

# 2023.2.14 Group meeting

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# Outline

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Validity test of surface approximation  
with a full quantum mechanical reaction model

What is surface approximation (SA)?

Physical content of SA?

What is the suitable SA for IAV model?

Applicable conditions of SA and the physical justifications of these conditions?

# Surface approximation

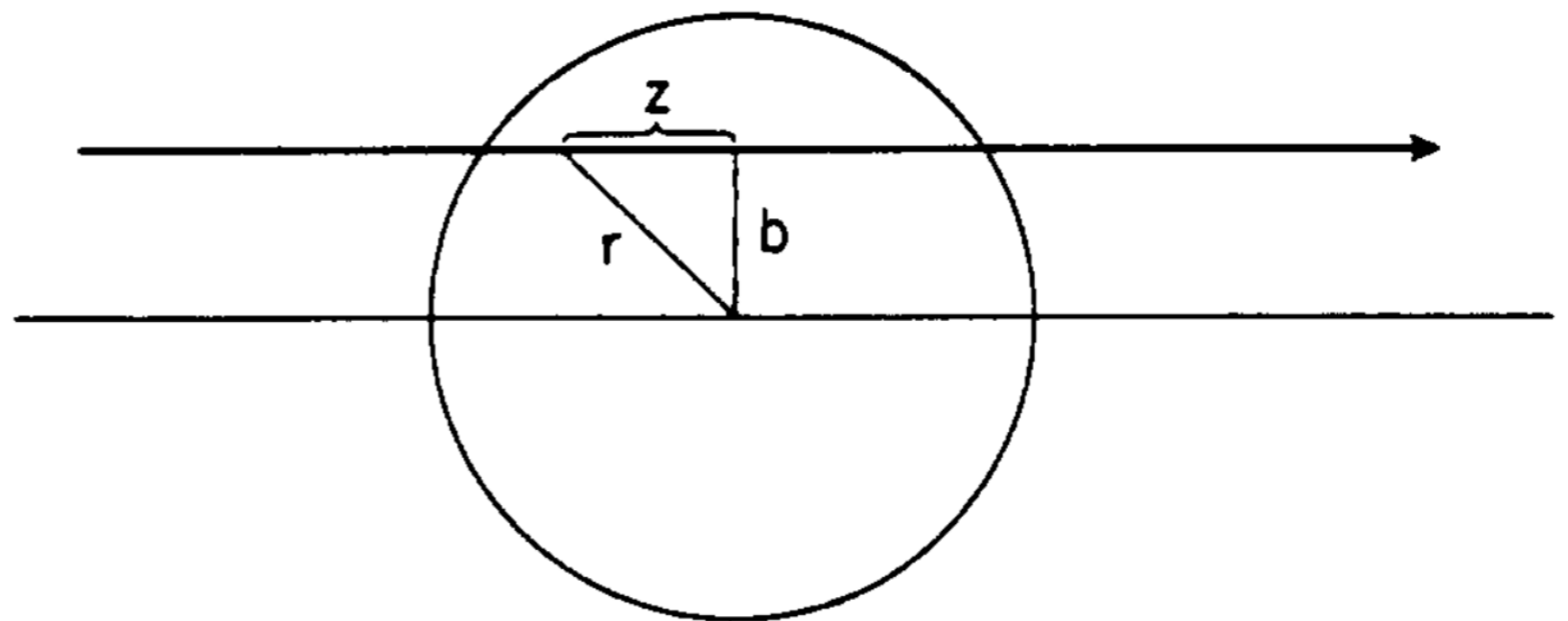
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Using the asymptotic behavior or the boundary condition of the wave function to calculate cross section in a semi-classical picture.

Phase shift / S matrix

In a semi-classical picture, some reactions only take place on the surface of nuclei, because two nuclei will not go into each other.

Classical trajectory



# Theoretical framework

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Post form of the NEB cross section in IAV model

$$\left. \frac{d^2\sigma}{d\Omega_b dE_b} \right|_{\text{post}}^{\text{NEB}} = \frac{-2}{\hbar v_a} \rho(E_b) \langle \rho_b(\mathbf{r}_b) | G_x^\dagger W_x G_x | \rho_b(\mathbf{r}_b) \rangle$$

Definition of the source term

$$\langle \mathbf{r}_b | \rho_b(\mathbf{r}_b) \rangle = \langle \mathbf{r}_b \chi_b^{(-)} | V_{\text{post}} | \chi_a^{(+)} \phi_a \Phi_A \rangle$$

How to introduce the surface approximation in IAV model?

