Resonance phenomena: from compound nucleus decay to proton radioactivity
(EPJ manuscript No.)

Jizheng,Bo

September 26, 2023

Index

1. Introduction
2. Resonance properties
   - Single model
   - Gamow complex energy
   - P_{well} and phase shift
3. Symmetry dependence of mean field potential
4. Summary

Introduction

Intro

- The low-lying excited states of exotic nuclei near the drip line are in the continuum.
- Even the ground states are resonances when beyond the drip line.
- Resonances play an important role in the structure of exotic nuclei.

Resonance properties

single model

One-dimensional model

Consider a simple one-dimensional potential with some important features:

- A potential well with a finite depth V_0. $(x < x_0)$
- A barrier with a height of V_b. $(x_0 < x < x_b)$
- A continuum → the potential out of the barrier equal to zero. $(x > x_b)$

- The solutions in different areas respectively:

$$\psi(x) = \mathcal{A}sin(k_0 x) + \mathcal{B}cos(k_0 x) = \mathcal{A}sin(k_0 x) \quad x < x_0 \tag{1}$$

$$\psi(x) = \mathcal{E}exp(\kappa_\infty x) + \mathcal{F}exp(-\kappa_\infty x) = \mathcal{F}exp(-\kappa_\infty x) \quad x > x_b \tag{2}$$

$$\psi(x) = \mathcal{C}exp(\kappa_b x) + \mathcal{D}exp(-\kappa_b x) \quad x_0 < x < x_b \tag{3}$$

with $k_0 = \sqrt{2m(E+V_0)}/\hbar$, $\kappa_\infty = \sqrt{2m|E|}/\hbar$, $\kappa_b = \sqrt{2m(V_b - E)}/\hbar$.

Resonance properties

single model

Solution of the *Schrödinger* equation

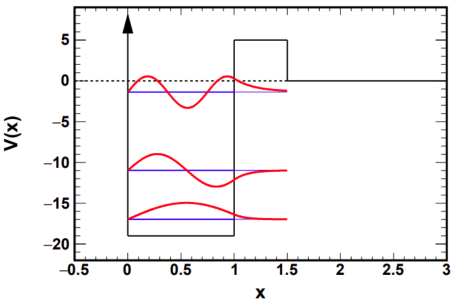

Figure 1: The solution with the example simple potential mentioned before

- blue horizontal lines: bound energies
- red lines: the corresponding wavefunctions

Resonance properties

from bound to resonance

What if we decrease the depth of the well V_0?

$\rightarrow$ The least-bound level will cross zero energy, what kind of state?

Resonance properties

from bound to resonance

Quantum tunneling

- $0 < E < V_0$
- Each time the nucleon attacks the barrier, there is a probability of penetrating the barrier.
- The probability for the particle staying behind the barrier is reduced by a constant decreasing exponentially with time.

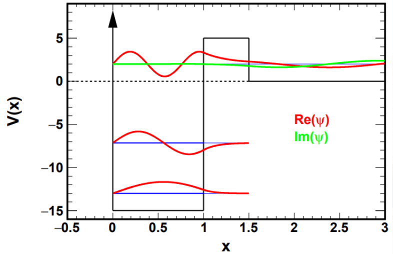

Figure 2: With a shallower well, the least-bound state becomes a reasonance state.

Resonance properties

Gamow complex energy

Exponential-decaying solutions of *Schrödinger* equation(by Gamow)

- time-dependent *Schrödinger* equation

$$i\hbar\frac{\partial}{\partial t}\Psi(x,t) = [-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(r)]\Psi(x,t)$$

- use a complex energy $E = E_r - i\Gamma/2$, the modulus squared of the wave function.

$$|\Psi(x,t)|^2 = exp(\frac{-\Gamma t}{\hbar})|\psi(x)|^2$$

- by Heisenberg's uncertainty principle we can define the width and lifetime.

$$\Delta E_r \Delta t \sim \hbar$$

- the state has a finite lifetime $\tau = \Delta t = \frac{\hbar}{\Gamma}$

Resonance properties

P_{well} and phase shift

The probability of being in the well

- Considering the potential in Figure.2, we define the probability as

$$P_{well} = \int_0^{x_0} |\psi(x)|^2 dx$$

- In addition, the wavefunction in the exterior region($x > x_b$)

$$\psi(x) = \frac{\sqrt{\mathcal{A}^2 + \mathcal{B}^2}}{2i} exp(-\delta)[exp(ik_\infty x + 2\delta) - exp(-ik_\infty x)]$$

- This leads to a phase shift of 2δ between the incoming and outgoing waves.

