Group Meeting 02.14

Benchmark with Glauber model

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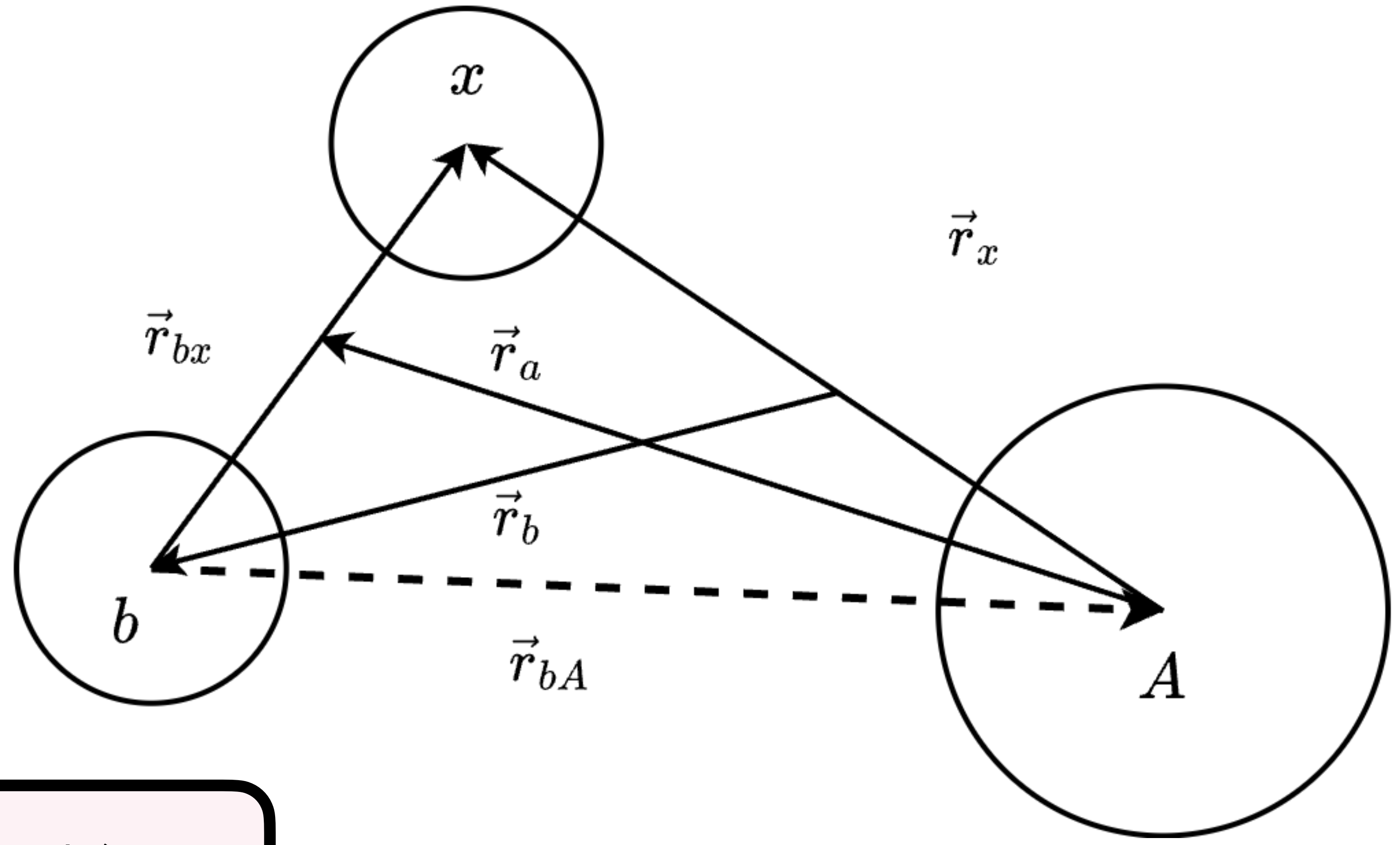
Overview

Consider the reaction:



$$a(=b+x)+A \rightarrow b+B^*(=x+A)$$

There are two methods to handle this question.

One is fully quantum mechanical, the IAV model

The semiclassical method called the Glauber model (or Eikonal model).

Glauber Model

The Glauber model assumes the energy is high and the projectile will move on a straight-line trajectory.

