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Predicting hypernuclei based on chiral interactions

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Seminar at Tongij University, Shanghai, China

- Motivation
- YN & YY interactions
- J-NCSM & SRG evolution of (hyper-)nuclear interactions
- Determination of CSB contact interactions and Λn scattering length & application to $A = 7$ and 8 hypernuclei
- Uncertainty of Λ separation energies & chiral YNN interactions
- $S = -2$ hypernuclei: predictions for $A \leq 6$
- SRG & long-range corrections of two-nucleon densities
- Conclusions & Outlook

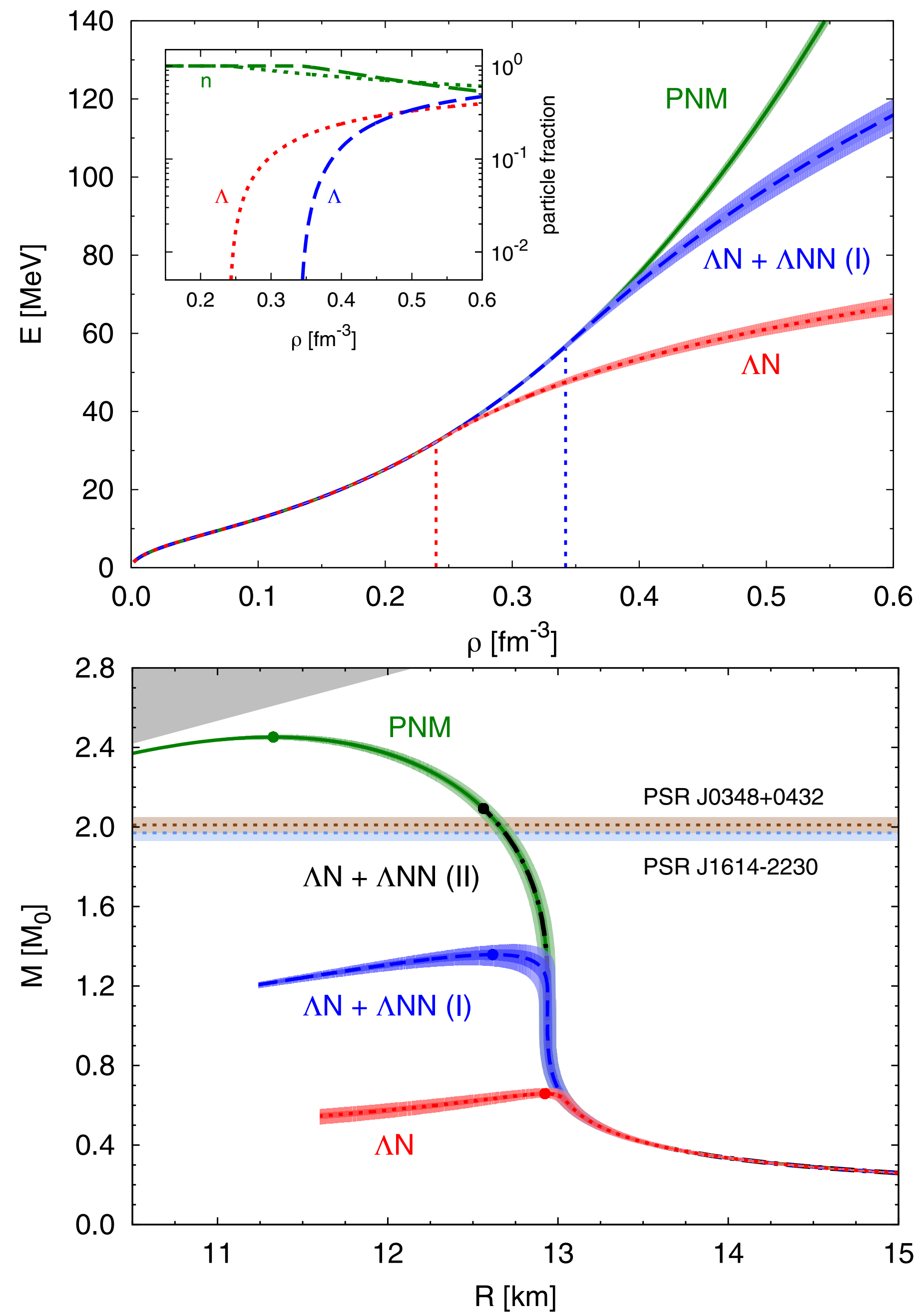
in collaboration with Johann Haidenbauer, Hoai Le, Ulf Meißner, **Xiang-Xiang Sun**

