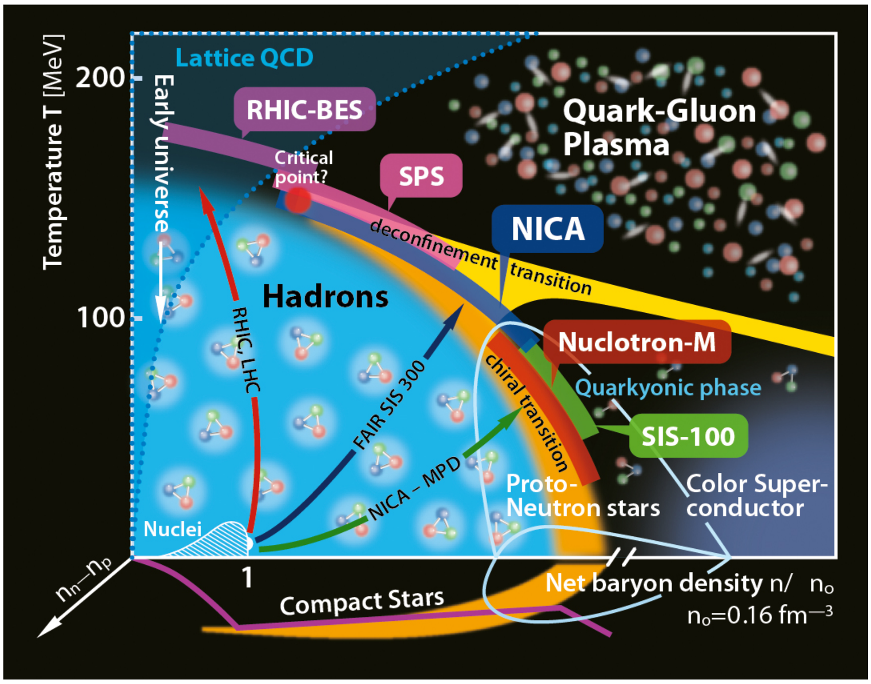
Nuclear Equation of State

Introduction

The nuclear equation of state (**EoS**) describes the **energy per nucleon** as a function of the neutron (ρ_n) and proton (ρ_p) densities of an **uniform and infinite system at zero temperature** that interact only via the residual strong interaction, or nuclear force. (*Neglect the Coulomb interaction*)

Ideal system : in the interior of nuclei and in cold neutron stars.



Nuclear energy density functional (**EDF**) constitutes a unique tool to reliably and consistently access bulk ground state and collective excited state properties of atomic nuclei along the nuclear chart as well as the **EoS around saturation density**.

Phenomenology

saturation density $\rho_0 = 0.16\ \mathrm{fm}^{-3}$

Ground state

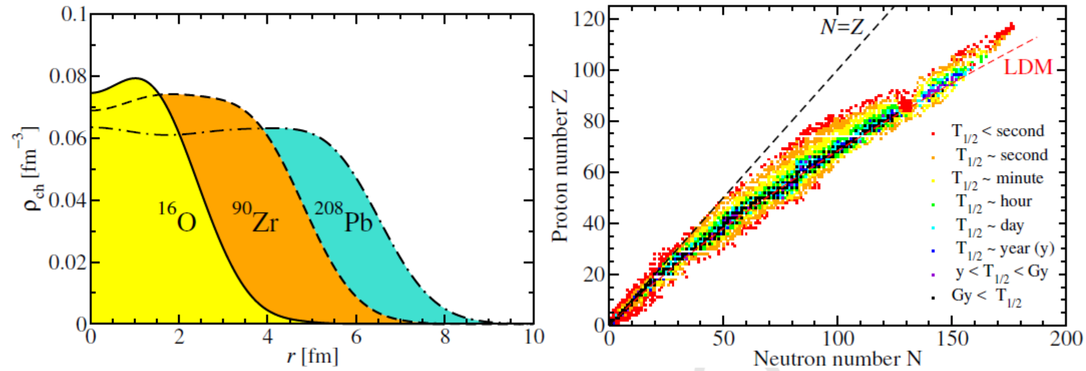


Figure 1: Left panel: experimental charge density ($\rho_{\rm ch}$) as a function of the distance to the center of the nucleus (r, assuming spherical symmetry) derived from available data on elastic electron scattering for ^{16}O, ^{90}Zr and ^{208}Pb [16]. Right panel: Chart of nuclides classified by half-life. Data taken from NUBASE16 [17].

