Unparticle physics and universality

Hans-Werner Hammer
Technische Universität Darmstadt

Tangram Nuclear Theory Seminar, Feb. 17, 2022

Outline

TECHNISCHE UNIVERSITÄT DARMSTADT

- Universality and the unitary limit
- Schrödinger symmetry
- Nuclear reactions with neutrons
- Neutral charm mesons and the $X(3872)$
- Summary and Outlook

References:

HWH, **D.T. Son**, Proc. Nat. Acad. Sci. **118**, e2108716118 (2021) [arXiv:2103.12610]
Braaten, HWH, Phys. Rev. Lett. **128**, 032002 (2022) [arxiv:2107.02831]

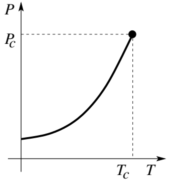
Universality

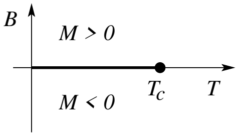
Universality: Physical systems with different short-distance behavior exhibit identical behavior at large distances

Universality

Universality: Physical systems with different short-distance behavior exhibit identical behavior at large distances

- Condensed matter systems near critical point





$$\rho_{liq/gas}(T) - \rho_c \longrightarrow \pm A(T_c - T)^{\beta}$$

Liquid-gas system

$$M_0(T) \longrightarrow A'(T_c - T)^{\beta}$$

Ferromagnet (one easy axis)

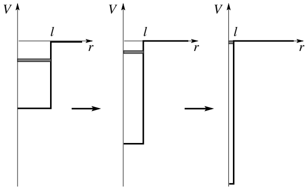
- Universality class determines critical exponents: $\beta = 0.325$
- Scale invariance (often conformal invariance)

Physics Near the Unitary Limit

TECHNISCHE UNIVERSITÄT DARMSTADT

- Consider short-ranged, resonant S-wave interactions
- Unitary limit: $a \to \infty, \ell \sim r_e \to 0$

$$\mathcal{T}_2(k,k) \propto \Big[\underbrace{k \cot \delta}_{-1/a + r_e k^2/2 + \ldots} - ik \Big]^{-1} \sim i/k$$



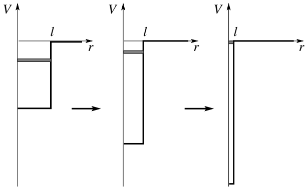
  - Scattering amplitude scale invariant, saturates unitarity bound

Physics Near the Unitary Limit

TECHNISCHE UNIVERSITÄT DARMSTADT

- Consider short-ranged, resonant S-wave interactions
- Unitary limit: $a \to \infty, \ell \sim r_e \to 0$

$$\mathcal{T}_2(k,k) \propto \Big[\underbrace{k \cot\delta}_{-1/a + r_e k^2/2 + \ldots} - ik \Big]^{-1} \sim i/k$$



  - Scattering amplitude scale invariant, saturates unitarity bound

- Many-body challenge (Bertsch, 1999)

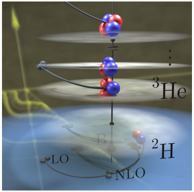
  Ground state of a many-body system of spin-1/2 fermions in unitary limit? Stability?

- Density $n = \frac{k_F^3}{3\pi^2}$ is only scale $\Rightarrow$ $E = \xi E_F\,,$ $\xi \approx 0.37$

Physics Near the Unitary Limit

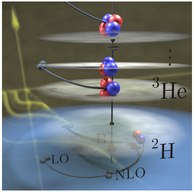
TECHNISCHE UNIVERSITÄT DARMSTADT

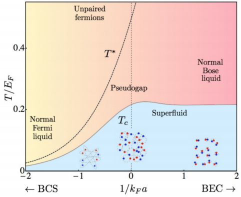
- Unitary limit is relevant for many physical systems
  - Ultracold atoms (tunable interaction)
    cf. Braaten, HWH, Phys. Rep. **428**, 259 (2006)
  - Light nuclei and halos
    cf. König, Grießhammer, HWH, van Kolck,
    PRL **118**, 202501 (2017)



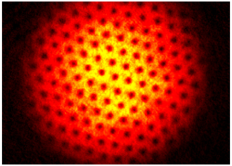
Physics Near the Unitary Limit

- Unitary limit is relevant for many physical systems
  - Ultracold atoms (tunable interaction)
    cf. Braaten, HWH, Phys. Rep. **428**, 259 (2006)
  - Light nuclei and halos
    cf. König, Grießhammer, HWH, van Kolck,
    PRL **118**, 202501 (2017)
  - BEC/BCS crossover, neutron matter, ...
    cf. Schäfer, Baym, PNAS **118**, e2113775118 (2021)