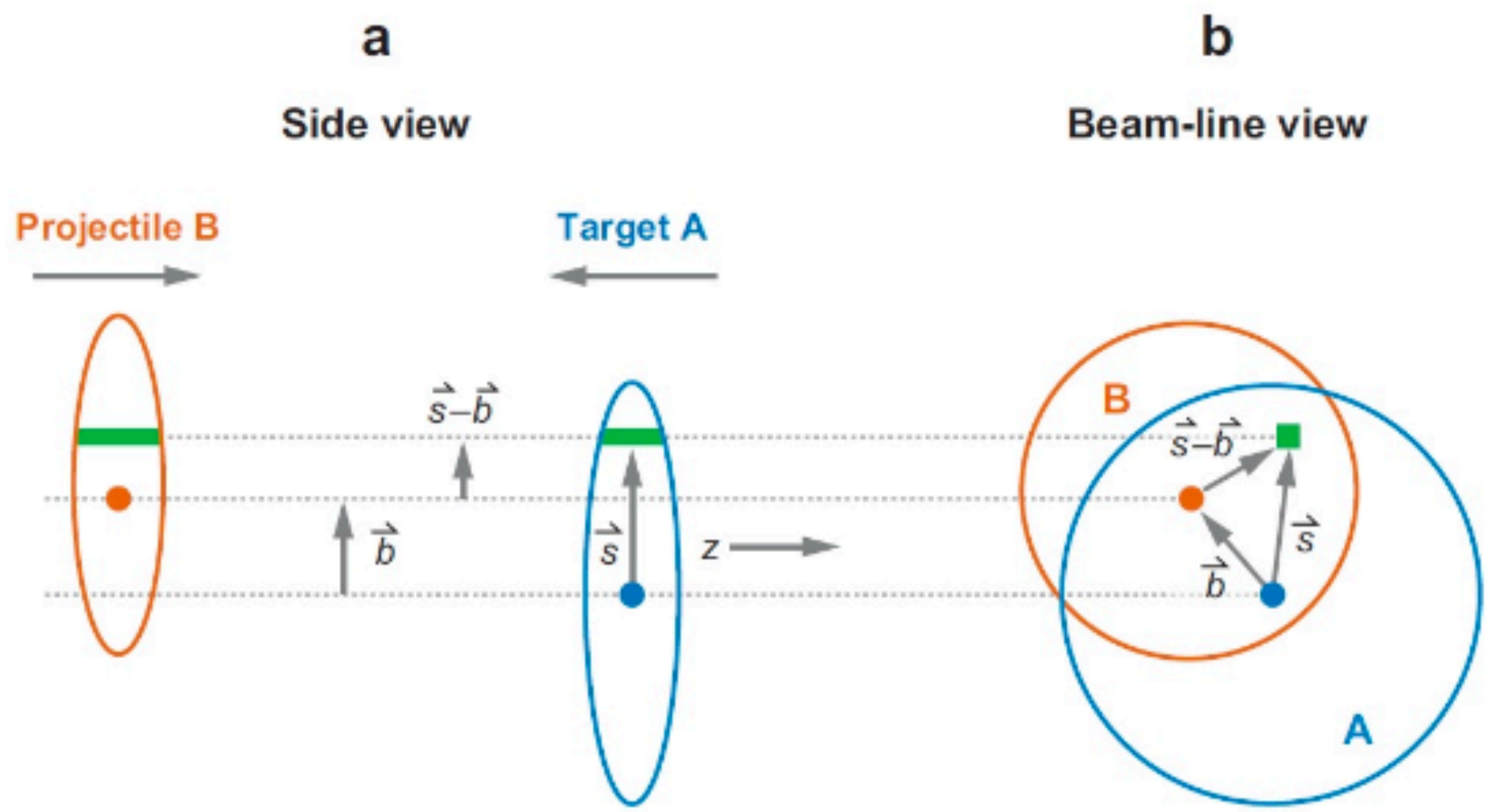


Fig.1 Glauber model[1]

The wave function was assumed

$$\Psi(\overrightarrow{R}) = e^{ikz}\phi(\overrightarrow{b}, z)$$

where $\phi(\overrightarrow{b}, z)$ should be a slowly varying function of both variables in high energy.