**Replace the wave function with its asymptotic form?**

# Theoretical framework

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Replace the wave function with its asymptotic form?

$$u_l(r) = H_l^{(-)} - S_l H_l^{(+)}$$

The linear combination of two coulomb functions will be **irregular** at the origin, thus can not be handle numerically.

"Due to the strong absorption properties of the optical model wave functions, it is expected that the matrix element, eq. (5), is insensitive to the details of this part of the wave function. So one could either introduce a **radial cut-off** or some **suitable continuation of the function** given by eq. (6) into the interior region (see eq. (26) of ref. [3] )."

[1] Baur, G. (1986). Breakup reactions as an indirect method to investigate low-energy charged-particle reactions relevant for nuclear astrophysics. *Physics Letters B*, 178(2-3), 135-138.

# Radial cutoff

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Introduce the radial cutoff

$$u_{l_a}(r) = u_{l_b}(r) = 0 \quad r < R_{\text{cut}}$$

Selection of  $R_{\text{cut}}$  is crucial:

- Too big: the wave function will soon get oscillated outside the interaction region, thus has no contribution to the integral.
- Too small: only a tiny part of information is thrown away. The result will stay the same undoubtedly.

Choose  $R_{\text{cut}}$  according to the interaction region radius

$$r_i = r_{0i} \times A^{1/3}$$

# Calculation

Consider  $^{208}\text{Pb}(^6\text{Li}, \alpha X)$  at  $E_{\text{lab}} = 100 \text{ MeV}$

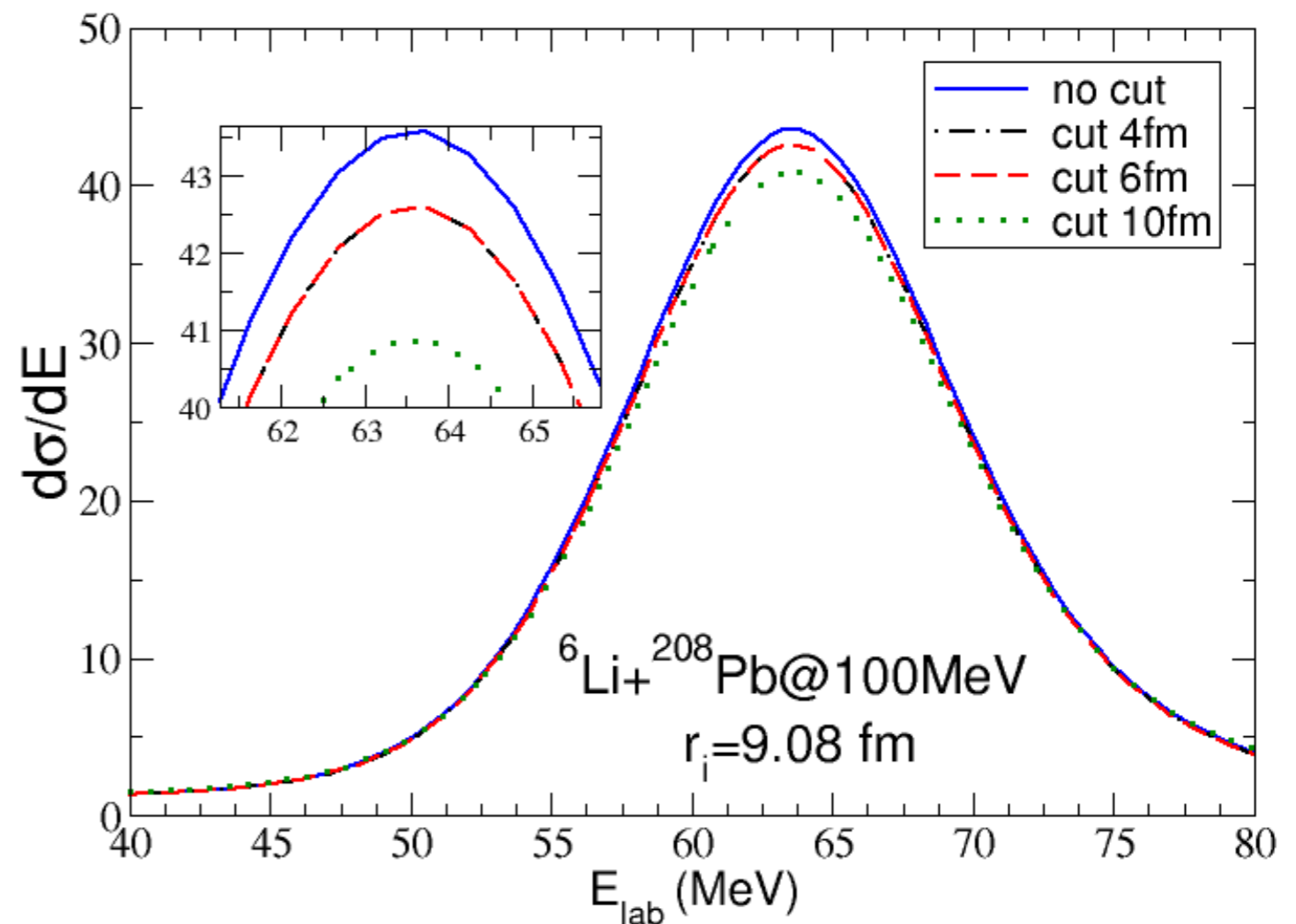
According to the optical potential,  $r_i = 9.08 \text{ fm}$ .