March 11th, 2025

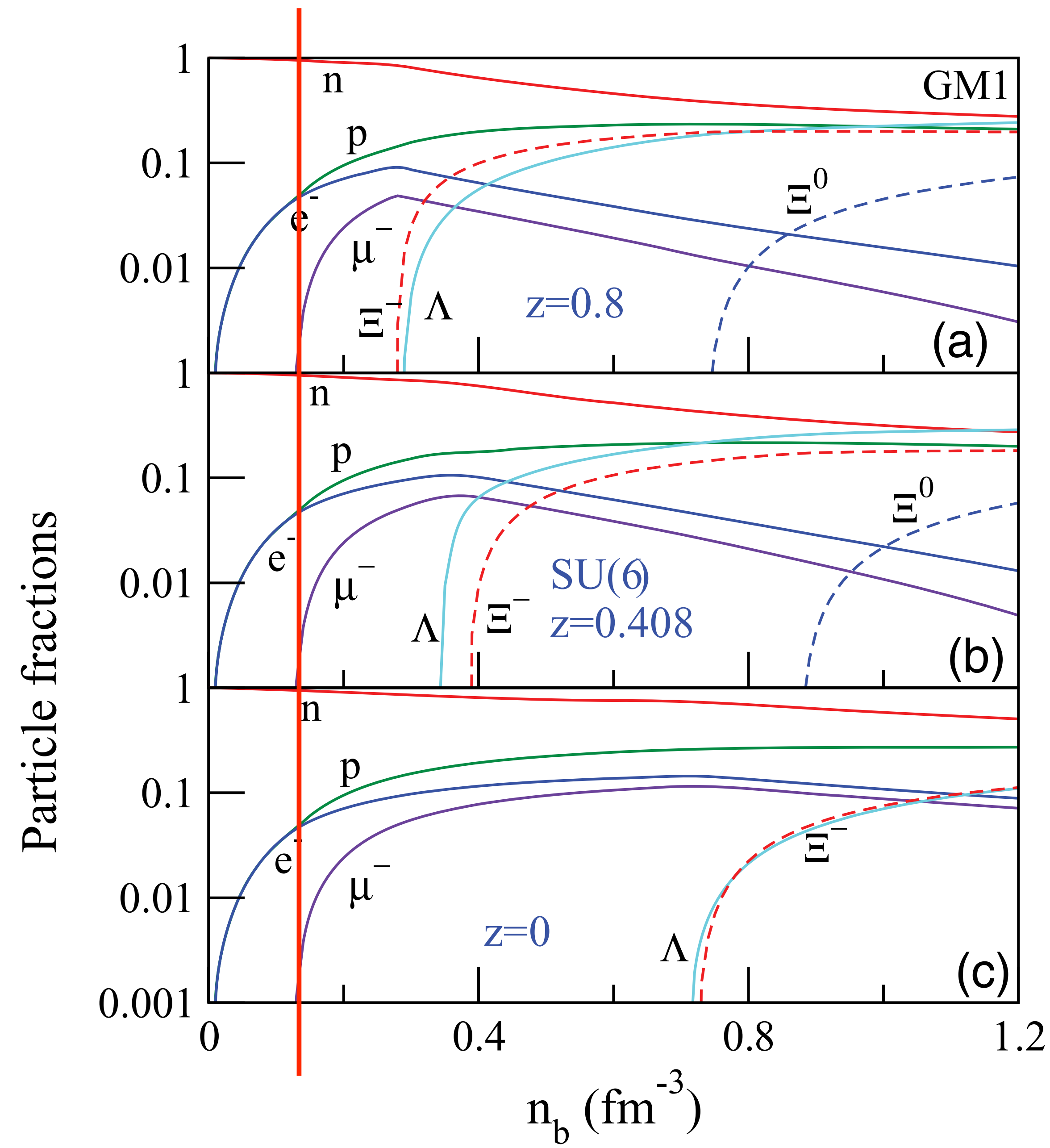
Hypernuclear interactions

Why is understanding hypernuclear interactions interesting?