Resonance properties

P_{well} and phase shift

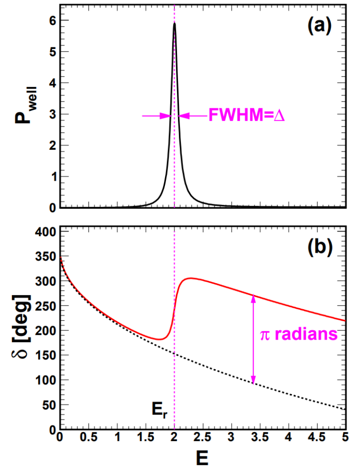

Figure 3: Energy dependence of the phase shift and P_{well}(the relative probability). At the resonance energy, the phase shift shows a sharp jump of π radians.

Resonance properties

P_{well} and phase shift

Breit-Wigner resonance

- The relative probability of getting a particle inside of the potential well has a strong peak → resonance.
- For narrow peaks, the centroid of a peak has the same value as E_r
- The peak's shape is given by Breit-Wigner form:

$$P_{well} \propto \frac{\Gamma^2}{(E - E_r)^2 + (\Gamma/2)^2}$$

- Γ: the full width at half maximum (FWHM) of the peak.
- The 'Γ' here is the same as that of the imaginary part of complex energy('$-\Gamma/2$').
- The phase shift shows a sudden change of magnitude π radians relative to the slowly varying background.

Symmetry dependence of mean field potential

We take the different kinds of nucleons into consideration.

the modification of the potential deriving from the two nucleon types

- keep the same number of nucleons but change the ratios of p to n
- the depth of the mean field

$$V_0 = V' + v_{sym}\frac{N-Z}{A}protons \quad (4)$$

$$= V' - v_{sym}\frac{N-Z}{A}neutrons \quad (5)$$

- This kind of symmetry dependence contributes about 50% of the symmetry energy.(emperical)
- The binding energy

$$E_{binding}(Z,A) = a_{vol}A - a_{sur}A^{2/3} - a_{Coul}\frac{Z^2}{A^{1/3}} - a_{asy}\frac{(N-Z)^2}{A}$$

involves the volume, surface, Coulomb, and symmetry terms.

Symmetry dependence of mean field potential

the importance of the symmetry force for the single-particle energies

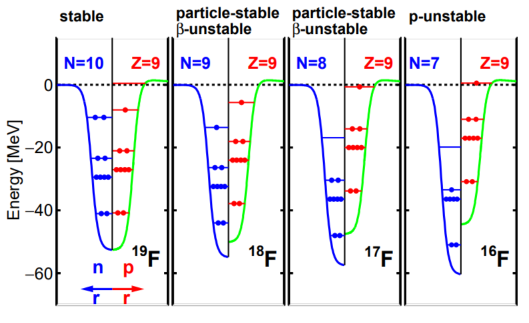

Figure 4: Schematic showing the evolution of neutron and proton single-particle levels with mass number A for fluorine isotopes.

- With the decrease of the number of neutrons, the energy level of protons increase. → making a proton resonance.

Summary

Main theoretical part of this article

1. A resonance state will decay with an exponential time distribution with a lifetime of $\hbar/\Gamma$.
2. Resonance can be detected in scattering experiments in that the cross section corresponds to the P_{well} with a FWHM of Γ.
3. A resonance has a sharp variation (which is a jump of π radians) on the phase shift when the energy scans across the resonance energy.
4. We can modified the number of one type of nucleons thus changing the mean field of each nucleon, making a proton or a neutron resonance.

- Next time we will cover the left part of this article.

Thank You!