Kinetic operator: $\hat{T}_R = -\frac{\hbar^2}{2\mu}[\nabla_b^2 + \frac{\partial^2}{\partial z^2}]$

[1]Miller, M.L., Reygers, K., Sanders, S.J., Steinberg, P., 2007. Annual Review of Nuclear and Particle Science 57, 205–243

Glauber Model

$$[\hat{T}_R + V(\overrightarrow{R})]\Psi = E\Psi$$

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$$-\frac{\hbar^2}{2\mu}[\nabla_b^2 + \frac{\partial^2}{\partial z^2}]e^{ikz}\phi(\overrightarrow{b}, z) + V(\overrightarrow{R})e^{ikz}\phi(\overrightarrow{b}, z) = Ee^{ikz}\phi(\overrightarrow{b}, z)$$

$$e^{ikz}\nabla_b^2\phi + \phi(\overrightarrow{b}, z)(ik)^2e^{ikz} + 2ik\frac{\partial\phi}{\partial z} + e^{ikz}\frac{\partial^2\phi}{\partial z^2} - \frac{2\mu}{\hbar^2}(V(\overrightarrow{R} - E)e^{ikz}\phi(\overrightarrow{b}, z) = 0$$

$$\nabla_b^2\phi + 2ik\frac{\partial\phi}{\partial z} + \frac{\partial^2\phi}{\partial z^2} - \frac{2\mu}{\hbar^2}V(\overrightarrow{R})\phi(\overrightarrow{b}, z) = 0$$

Glauber Model

$$2ik\frac{\partial \phi}{\partial z} - \frac{2\mu}{\hbar^2}V(\overrightarrow{R})\phi(\overrightarrow{b}, z) = 0$$

$$\phi = exp(-\frac{i}{\hbar v_p}\int_{-\infty}^{z} V(\overrightarrow{b}, z)dz)$$

where $v_p = \hbar k/\mu$. And the boundary condition satisfies $\phi(b, -\infty) = 1$.

The eikonal phase was defined,

$$\chi(\overrightarrow{b}, z) = -\frac{1}{\hbar v_p}\int_{-\infty}^{z} V(\overrightarrow{b}, z')dz'$$

The wave function can be written by eikonal phase,

$$\Psi(\overrightarrow{b}, z) = \exp[i(kz + \chi(\overrightarrow{b}, z))]$$

With plane wave matrix, the scattering amplitude,

$$f(\theta) = -\frac{\mu}{2\pi\hbar^2}\int \mathrm{d}\overrightarrow{R}\ \exp(-i\overrightarrow{K}' \cdot \overrightarrow{R})V(\overrightarrow{R})\Psi_{K_0}(\overrightarrow{R})$$

$$\overrightarrow{R} = z\hat{n} + \overrightarrow{b}$$

$$\overrightarrow{q} = \overrightarrow{K}_0 - \overrightarrow{K}'$$

Glauber Model

$$f(\theta) = -\frac{\mu}{2\pi\hbar^2}\int \mathrm{d}^2b\ \exp(-i\vec{q}\cdot\vec{b})\int \mathrm{d}z\,\exp(-iz\vec{q}\cdot\hat{n})V(\vec{b},z)\exp(-\frac{1}{\hbar v_p}\int_{-\infty}^{z}V(\vec{b},z')dz')$$

where θ is the angle between $\vec{K}_0$ and $\vec{K}'$. We assume $q \ll K$, so $\vec{q}\cdot\hat{n} \approx 0$

$$\int \mathrm{d}z V(\mathbf{b},z)\exp\left[-\frac{\mathrm{i}}{\hbar v_p}\int_{-\infty}^{s} V(\mathbf{b},z')\,\mathrm{d}z'\right] = \mathrm{i}\hbar v_p\left(\mathrm{e}^{\mathrm{i}\chi(\mathbf{b})}-1\right)$$

the scattering amplitude can be simplified to

$$f(\theta) = -\frac{iK_0}{2\pi}\int d^2b e^{iq\cdot b}\left(e^{i\chi(b)}-1\right)$$

Glauber Model

Push to the few-body question,

$$f_{fi}(\theta) = -\frac{\mathrm{i}K_0}{2\pi}\int \mathrm{d}^2 b \mathrm{e}^{\mathrm{i}\mathrm{q}\cdot\mathrm{b}} \left\langle \Phi_f(\mathbf{r}) \middle| \mathrm{e}^{\mathrm{i}\chi(\mathbf{b}-\mathbf{b}_r)} - 1 \middle| \Phi_i(\mathbf{r}) \right\rangle$$

