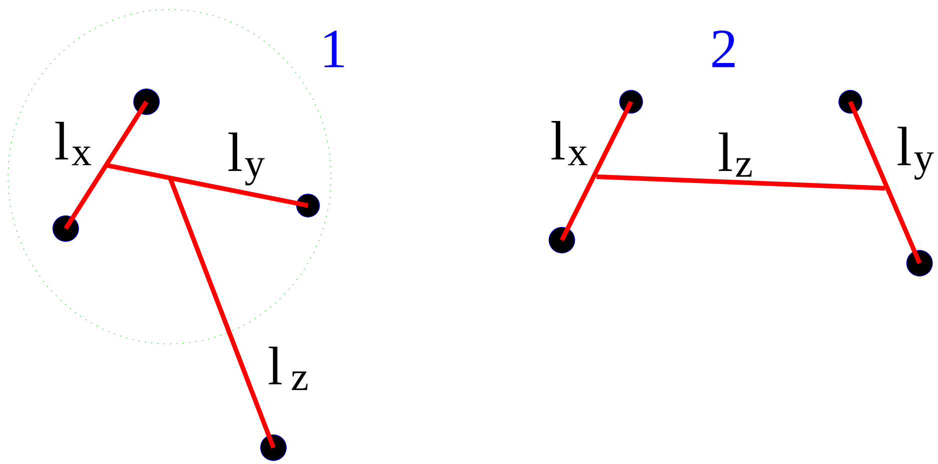
$$T_{3i} = s_{3i} \langle \phi_3 | [(1 + \zeta P_{34}) U_1 G_0 t G_0 \, \mathcal{U}_{1i} + U_2 G_0 t G_0 \, \mathcal{U}_{2i}] | \phi_i \rangle$$

4-cluster breakup/recombination:

$$T_{4i} = s_{4i} \{ \langle \phi_4 | [1 + (1 + P_1) \zeta P_{34}] (1 + P_1) t G_0 U_1 G_0 t G_0 \, \mathcal{U}_{1i} | \phi_i \rangle$$
$$+ \langle \phi_4 | (1 + P_1)(1 + P_2) t G_0 U_2 G_0 t G_0 \, \mathcal{U}_{2i} | \phi_i \rangle \}$$

Solution of 4N AGS equations

$$\mathcal{U}_{12}|\phi_2\rangle = G_0^{-1}P_2|\phi_2\rangle - P_{34}U_1 G_0\, t\, G_0\, \mathcal{U}_{12}|\phi_2\rangle + U_2 G_0\, t\, G_0\, \mathcal{U}_{22}|\phi_2\rangle$$