- *hyperon contribution to the EOS, neutron stars, supernovae*
- *"hyperon puzzle"*
- *Λ as probe to nuclear structure*
- ***flavor dependence of baryon-baryon interactions***



(Lonardoni et al. (2015))

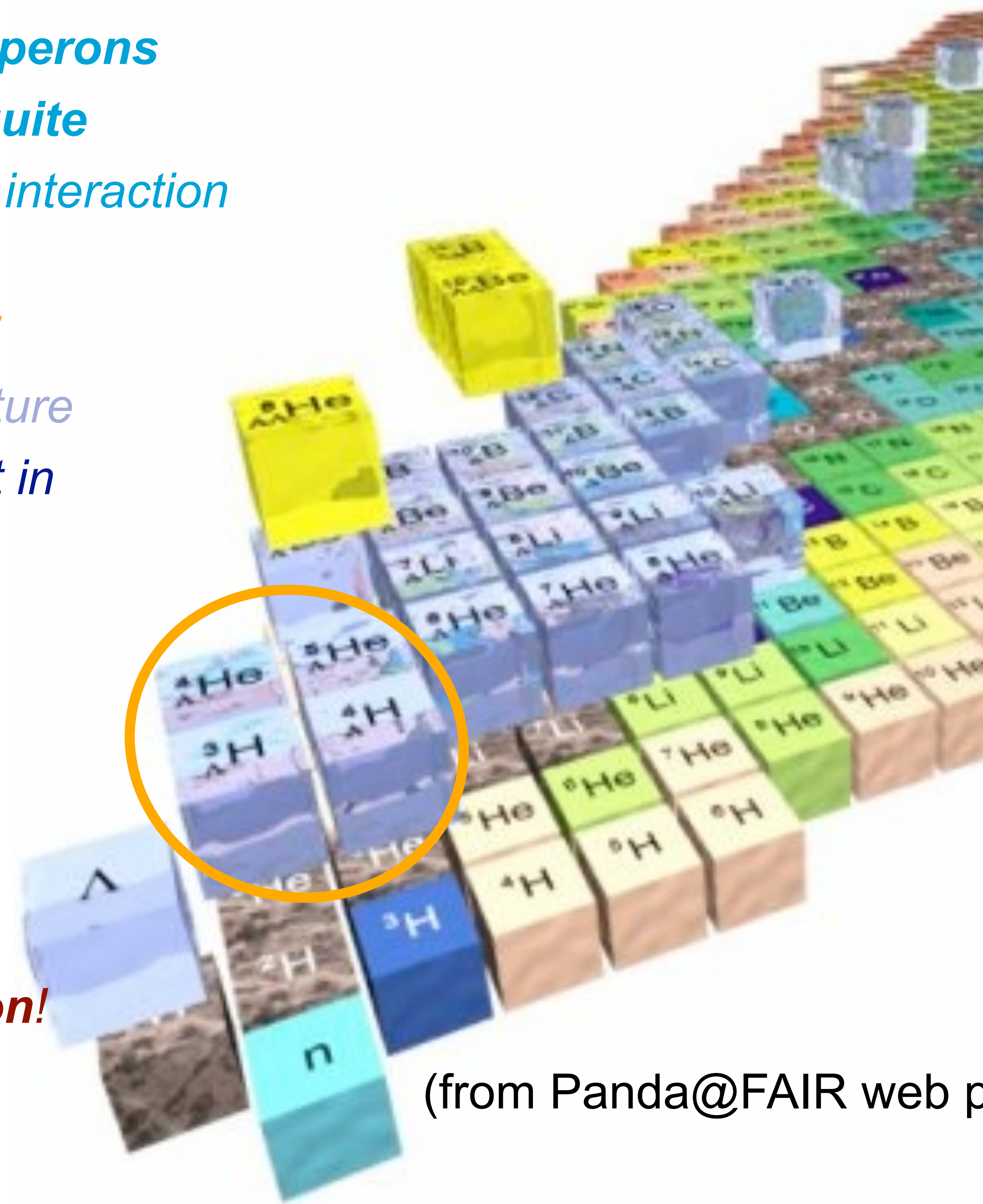


(Weissenborn et al. 2012)