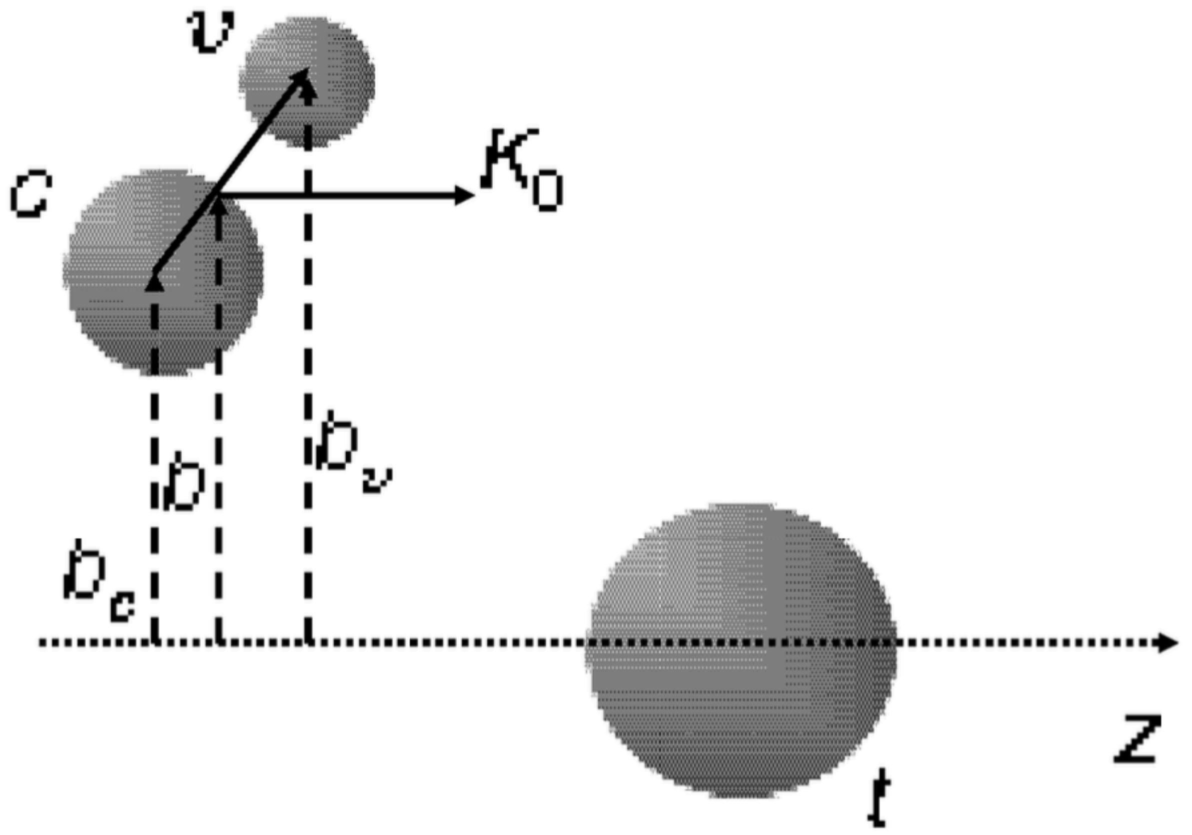
When f=i, it's the elastic process. And here, define the few-body eikonal phase

$$\chi\left(\mathbf{b}-\mathbf{b}_r\right) \equiv \sum_{j=1}^{n}\chi\left(\mathbf{b}-\mathbf{b}_{r_j}\right)$$

For one particle, it can written by

$$S_i(\mathbf{b_i}) = \exp(i\chi(\mathbf{b_i}))$$

S matrix of (d, px) is given by



$$S(b) = S_p(b)S_n(b)$$

$$\chi(b) = \chi_{p\,\mathrm{A}}\left(b_p\right) + \chi_{n\,\mathrm{A}}\left(b_n\right)$$

Where $\overrightarrow{b}_p = \overrightarrow{b} + \frac{1}{2}\vec{r}_\perp$ and $\overrightarrow{b}_n = \overrightarrow{b} - \frac{1}{2}\vec{r}_\perp$

So the cross section can be written by,

Glauber Model

$$\sigma_R = \int d^2\overrightarrow{b}\left[1 - \left|\int d^3\vec{r}\left|\psi_{00}(\vec{r})\right|^2 S_p\left(b_p\right) S_n\left(b_n\right)\right|^2\right]$$

$$\sigma_{\mathrm{STR}}^p = \int d^2\overrightarrow{b}\int d^3\vec{r}\left|\psi_{00}(\vec{r})\right|^2\left|S_n\left(b_n\right)\right|^2\left(1 - \left|S_p\left(b_p\right)\right|^2\right)$$

$$\sigma_{\mathrm{STR}}^n = \int d^2\overrightarrow{b}\int d^3\vec{r}\left|\psi_{00}(\vec{r})\right|^2\left|S_p\left(b_p\right)\right|^2\left(1 - \left|S_n\left(b_n\right)\right|^2\right)$$

$$\sigma_{\mathrm{EB}} = \int d^2\overrightarrow{b}\left[\int d^3\vec{r}\left|\psi_{00}(\vec{r})\right|^2\left|S_p\left(b_p\right) S_n\left(b_n\right)\right|^2 - \left|\int d^3\vec{r}\left|\psi_{00}(\vec{r})\right|^2 S_p\left(b_p\right) S_n\left(b_n\right)\right|^2\right]$$