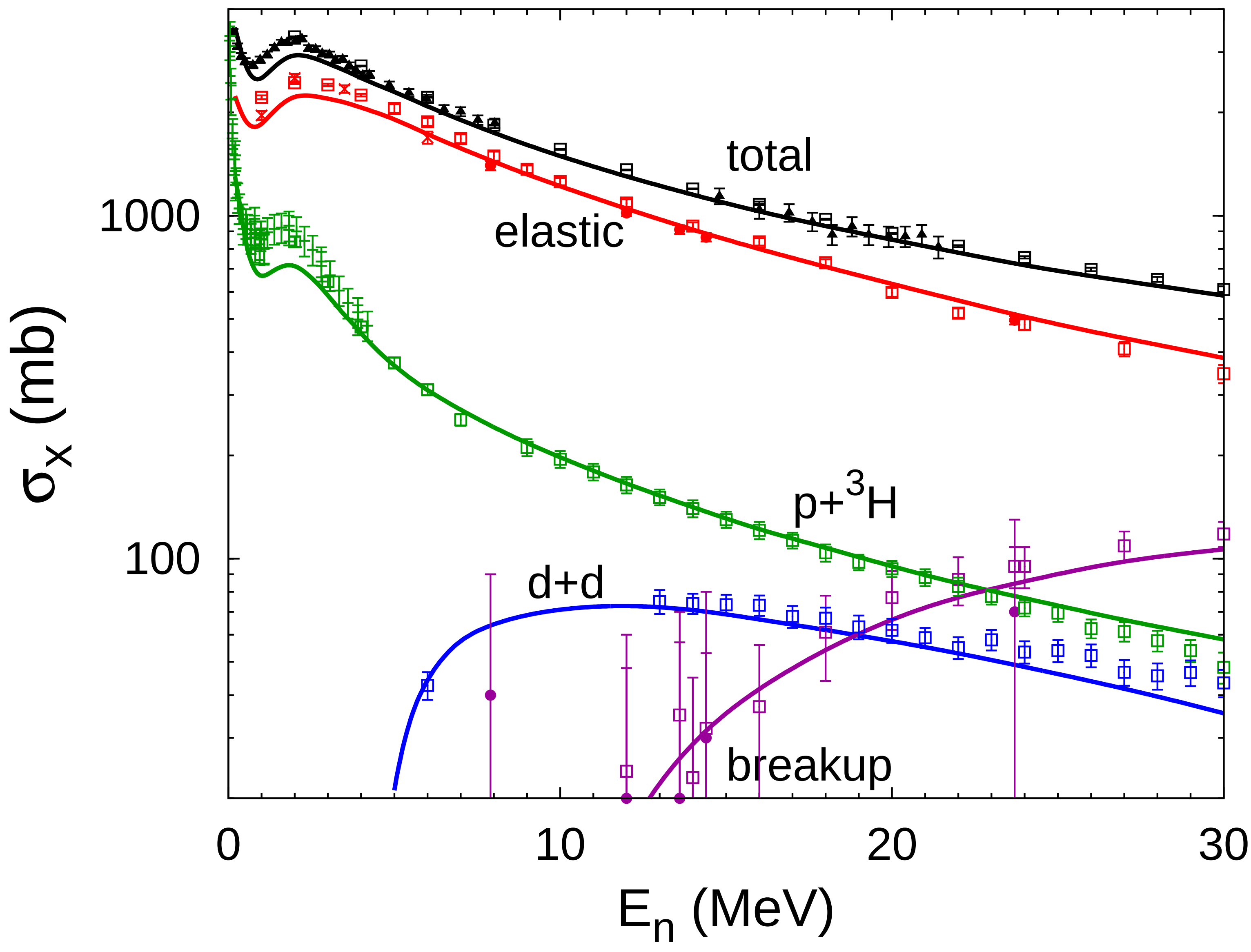
- momentum-space partial-wave basis

  $|k_x k_y k_z [l_z(\{l_y[(l_x S_x) j_x s_y] S_y\} J_y s_z) S_z] JM, [(T_x t_y) T_y t_z] T M_T\rangle_1$

  $|k_x k_y k_z [l_z\{(l_x S_x) j_x [l_y(s_y s_z) S_y] j_y\} S_z] JM, [T_x(t_y t_z) T_z] T M_T\rangle_2$