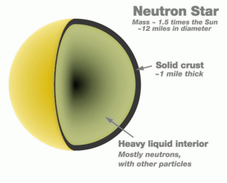




BCS BEC Crossover - Randeria, Taylor, 2014



Zwierlein group



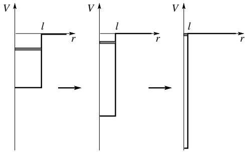
Robert Schulze CC BY-SA 3.0, via Wikimedia Commons

Physics Near the Unitary Limit

TECHNISCHE UNIVERSITÄT DARMSTADT

- Consider short-ranged, resonant S-wave interactions
- Unitary limit: $a \to \infty, \ell \sim r_e \to 0$

$$\mathcal{T}_2(k,k) \propto \Big[\underbrace{k \cot \delta}_{-1/a + r_e k^2/2 + \ldots} - ik \Big]^{-1} \sim i/k$$



  - Scattering amplitude scale invariant, saturates unitarity bound

- System has also (non-relativistic) conformal symmetry
  Mehen, Stewart, Wise, PLB **474**, 145 (2000); Nishida, Son, PRD **76**, 086004 (2007); ...
- Exploit approximate conformal symmetry for nuclear reactions with neutrons

$$\underbrace{1/(ma^2)}_{0.1\,\text{MeV}} \ll E_n^{cms} \ll \underbrace{1/(mr_e^2)}_{5\,\text{MeV}}$$

Schrödinger symmetry

TECHNISCHE UNIVERSITÄT DARMSTADT

- Non-relativistic conformal symmetry: Schrödinger symmetry
  - Galilei symmetry

    space + time translations

    rotations

    Galilei boosts
  - Scale transformations

$$\boldsymbol{x} \to e^{\lambda}\boldsymbol{x}, \qquad t \to e^{2\lambda}t, \qquad \psi \to e^{-\lambda\Delta}\psi$$

  - Special conformal transformations

$$\boldsymbol{x} \to \frac{\boldsymbol{x}}{1+\xi t}, \qquad t \to \frac{t}{1+\xi t}, \qquad \psi \to \psi' = \ldots$$

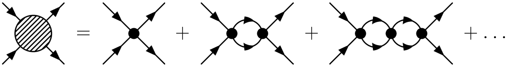
$\Rightarrow$ preserves angles

- 12 Parameters
- Generators: H, $\boldsymbol{P}$, $\boldsymbol{L}$, $\boldsymbol{K}$, D, C, satisfy Schrödinger algebra

Unitary limit

TECHNISCHE UNIVERSITÄT DARMSTADT

- Spin-1/2 Fermions with zero-range interactions ($|a| \gg r_e$)



- Renormalization group equation: $\Lambda \frac{d}{d\Lambda}\tilde{g}_2 = \tilde{g}_2(1+\tilde{g}_2)$
- Two fixed points:

$-\ \tilde{g}_2 = 0 \Leftrightarrow a = 0 \quad \Rightarrow$ no interaction

$-\ \tilde{g}_2 = -1 \Leftrightarrow 1/a = 0 \quad \Rightarrow$ unitary limit

$\Rightarrow$ conformal/Schrödinger symmetry

(Mehen, Stewart, Wise, PLB **474**, 145 (2000); Nishida, Son, PRD **76**, 086004 (2007); ...)

- Neutrons: $a \approx -18.6$ fm, $\quad r_e \approx 2.8$ fm

$\Rightarrow$ neutrons are approximately conformal

Unparticle physics

TECHNISCHE UNIVERSITÄT DARMSTADT

- (Relativistic) Unparticle (Georgi, Phys. Rev. Lett. **98**, 221601 (2007))
  - field ψ in relativistic conformal field theory
  - ψ characterized by scaling dimension Δ, massless
  - hidden conformal symmetry sector beyond Standard model (weakly coupled)
  - no evidence at LHC so far
    (CMS Coll., EPJC **75**, 235 (2015), PRD **93**, 052011, JHEP **03**, 061 (2017))
- (Non-relativistic) unparticle/unnucleus
  - non-relativistic analog of Georgi's unparticle
  - field ψ in non-relativistic conformal field theory
    (cf. Nishida, Son, Phys. Rev. D **76**, 086004 (2007))
  - ψ characterized by scaling dimension Δ and mass M
  - free field has $\Delta = 3/2 \quad \Longleftrightarrow \quad$ mass dimension