Hypernuclei

Only few YN data. Hypernuclear data provides additional constraints.

- *ΛN interactions are generally weaker than the NN interaction*
  - *naively:* ***core nucleus + hyperons***
  - *„separation energies" are* ***quite*** *independent from NN(+3N) interaction*
- *no Pauli blocking of Λ in nuclei*
  - *good to study nuclear structure*
  - *even light hypernuclei exist in* ***several spin states***
- ***non-trivial constraints*** *on the YN interaction even from lightest ones*
- *size of* ***YNN*** *interactions? need to include* ***Λ-Σ conversion****!*



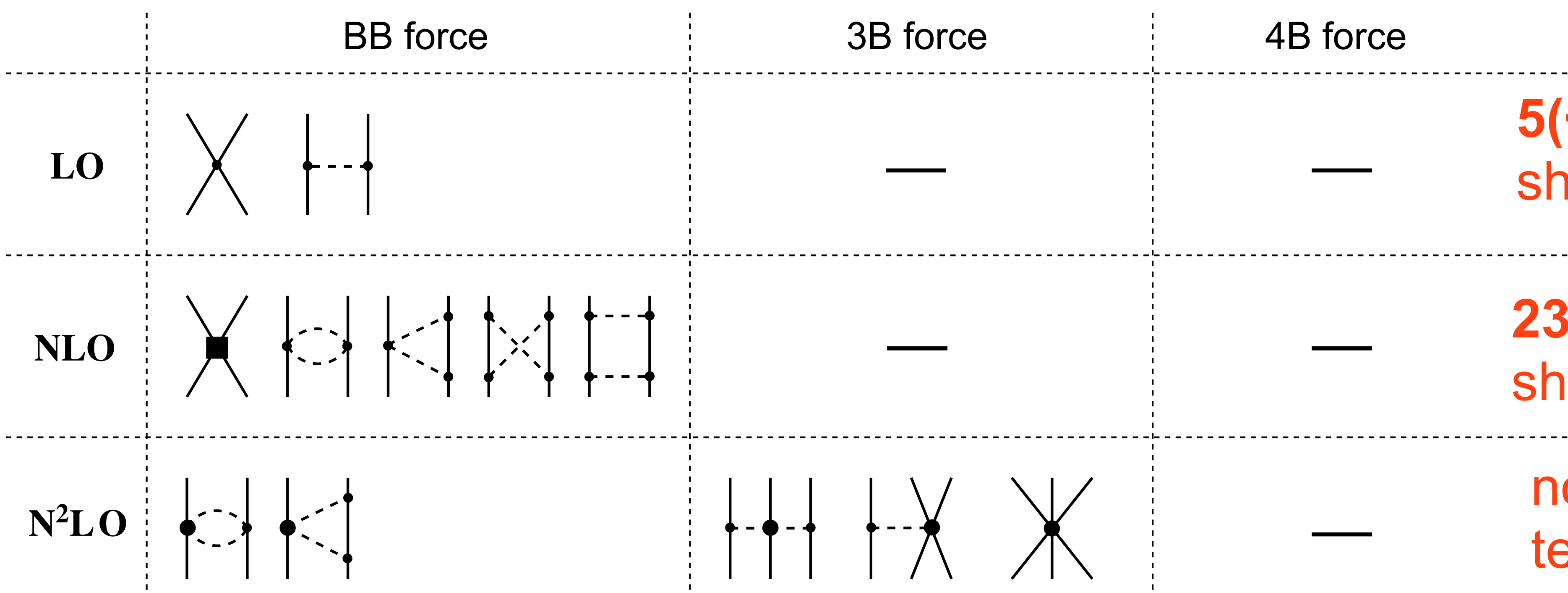
(from Panda@FAIR web page)