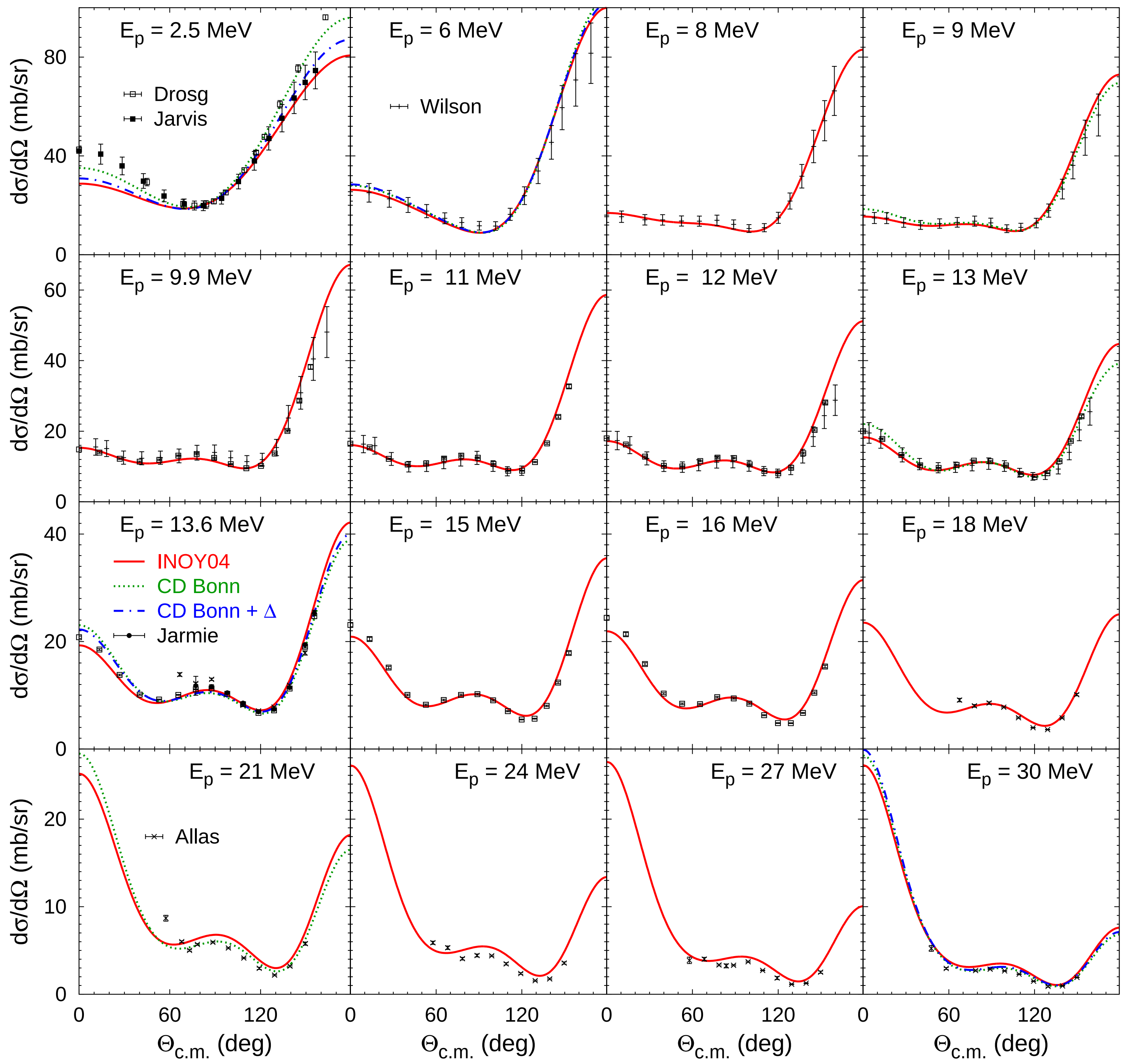
- large system (up to 30000) of coupled 3-variable integral equations with integrable singularities
- Coulomb interaction: screening and renormalization [PRC 75, 014005, PRL 98, 162502]