Glauber Model

The double differential cross section is given by[2-4]

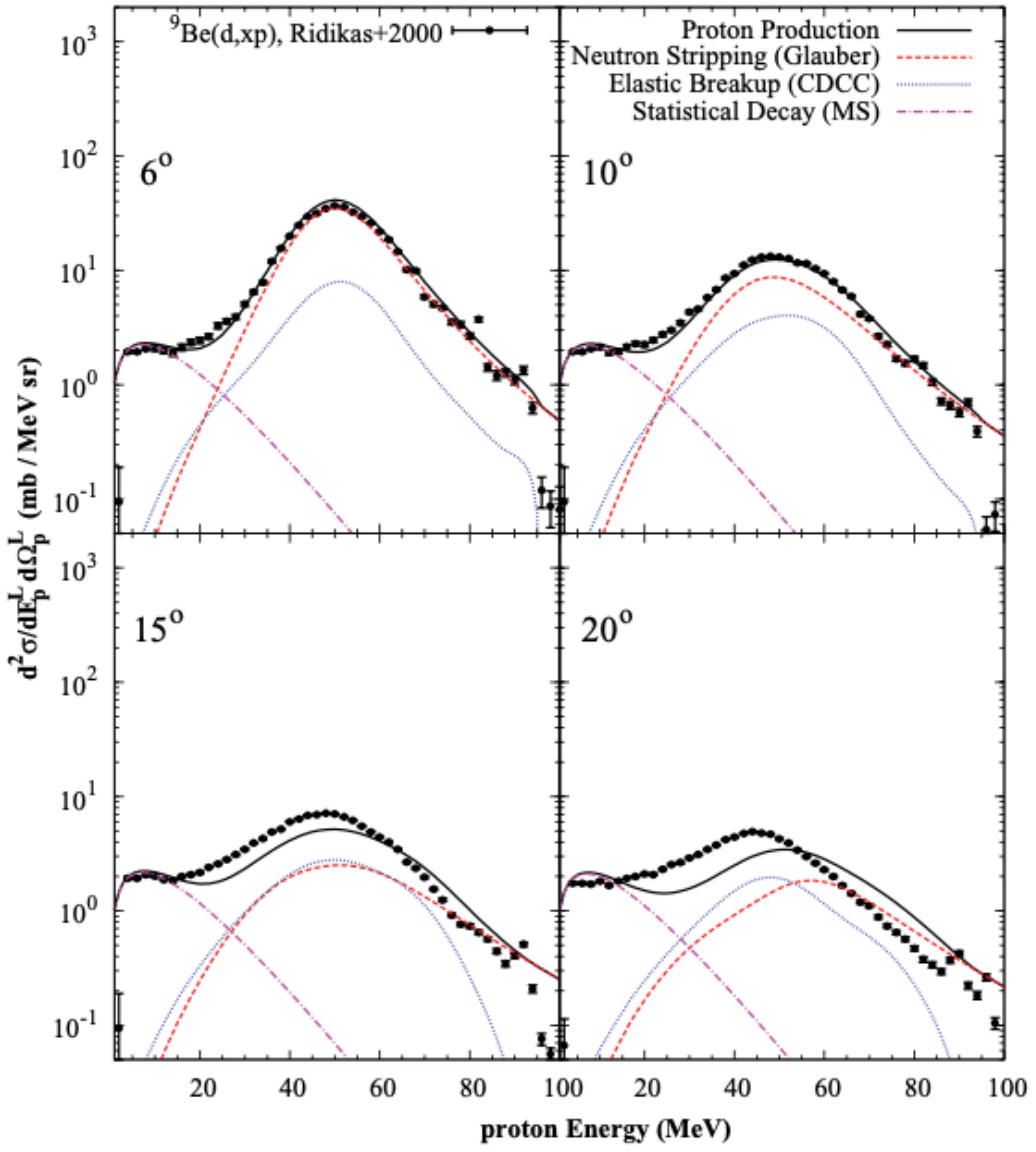
$$\frac{d^3\sigma^p_{\mathrm{STR}}}{d^3k^{\mathrm{C}}_n} = \frac{1}{(2\pi)^3}\int d^2\overrightarrow{b}_p \left\{ \left[1 - \left| S_p\left(b_p\right)\right|^2 \right] \times \left| \int d^3\vec{r} e^{-i\vec{k}^c_n\cdot\vec{r}} S_n\left(b_n\right)\psi_{00}(\vec{r}) \right|^2 \right\}$$