So we choose 4 fm, 6 fm, 10 fm to be cutoff radii.

Overall agreement

Energy dependent

Sensitivity to  
interior wave function



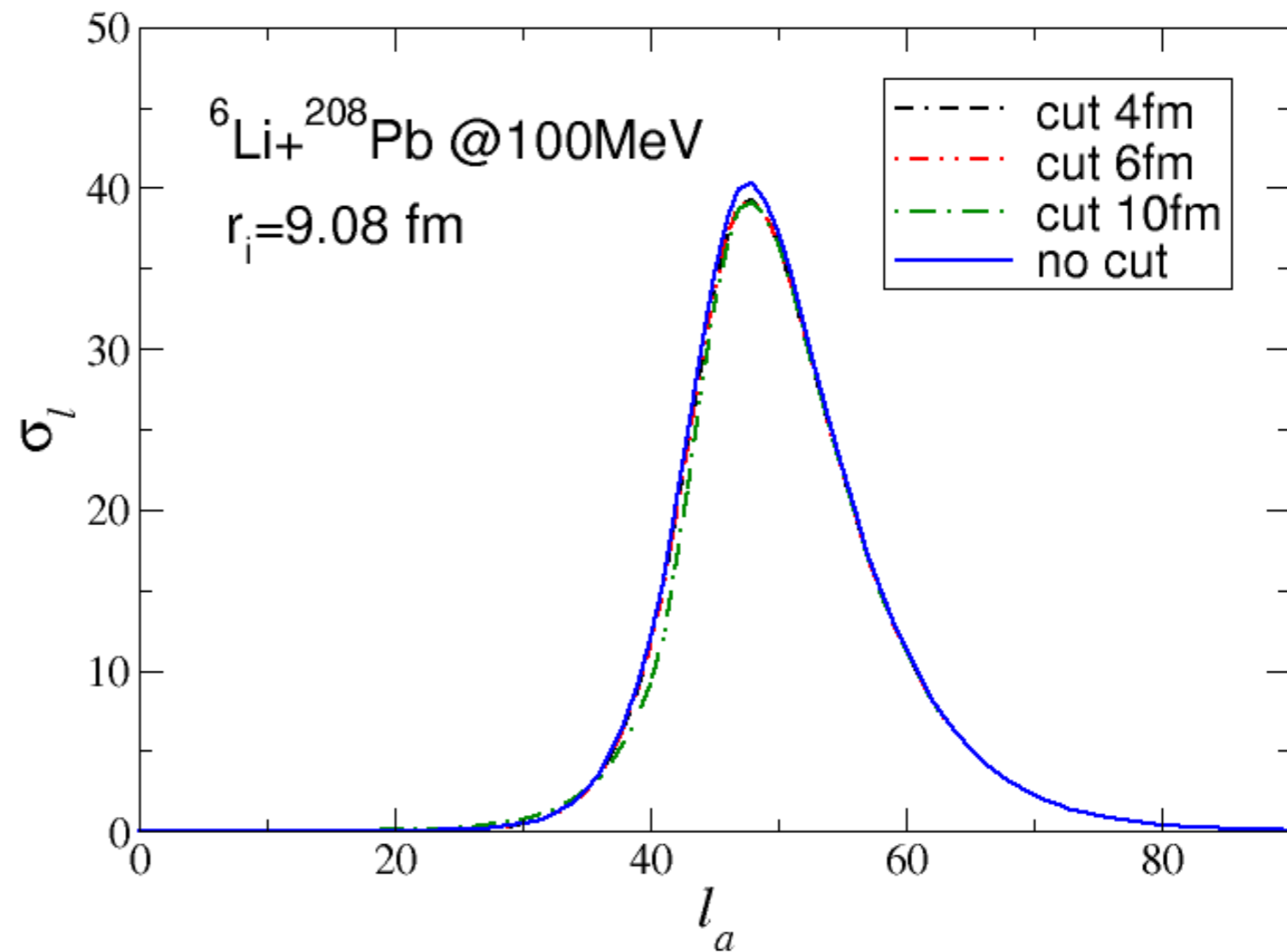
# Calculation

Consider  $^{208}\text{Pb}(^6\text{Li}, \alpha X)$  at  $E_{\text{lab}} = 100 \text{ MeV}$