$n+^3He$ total and partial cross sections

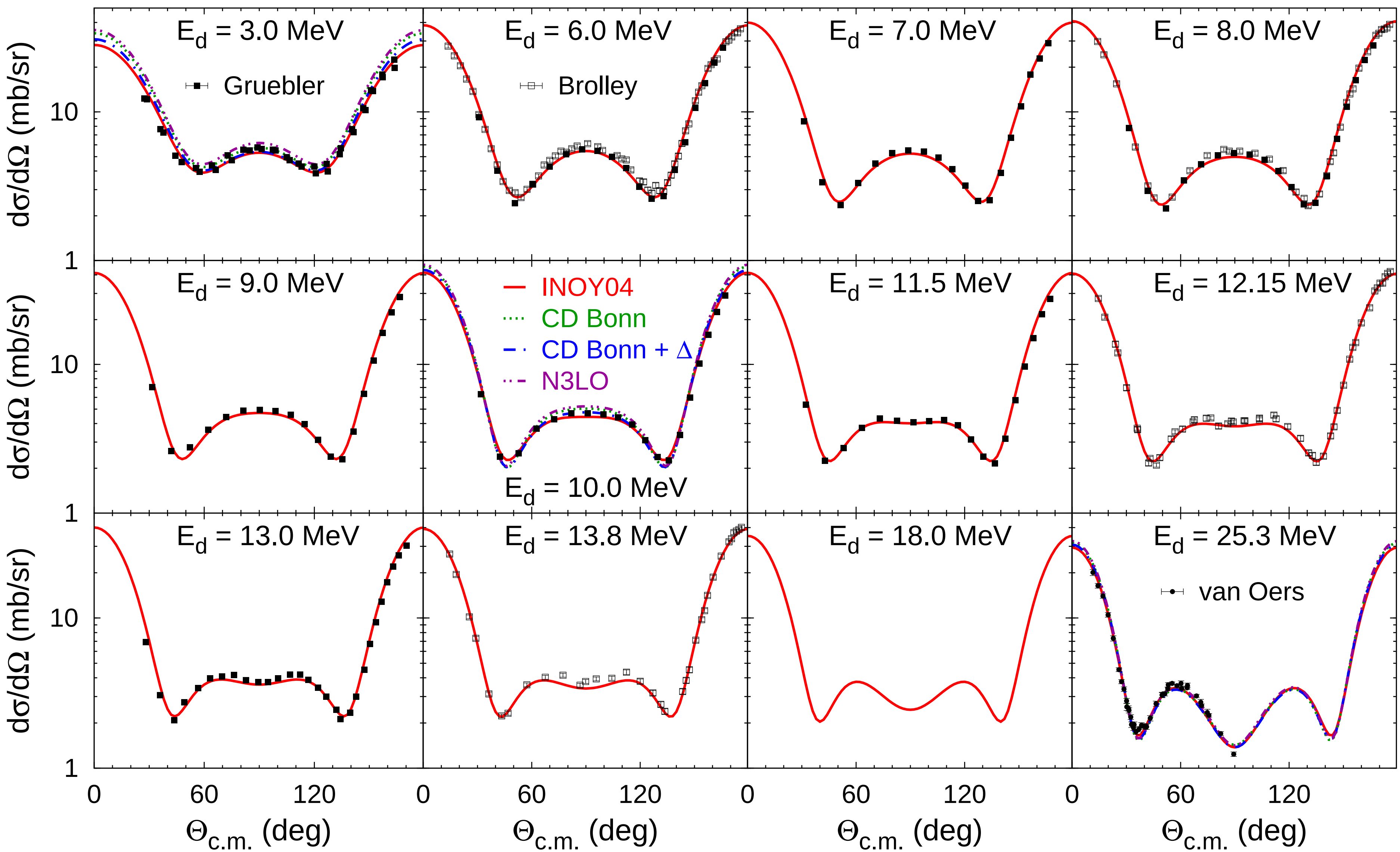


[PRL 113, 102502; PRC 90, 044002]