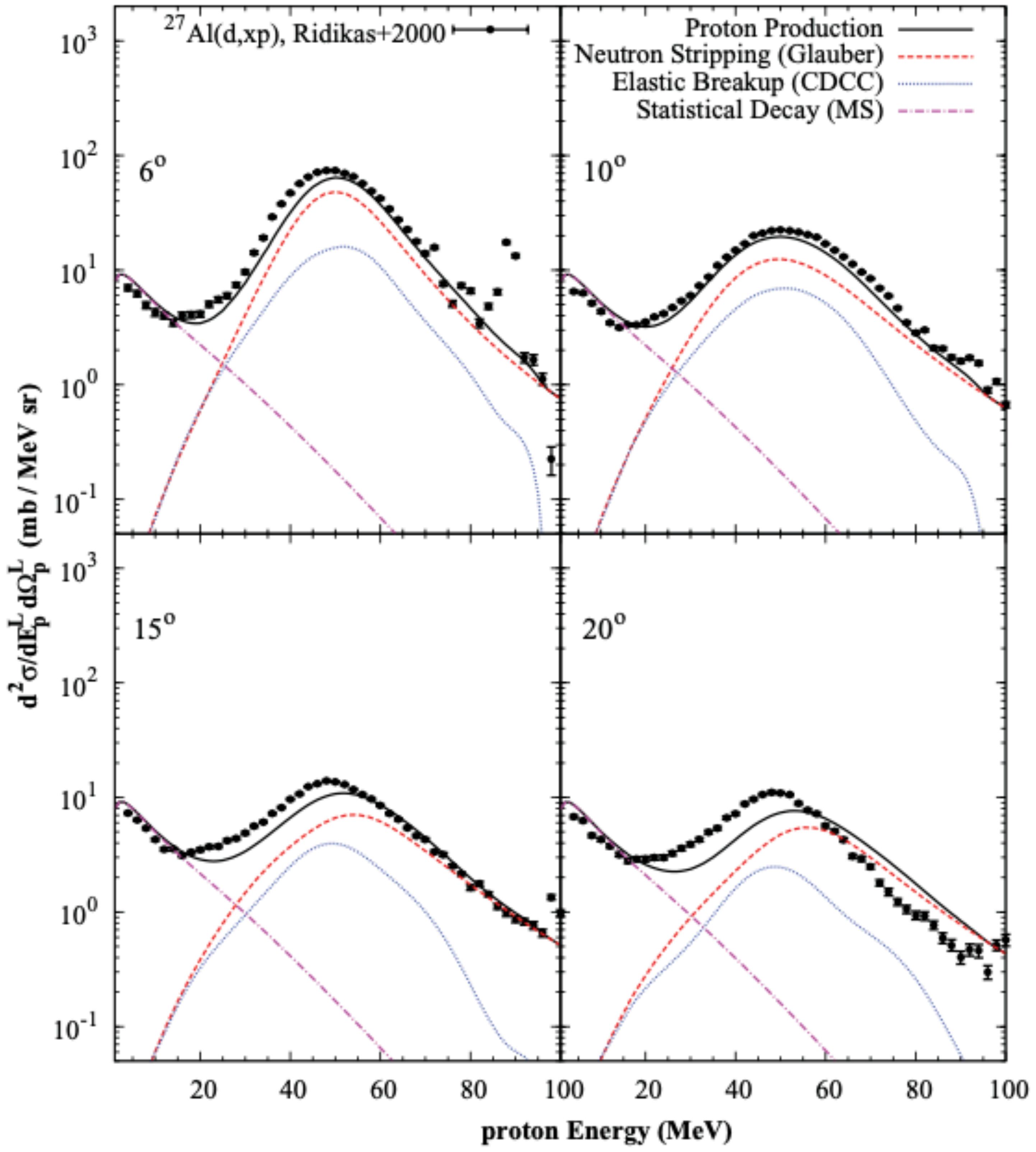
$$\left.\frac{d^2\sigma^p_{\mathrm{STR}}}{dE^L_n d\Omega^L_n}\right|_{\mathrm{Glauber}} = \frac{m_n k^L_n}{\hbar^2}\frac{d^3\sigma^p_{\mathrm{STR}}}{d^3k^C_n}$$