Chiral NN & YN & YY interactions

EFT based approaches

Chiral EFT implements **chiral symmetry of QCD**

- symmetries constrain exchanges of Goldstone bosons
- relations of two- and three- and more-baryon interactions
- breakdown scale $\approx 600 - 700\,\mathrm{MeV}$
- Semi-local momentum regularization (SMS) up to N²LO (for YN)



5(+1) NN/YN (YY) short range parameters

23(+5) NN/YN (YY) short range parameters

no additional contact terms in NN/YN

(adapted from Epelbaum, 2008)

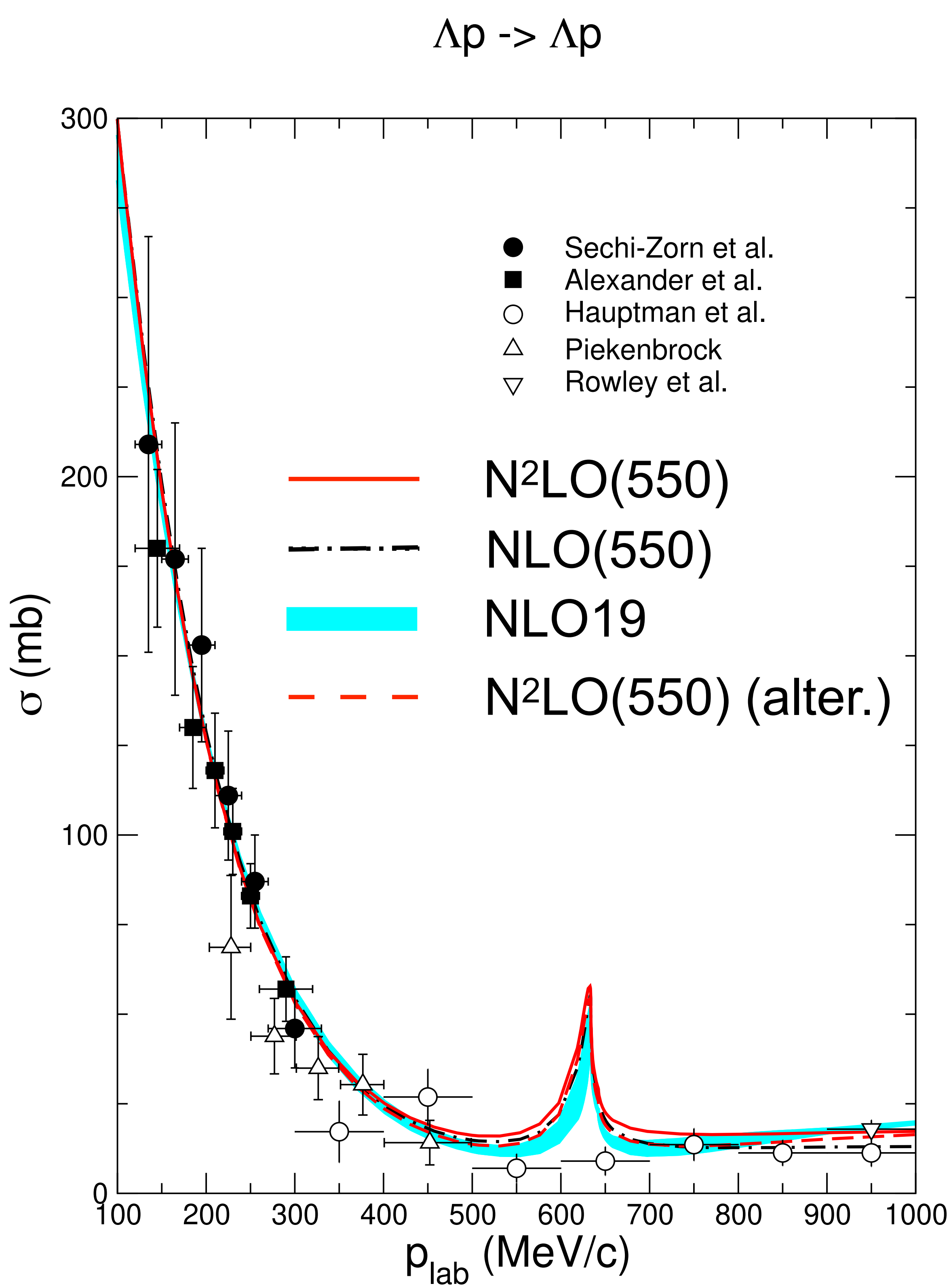
Retain flexibility to adjust to data due to counter terms