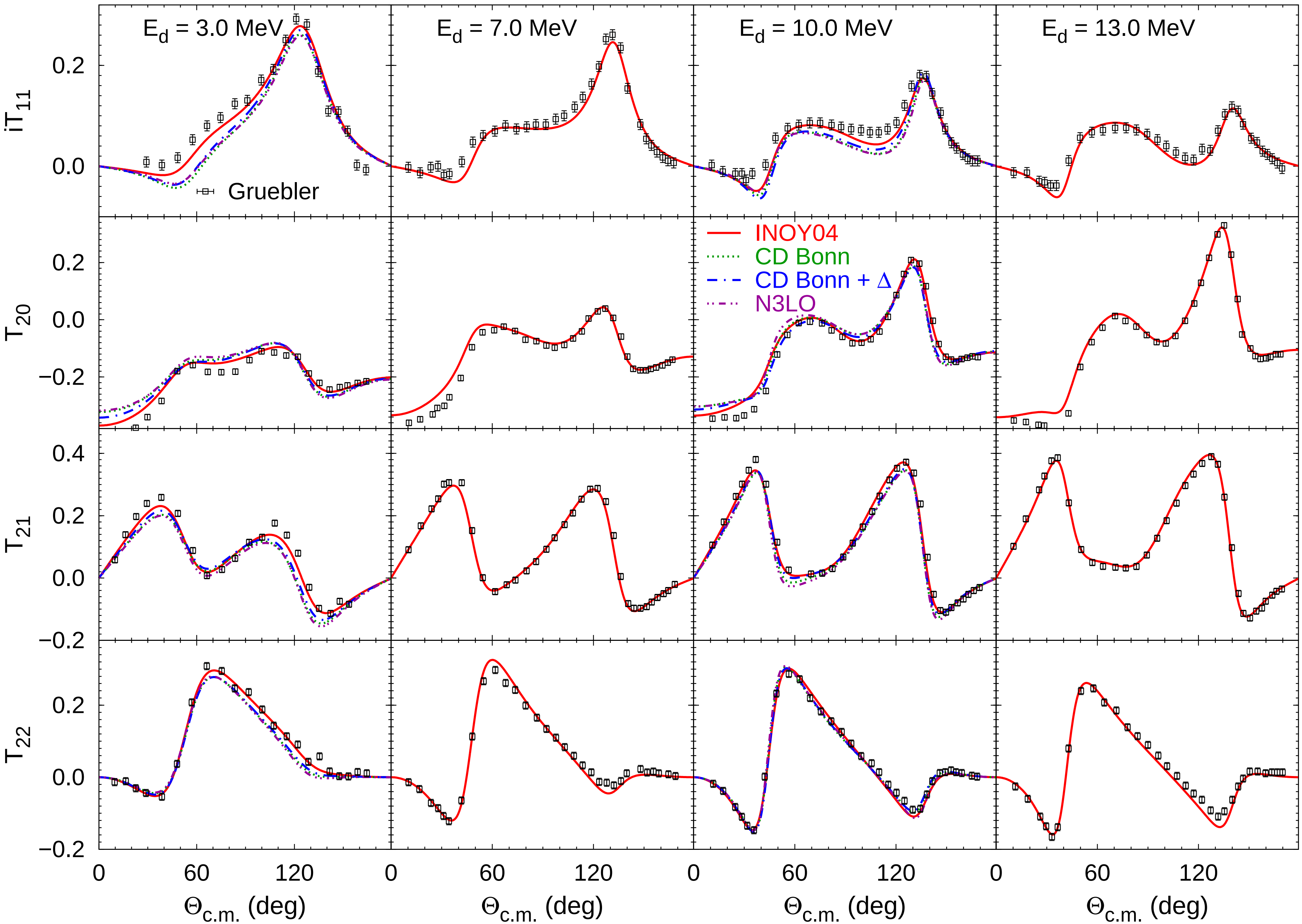
Charge exchange reaction $^3\mathrm{H}(p,n)^3\mathrm{He}$