    $\Rightarrow$ lowest possible value (unitarity)
  - N neutrons are (approximate) unparticle with mass Nm_N and scaling dimension $\Delta = ?$

Conformal field theory

TECHNISCHE UNIVERSITÄT DARMSTADT

- Two-point function of primary field operator $\mathcal{U}$ ("unnucleus")

$$G_{\mathcal{U}}(t,\boldsymbol{x}) = -i\langle T\mathcal{U}(t,\boldsymbol{x})\mathcal{U}^\dagger(0,\boldsymbol{0})\rangle = C\,\frac{\theta(t)}{(it)^\Delta}\exp\left(\frac{iM\boldsymbol{x}^2}{2t}\right)$$

- Determined by symmetry up to overall constant C
- Two-point function in momentum space

$$G_{\mathcal{U}}(\omega,\boldsymbol{p}) = -C\left(\frac{2\pi}{M}\right)^{3/2}\Gamma\left(\frac{5}{2}-\Delta\right)\left(\frac{\boldsymbol{p}^2}{2M}-\omega\right)^{\Delta-\frac{5}{2}}$$

  - pole only for $\Delta = 3/2$ (free field)
  - branch cut for $\Delta > 3/2$
- General unnucleus (unparticle) does not behave like a particle

$\Rightarrow$ continuous energy spectrum

Conformal field theory

TECHNISCHE UNIVERSITÄT DARMSTADT

- Imaginary part of propagator

$$\mathrm{Im}\, G_{\mathcal{U}}(\omega, \boldsymbol{p}) \sim \begin{cases} \delta\left(\omega - \frac{\boldsymbol{p}^2}{2M}\right), & \Delta = \frac{3}{2}\,, \\ \left(\omega - \frac{\boldsymbol{p}^2}{2M}\right)^{\Delta - \frac{5}{2}} \theta\left(\omega - \frac{\boldsymbol{p}^2}{2M}\right), & \Delta > \frac{3}{2} \end{cases}$$

- Examples of un-nuclei
  - free field: $\mathcal{U} = \psi, \quad M = m_\psi, \quad \Delta = 3/2$
  - N free fields: $\mathcal{U} = \psi_1 \ldots \psi_N, \quad M = N m_\psi, \quad \Delta = 3N/2$
  - N interacting fields: $\mathcal{U} = \psi_1 \ldots \psi_N, \quad M = N m_\psi, \quad \Delta > 3/2$
- In our case: un-nucleus is strongly interacting multi-neutron state with

$$\underbrace{1/(ma^2)}_{0.1\,\mathrm{MeV}} \ll E_n^{cms} \ll \underbrace{1/(mr_e^2)}_{5\,\mathrm{MeV}}$$

Scaling dimension

TECHNISCHE UNIVERSITÄT DARMSTADT

- How to calculate scaling dimension Δ?

  (1) Δ can be obtained from field theory calculation

  (2) Δ can be obtained from operator state correspondence

Δ of primary operator = (Energy of state in HO)/$\hbar\omega$

(Nishida, Son, Phys. Rev. D **76**, 086004 (2007))

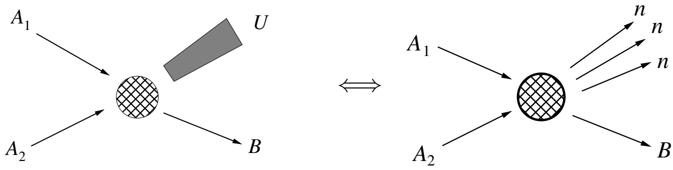
N	S	L	$\mathcal{O}$	Δ
2	0	0	$\psi_1\psi_2$	2
3	1/2	1	$\psi_1\psi_2\nabla_j\psi_2$	4.27272
3	1/2	0	$\psi_1\nabla_j\psi_2\nabla_j\psi_2$	4.66622
4	0	0	$\psi_1\psi_2\nabla_j\psi_1\nabla_j\psi_2$	5.07(1)
5	1/2	1	$\ldots$	7.6(1)

$\Rightarrow$ connection between Δ and energy of particles in a trap

Reactions with neutrons

TECHNISCHE UNIVERSITÄT DARMSTADT

- Application: High-energy nuclear reaction with final state neutrons



$$E_{\text{kin}} = (M_{A_1} + M_{A_2} - M_B - M_{\mathcal{U}})c^2 + \frac{p_{A_1}^2}{2M_{A_1}} + \frac{p_{A_2}^2}{2M_{A_2}} = E_B + E_{\mathcal{U}}$$