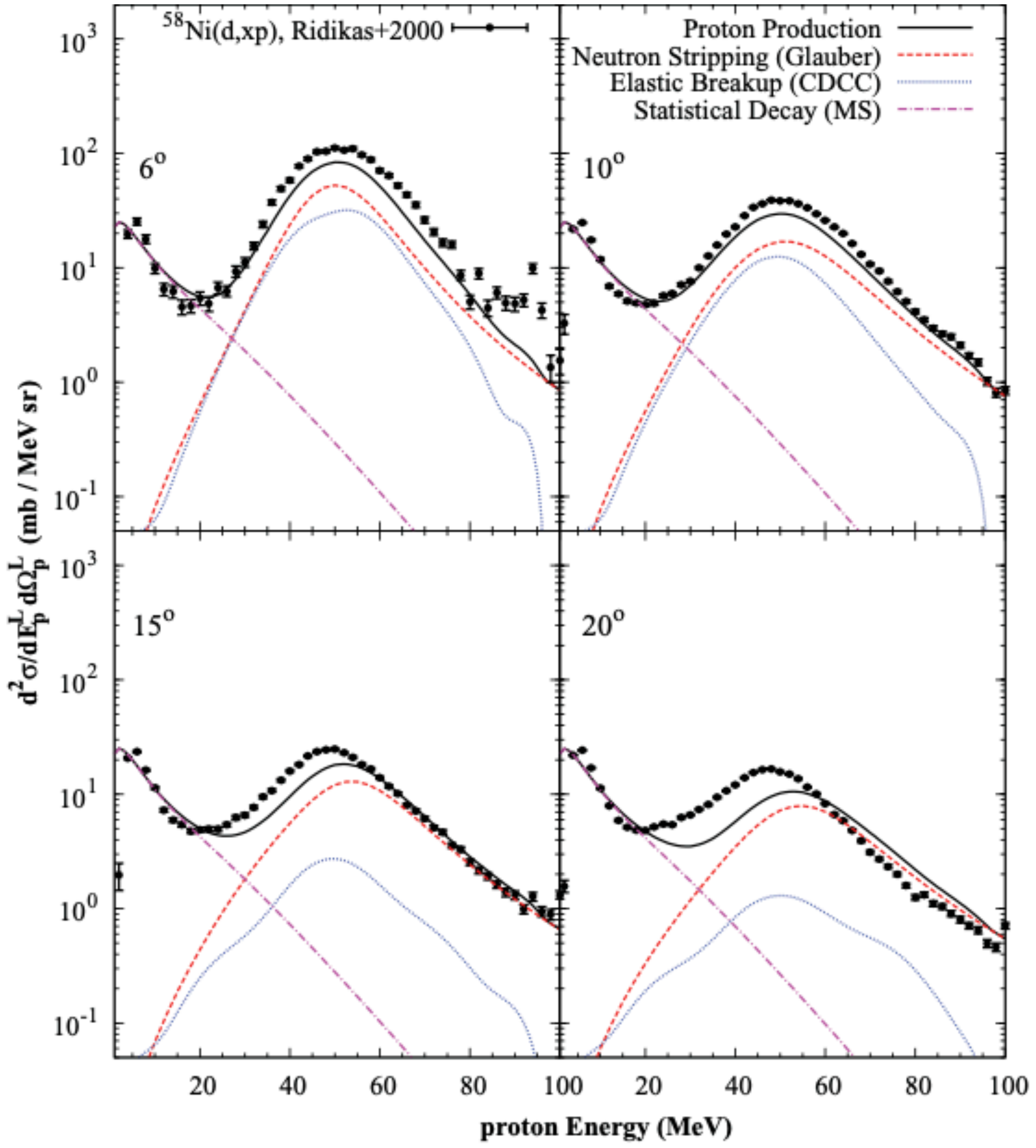
[2] K. Hencken, G. Bertsch, and H. Esbensen, Phys. Rev. C 54, 3043 (1996).
[3] M. S. Hussein and K. W. McVoy, Nucl. Phys. A445, 124 (1985).
[4] P. G. Hansen and J. A. Tostevin, Annu. Rev. Nucl. Part. Phys. **53**, 219 (2003).