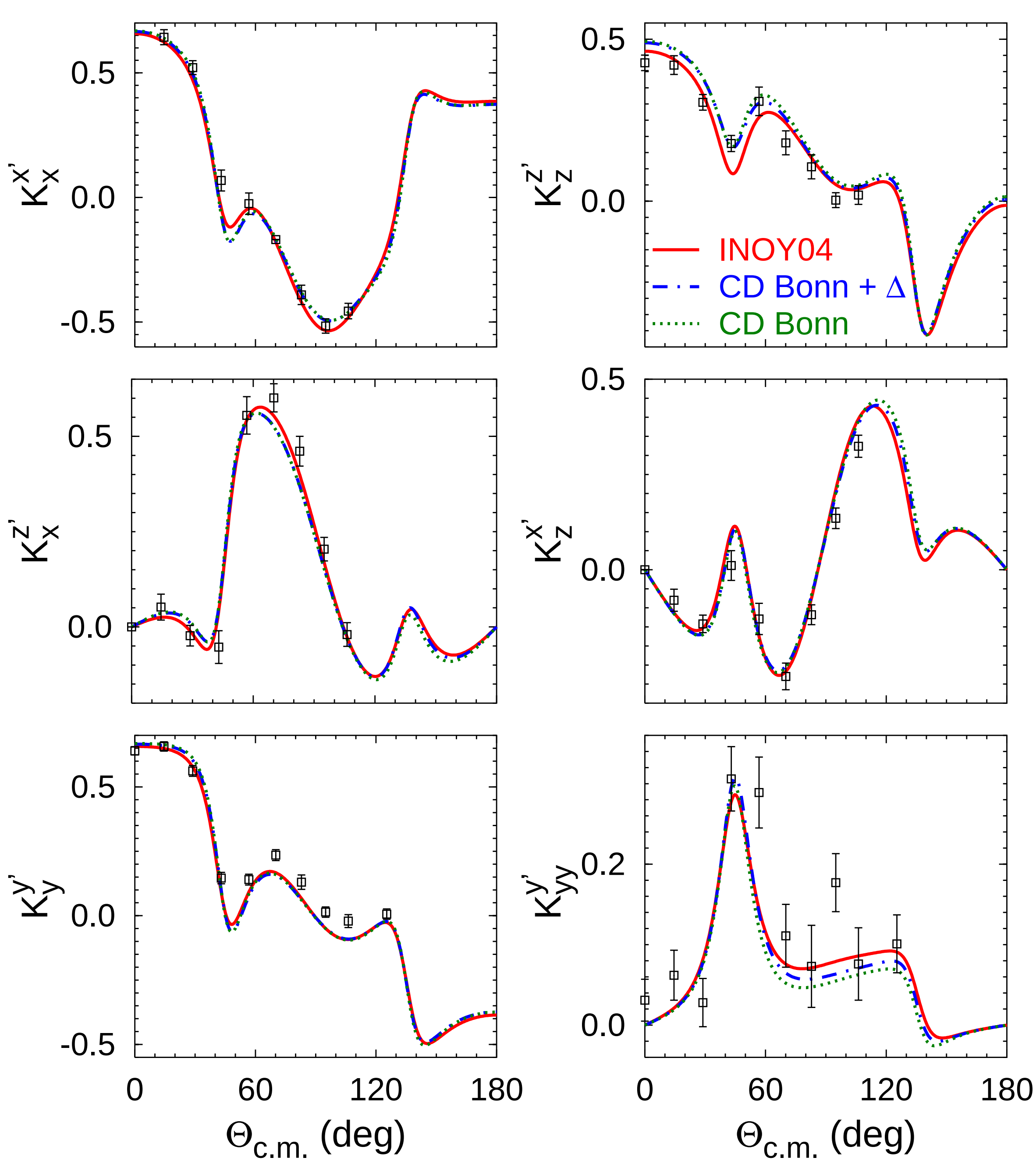
Transfer reaction $^2H(d,p)^3H$



Transfer reaction $^2\mathrm{H}(\vec{d},p)^3\mathrm{H}$: analyzing powers



Spin transfer in $^2\mathrm{H}(\vec{d},\vec{n})^3\mathrm{He}$ at 10 MeV



Few-body nuclear reactions

- 3-body AGS equations:
  extension including core excitation
- complicated CX effects in transfer reactions,
  no simple relation to SF
- 4-body AGS equations
  — (p,n), (d,p) and (d,n) reactions in 4N system