2023.3.14 组会

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Dynamical eikonal

对于projectile，考虑其两体结构由core和fragment构成。

弹核内部的自由度由内部哈密顿量描述：

$$H_0 = \frac{p^2}{2\mu_{cf}} + V_{cf}(\boldsymbol{r}), \quad H_0\phi_0 = E_0\phi_0$$

三体哈密顿量的形式：

$$H = \frac{P^2}{2\mu} + H_0 + V_{cT}\left(\boldsymbol{R} - \frac{m_f}{m_P}\boldsymbol{r}\right) + V_{fT}\left(\boldsymbol{R} + \frac{m_c}{m_P}\boldsymbol{r}\right)$$

目的是求解三体的薛定谔方程： $H\Psi = E_T\Psi$

Dynamical eikonal

Eikonal ansatz:

$$\Psi(\boldsymbol{R},\boldsymbol{r}) = e^{iKZ}\hat{\Psi}(\boldsymbol{R},\boldsymbol{r})$$

K为相对波矢:

$$\frac{\hbar^2 K^2}{2\mu} = E_T - E_0$$

代入三体哈密顿量之后得到约化后的薛定谔方程:

$$\frac{\hbar^2}{2\mu}\Delta_R\hat{\Psi} + i\hbar v\frac{\partial\hat{\Psi}}{\partial Z} = \left(H_0 + V_{cT} + V_{fT} - E_0\right)\hat{\Psi}$$

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$$\frac{\hbar^2}{2\mu}\Delta_R\hat{\Psi} + i\hbar v\frac{\partial\hat{\Psi}}{\partial Z} = \left(H_0 + V_{cT} + V_{fT} - E_0\right)\hat{\Psi}$$

传统的eikonal近似包含两项：1、忽略二阶导数。2、绝热近似。

$$i\hbar v\frac{\partial\hat{\Psi}}{\partial Z} = (V_{cT} + V_{fT})\hat{\Psi} \qquad \hat{\Psi} = \exp\left[\int_{-\infty}^{z}\frac{U_{cT} + U_{fT}}{i\hbar v}dz\right]$$

本质上还是两体问题

直接完全忽略了projectile的自由度

Dynamical eikonal

$$\frac{\hbar^2}{2\mu}\Delta_R\hat{\Psi} + i\hbar v\frac{\partial\hat{\Psi}}{\partial Z} = \left(H_0 + V_{cT} + V_{fT} - E_0\right)\hat{\Psi}$$

Dynamical eikonal只忽略二阶导数

$$i\hbar v\frac{\partial\hat{\Psi}}{\partial Z} = \left(H_0 + V_{cT} + V_{fT} - E_0\right)\hat{\Psi}$$

引入类似于时间的变量: $vdt = dz$

"... is formally equivalent to the semiclassical time-dependent Schrodinger equation with straight-line trajectories [1]."

$$i\hbar\frac{\partial\hat{\Psi}}{\partial t} = \left(H_0 + V_{cT} + V_{fT} - E_0\right)\hat{\Psi}$$

[1] Baye, D., Capel, P., & Goldstein, G. (2005). Collisions of halo nuclei within a dynamical eikonal approximation. *Physical review letters*, *95*(8), 082502.

Dynamical eikonal

直接计算T矩阵：

$$T_{fi} = \left\langle e^{iK\cdot R}\phi_0(\boldsymbol{r}) \middle| V_{cT} + V_{fT} \middle| \Psi(\boldsymbol{R}, \boldsymbol{r}) \right\rangle$$

$$T'_{fi} = \left\langle e^{iK\cdot R}\chi^{(-)}(\boldsymbol{r}) \middle| V_{cT} + V_{fT} \middle| \Psi(\boldsymbol{R}, \boldsymbol{r}) \right\rangle$$

转化为做二维傅立叶变换：

$$T_{fi} = i\hbar v \left\langle e^{i\boldsymbol{K}\cdot\boldsymbol{R}}\phi_0(\boldsymbol{r}) \mid e^{iKZ}\frac{\partial}{\partial Z}\hat{\Psi}(\boldsymbol{R}, \boldsymbol{r}) \right\rangle$$

$$\approx i\hbar v \int d\boldsymbol{R} e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \frac{\partial}{\partial Z} \left\langle \phi_0(\boldsymbol{r}) \mid \hat{\Psi}(\boldsymbol{R}, \boldsymbol{r}) \right\rangle$$

Hussein-McVoy 模型

NEB的形式为:

$$\left.\frac{d^2\sigma}{dE_b d\Omega_b}\right|_{\mathrm{NEB}}^{\mathrm{HM}} = -\frac{2}{\hbar v_a}\rho_b\left(E_b\right)\left\langle \varphi_x^{\mathrm{HM}} \middle| W_x \middle| \varphi_x^{\mathrm{HM}} \right\rangle$$

x-A波函数的形式:

$$\varphi_x^{\mathrm{HM}}\left(\mathbf{r}_x\right) = \left\langle \mathbf{r}_x \mid \varphi_x^{\mathrm{HM}} \right\rangle = \left\langle \mathbf{r}_x \chi_b^{(-)} \mid \chi_a^{(+)} \phi_a \right\rangle$$

不需要求解耦合方程/计算source term

对整个模型空间进行了简化

Hussein-McVoy 模型

NEB的形式为:

$$\sigma_{\mathrm{NEB}}^{\mathrm{EHM}} = \frac{2}{v_{\mathrm{a}}}(2\pi)^3 \frac{E_x}{\hbar k_x} \int \mathrm{d}^3\mathbf{r}_b \, \mathrm{d}^3\mathbf{r}_x \left| \phi_a\left(\mathbf{r}_{bx}\right) \right|^2 \times \left| S_{bA}\left(b_b\right) \right|^2 \left[1 - \left| S_{xA}\left(b_x\right) \right|^2 \right].$$

"This equation has an appealing and intuitive form: the integrand contains the product of the probabilities for the core being elastically scattered by the target, $|S_{bA}|^2$, times the probability of the valence particle being absorbed,$(1 - |S_{xA}|^2)$."[2]

"depends only on the asymptotic properties, this is, the *S* matrices, of the interaction of *b* and *x* with the target. There is no sensitivity to the wave- functions in the interaction region. This is a result of the eikonal approximation..."

[2] Gómez-Ramos, M., Gómez-Camacho, J., Lei, J. *et al.* The Hussein–McVoy formula for inclusive breakup revisited. *Eur. Phys. J. A* **57**, 57 (2021).

Transfer to continuum 模型

"There it was shown to hold under the hypothesis that the breakup process is limited to peripheral projectile-target trajectories and that it is due mainly to the neutron interaction with the target potential."[2]

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[2] Gómez-Ramos, M., Gómez-Camacho, J., Lei, J. *et al.* The Hussein–McVoy formula for inclusive breakup revisited. *Eur. Phys. J. A* **57**, 57 (2021).