L-distribution

Bell-like shape

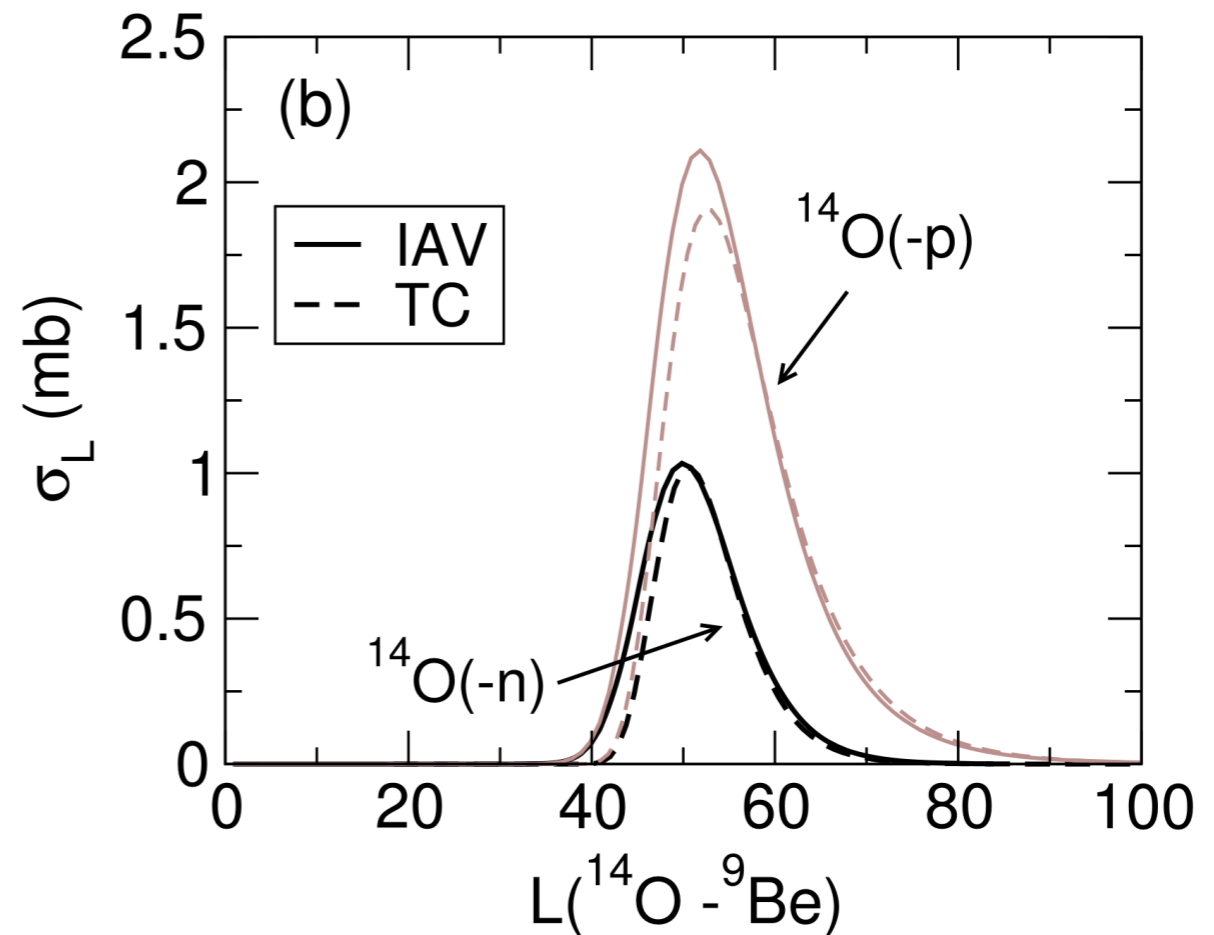
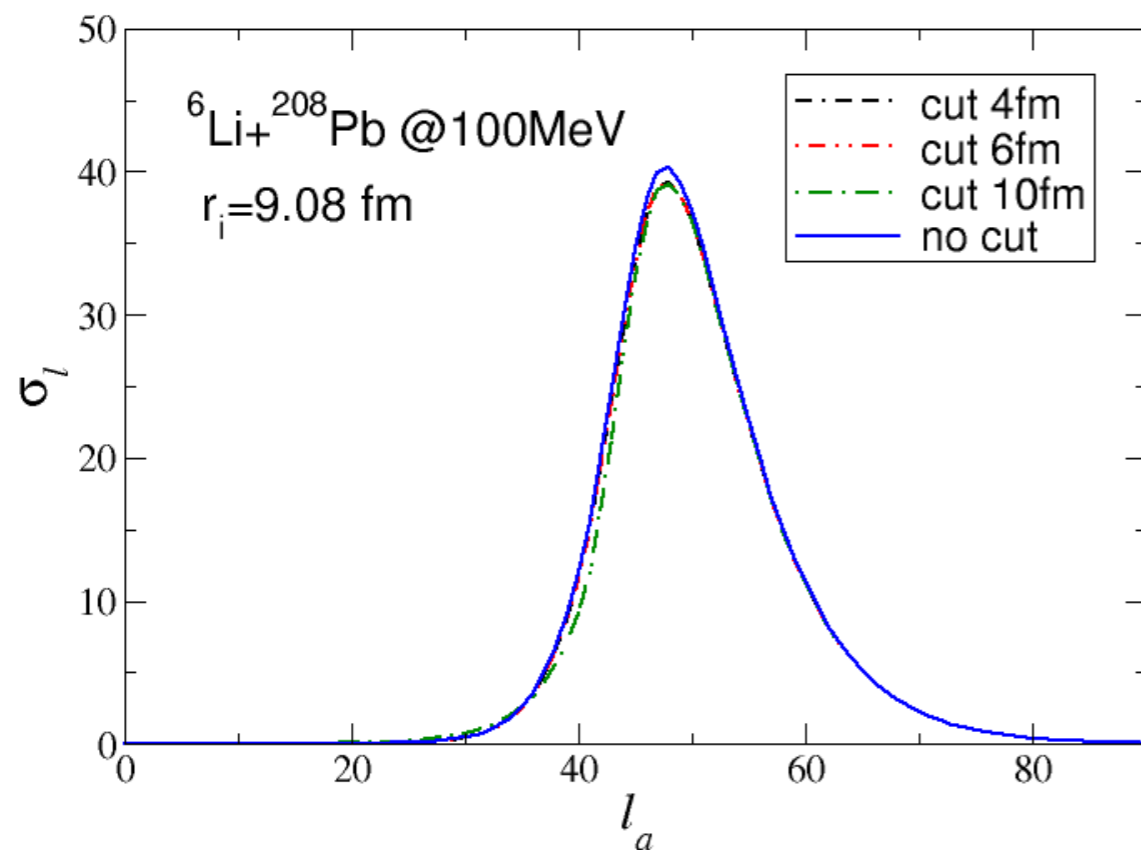
Comparison with  
TC model





# Calculation

Consider  $^{208}\text{Pb}(^6\text{Li}, \alpha X)$  at  $E_{\text{lab}} = 100 \text{ MeV}$



[2] Lei, J., & Bonaccorso, A. (2021). Comparison of semiclassical transfer to continuum model with Ichimura-Austern-Vincent model in medium energy knockout reactions. *Physics Letters B*, 813, 136032.