- Assumption: energy scale of primary reaction $\gg E_{\mathcal{U}} - \frac{\boldsymbol{p}^2}{2M_{\mathcal{U}}} = E_n^{cms}$
- Factorization: $\frac{d\sigma}{dE} \sim |\mathcal{M}_{primary}|^2 \, \mathrm{Im}\, G_{\mathcal{U}}(E_{\mathcal{U}}, \boldsymbol{p})$
- Reproduces Watson-Migdal treatment of FSI for $2n$

  (Watson, Phys. Rev. **88**, 1163 (1952); Migdal, Sov. Phys. JETP **1**, 2 (1955))

Reactions with neutrons

TECHNISCHE UNIVERSITÄT DARMSTADT

- Two ways to do experiments
  - (a) detect recoil particle B

$$\frac{d\sigma}{dE} \sim (E_0 - E_B)^{\Delta - 5/2}, \qquad E_0 = (1 + M_B/M_{\mathcal{U}})^{-1} E_{\text{kin}}$$

  - (b) detect all final state particles including neutrons

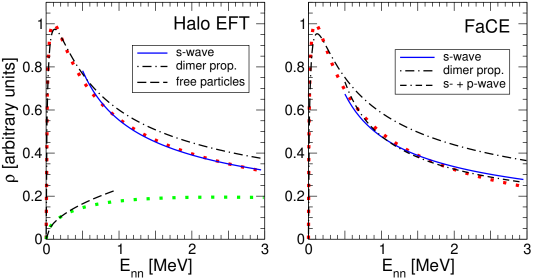
$$\frac{d\sigma}{dE} \sim (E_n^{cms})^{\Delta - 5/2}$$

- Consistent with previous experiments for ${}^3\mathrm{H}(\pi^-, \gamma)3n$
  (Miller et al., Nucl. Phys. A **343**, 347 (1980))
- Two few events in recent tetraneutron experiment: ${}^4\mathrm{He}({}^8\mathrm{He}, {}^8\mathrm{Be})4n$
  (Kisamori et al., Phys. Rev. Lett. **116**, 052501 (2016))

Reaction calculations

TECHNISCHE UNIVERSITÄT DARMSTADT

- Two-neutron spectrum for $^{6}\mathrm{He}(p, p\alpha)2n$ (Göbel et al., Phys. Rev. C **104**, 024001 (2021).)



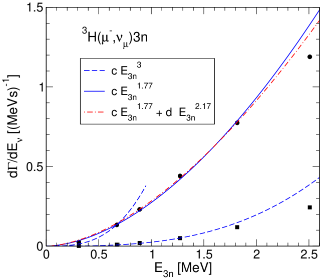
- Can be understood from dimer propagator ($\Delta = 2$)

$$G_d(E_{nn}, \mathbf{0}) \sim \frac{1}{1/a + i\sqrt{mE_{nn}}} \quad \Rightarrow \quad \mathrm{Im}\, G_d(E_{nn}, \mathbf{0}) \sim \frac{\sqrt{E_{nn}}}{(ma^2)^{-1} + E_{nn}}$$

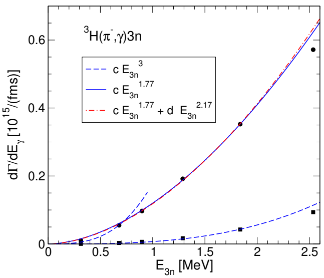
Reaction calculations

TECHNISCHE UNIVERSITÄT DARMSTADT

- Radiative muon/pion capture on the triton (AV18 + UIX)



Golak et al., PRC **98**, 054001 (2018)



Golak et al., PRC **94**, 054001 (2016)

- Un-nucleus behavior prediction

$$\frac{d\Gamma}{dE} \sim (E_{3n})^{4.27272-5/2} \sim (E_{3n})^{1.77272}, \qquad 0.1\,\text{MeV} \ll E_{3n} \ll 5\,\text{MeV}$$