The density in the interior of very different nuclei is almost the same $\rho_{0p} = 0.07$ fm^{-3} $\rightarrow$ a saturation mechanism (equilibrium) $\rightarrow \langle r_p^2\rangle^{1/2} \approx 1.2Z^{1/3}$.
Stable nuclei are not far to be symmetric + assuming the nuclear force is isospin invariant $\rightarrow \rho_{0p} \approx \rho_{0n} \rightarrow \langle r^2\rangle^{1/2} \approx 0.9A^{1/3}$

The most famous macroscopic model : **liquid drop model**
(*neglecting pairing effects among others*)

$$M(A,Z) = m_p Z + m_n(A-Z) - B(A,Z), \quad \text{where}$$
$$B(A,Z) = (a_V - a_S A^{-1/3})A - a_C \frac{Z(Z-1)}{A^{1/3}} - (a_A - a_{SA}A^{-1/3})\frac{(A-2Z)^2}{A}$$

Symmetry energy term

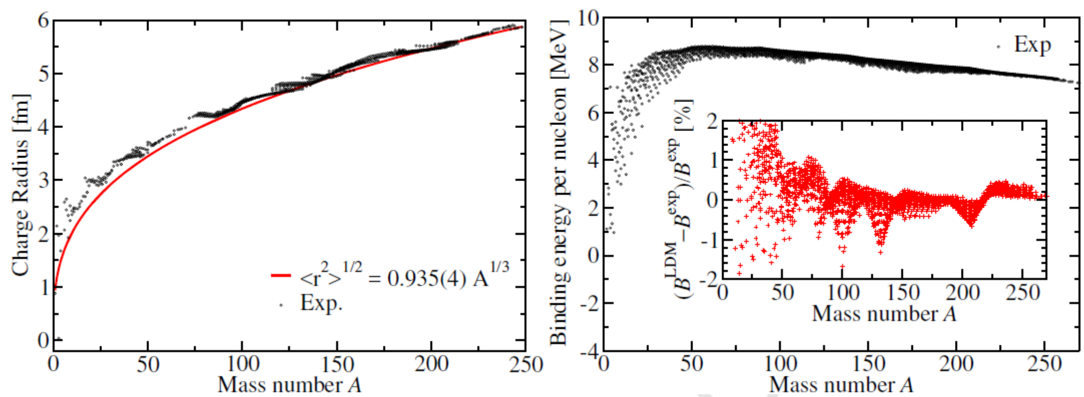


Figure 2: Left panel: root mean square charge radii of all measured nuclei as a function of the mass number A reported in Ref.[18]. Right panel: binding energy per particle of all measured nuclei by 2016 that where compiled in AME16 [21]. The inset shows the relative difference between the liquid drop model described in the text and the experimental data in %.

Liquid drop model $\rightarrow$ diffuse surface [1] $\rightarrow$ droplet model (DM) $\rightarrow$ prediction of neutron skin thickness ($\Delta r_{\rm np} \equiv \langle r_n^2\rangle^{1/2} - \langle r_p^2\rangle^{1/2}$)

$$\Delta r_{\rm np}^{\rm DM} = \frac{2\langle r^2\rangle^{1/2}}{3}\frac{a_{AS}}{a_A}(I - I_C) + \Delta r_{\rm np}^C$$

Relative neutron excess : $I \equiv (N-Z)/A$; Coulomb correction I_C; shift in the neutron skin due to the Coulomb interaction : Δr_{np}^C

As isospin asymmetries $\delta \equiv (\rho_n - \rho_p)/(\rho_n + \rho_p)$ are relatively small, an expansion for $\delta \to 0$:

$$e(\rho,\delta) = e(\rho,0) + S(\rho)\delta^2 + \mathcal{O}[\delta^4], \quad \text{where}$$

$$\text{EoS of symmetric matter: } e(\rho,0) = e(\rho_0,0) + \frac{1}{2}K_0\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \mathcal{O}[(\rho-\rho_0)^3],$$

$$\text{symmetry energy: } S(\rho) \equiv \frac{\partial e(\rho,\delta)}{\partial \delta}|_{\delta=0} = J + L\frac{\rho-\rho_0}{3\rho_0} + \frac{1}{2}K_{\mathrm{sym}}\left(\frac{\rho-\rho_0}{3\rho_0}\right)^2 + \mathcal{O}[(\rho-\rho_0)^3]$$

Incompressibility of symmetric nuclear matter K_0; symmetry energy at saturation $J \equiv S(\rho_0)$; the slope of the symmetry energy at saturation L; the incompressibility (or curvature) of the symmetry energy at saturation K_{sym}

Neutron matter EoS : $\delta = 1$, $S(\rho) \approx e(\rho,1) - e(\rho,0)$

References

1. W.D Myers and W.J Swiatecki. The nuclear droplet model for arbitrary shapes. Annals of Physics, 84(1):186 – 210, 1974. ↩