Glauber Model

There are two disadvantages in the Glauber model, one is in the big emitting angles.



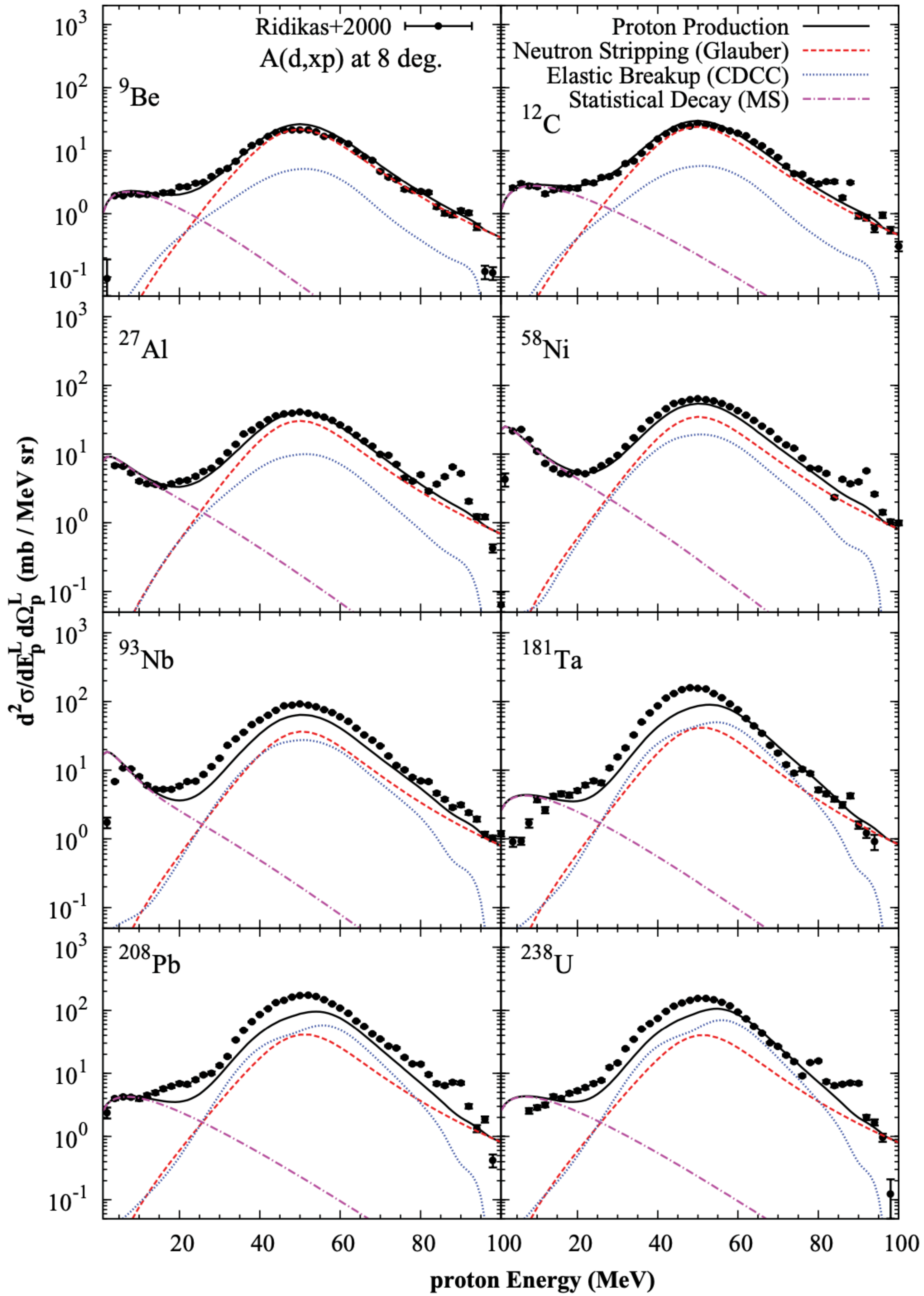




[5] T. Ye, S. Hashimoto, Y. Watanabe et al., Phys. Rev. C84, 054606 (2011).

Glauber Model

The other one is in the heavy elements. The Glauber model makes a low prediction about the double differential cross-section.



[5] T. Ye, S. Hashimoto, Y. Watanabe et al., Phys. Rev. C84, 054606 (2011).

Result

We compare the result with the Glauber model calculated by the Nakayama.