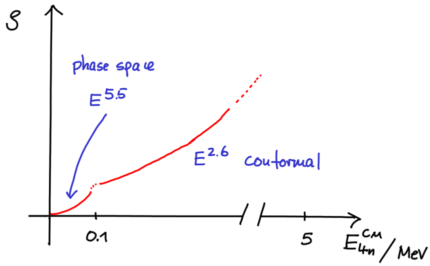
New experiments

- New experiments in complete kinematics at RIBF/RIKEN
- Measurement of a_{nn} in ${}^{6}\mathrm{He}(p, p\alpha)2n$
  (T. Aumann et al., NP2012-SAMURAI55R1 (2020))
- Search for tetraneutron resonances in ${}^{8}\mathrm{He}(p, p\alpha)4n$
  (S. Paschalis et al., NP1406-SAMURAI19R1 (2014))

  - unnucleus prediction for point source:

    $\rho \sim (E_{4n})^{5.07-5/2} \sim (E_{4n})^{2.57}$

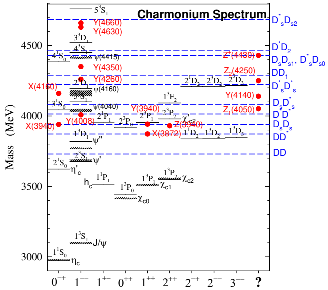
    $0.1\,\text{MeV} \ll E_{4n} \ll 5\,\text{MeV}$



Neutral charm mesons and $X(3872)$

TECHNISCHE UNIVERSITÄT DARMSTADT

- New $c\bar{c}$ states at B factories: X, Y, Z
  (cf. Godfrey, arXiv:0910.3409)
  - Challenge for understanding of QCD
  - Unitary limit relevant?
- $X(3872)$ (Belle, CDF, BaBar, D0, LHCb)
- Nature of $X(3872)$?
  - $\bar{D}^0 D^{0*}$-molecule, tetraquark, charmonium hybrid, ...



$m_X = (3871.65 \pm 0.06)$ MeV, $\Gamma = (1.19 \pm 0.21)$ MeV, $J^{PC} = 1^{++}$ (PDG 2021)

- Assumption: $X(3872)$ is weakly-bound D^0-$\bar{D}^{0*}$-molecule

$\Rightarrow$ $|X\rangle = (|D^0\bar{D}^{0*}\rangle + |\bar{D}^0 D^{0*}\rangle)/\sqrt{2}$, $B_X = (0.07 \pm 0.12)$ MeV $\approx 1/(2\mu_{DD^*} a^2)$

$\Rightarrow$ universal properties (Braaten et al., 2003-2008; ...)

Neutral charm mesons and $X(3872)$

TECHNISCHE UNIVERSITÄT DARMSTADT

- Approximate unparticles of three D^0/D^{0*} mesons
- Interaction of $X(3872)$ with $D^0, \bar{D}^0, D^{0*}, \bar{D}^{0*}$ determined by large a

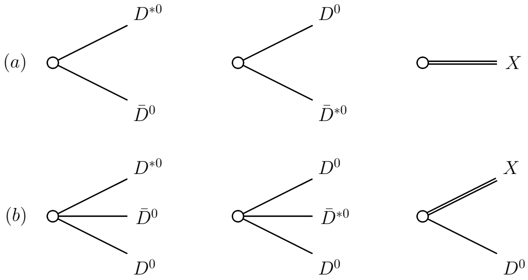
(Canham, HWH, Springer, PRD **80**, 014009 (2009))

$$a_{D^0X} = -9.7a \qquad a_{D^{*0}X} = -16.6\,a$$

- Richer structure because of $X(3872)$ (bound state)

two charm mesons

three charm mesons



Neutral charm mesons and $X(3872)$

TECHNISCHE UNIVERSITÄT DARMSTADT

- Approximate unparticles of three D^0/D^{0*} mesons
- Interaction of $X(3872)$ with D^0, $\bar{D}^0$, D^{0*}, $\bar{D}^{0*}$ determined by large a

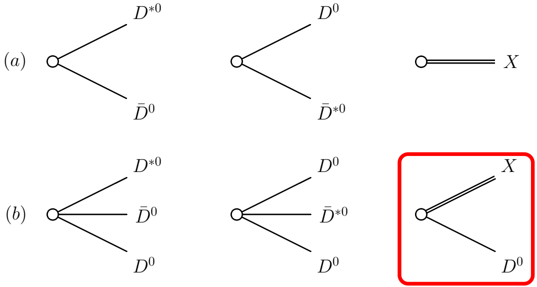
(Canham, HWH, Springer, PRD **80**, 014009 (2009))

$$a_{D^0X} = -9.7a \qquad a_{D^{*0}X} = -16.6\,a$$

- Richer structure because of $X(3872)$ (bound state)

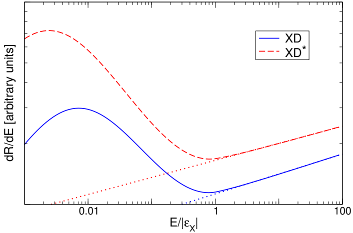
two charm mesons

three charm mesons

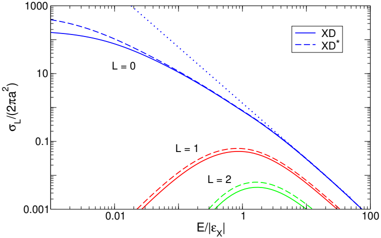


Scaling Behavior

- Universal scaling for unparticles of three neutral charm mesons
  (Braaten, HWH, Phys. Rev. Lett. **128**, 032002 (2022) [arXiv:2107.02831])



XD point production



XD elastic scattering

$$\frac{dR}{dE} \sim \left(E^{-(\Delta_1+\Delta_2-\Delta_3)/2}\right)^2 \sqrt{E} \approx E^{0.1}$$

$$\sigma \sim E^{-1.6}$$

$\Delta_1 = 3/2, \quad \Delta_2 = 2, \quad \Delta_3 \approx 3.10119/3.08697$

Summary and Outlook

TECHNISCHE UNIVERSITÄT DARMSTADT

- Universality in the unitary limit

  $\Rightarrow$ (approximate) conformal symmetry
  $\Rightarrow$ power law behavior of observables determined by Δ

- Application to high-energy nuclear reactions with neutrons
- Model-independent constraints on nuclear reactions
- Connection between reactions & properties of trapped particles

Summary and Outlook

TECHNISCHE UNIVERSITÄT DARMSTADT

- Universality in the unitary limit

  $\Rightarrow$ (approximate) conformal symmetry
  $\Rightarrow$ power law behavior of observables determined by Δ

- Application to high-energy nuclear reactions with neutrons
- Model-independent constraints on nuclear reactions
- Connection between reactions & properties of trapped particles
- Other applications & extensions
  - Two-component Fermions in ultracold atom physics
  - Neutral charm mesons
  - Systems with the Efimov effect?

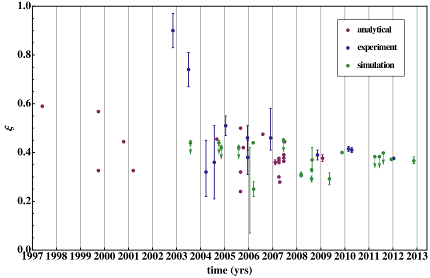
    $\Rightarrow$ bosonic atoms, nucleons, α particles
    $\Rightarrow$ complex scaling dimensions
    $\Rightarrow$ scale symmetry broken

Additional Slides

TECHNISCHE UNIVERSITÄT DARMSTADT

Physics Near the Unitary Limit

- Ground state energy: $E = \xi E_F \,, \qquad \xi \approx 0.37$
- Difficult non-perturbative problem
  - diagrammatic resummation, ϵ-expansion, fixed-node Greens function MC, auxilliary field MC, quantum simulation w/ ultracold atoms, ...

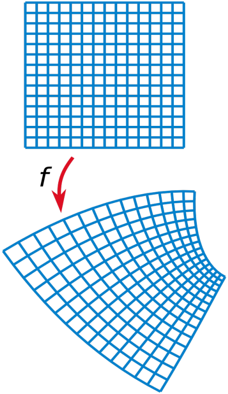


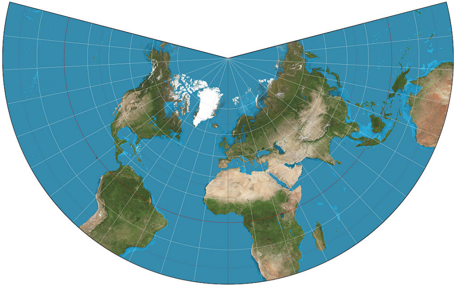
- Lattice Monte Carlo: $\xi = 0.366^{+0.016}_{-0.011}$

  Endres, Kaplan, Lee, Nicholson, Phys. Rev. A **87**, 023615 (2013)

Conformal mapping

TECHNISCHE UNIVERSITÄT DARMSTADT





Lambert conformal conic projection

Oleg Alexandrov, via Wikimedia Commons; Strebe CC BY-SA 3.0, via Wikimedia Commons