Regulator required — cutoff/different orders often used to estimate uncertainty

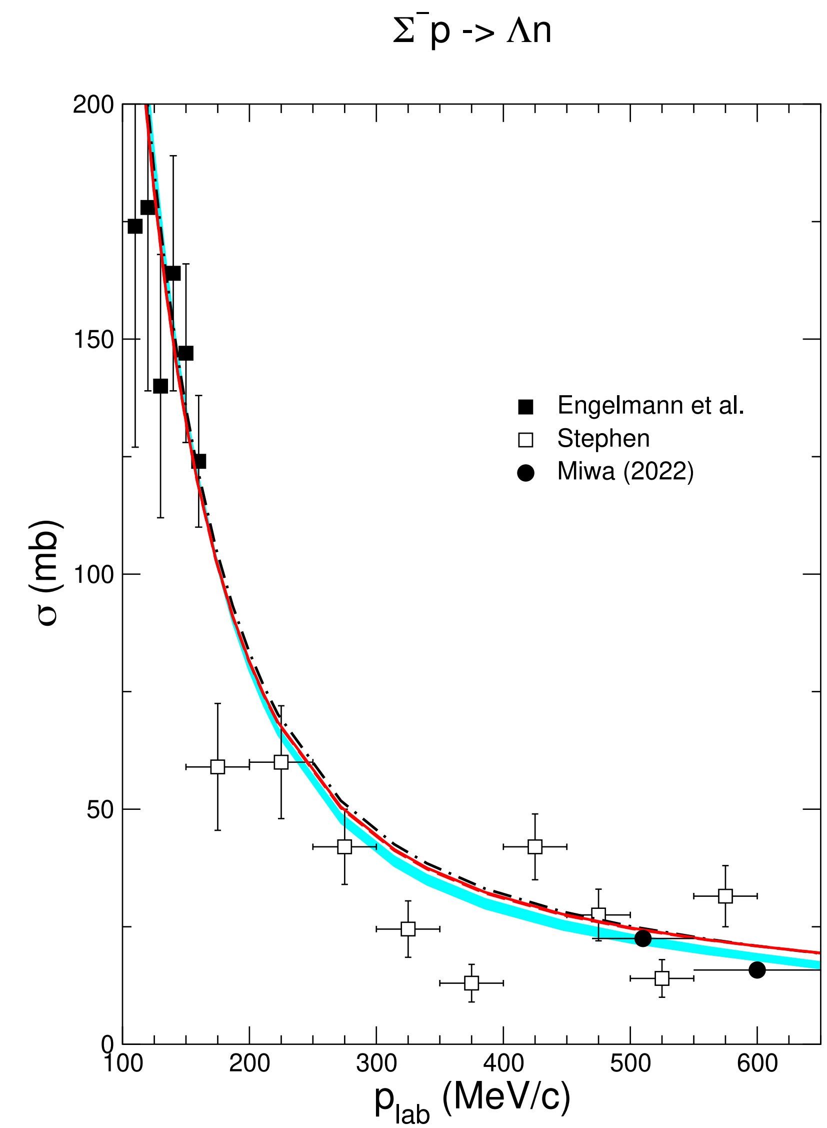
$\Lambda - \Sigma$ $(\Lambda\Lambda - \Sigma\Sigma - \Xi N)$ **conversion** is explicitly included (3BFs only in N²LO)

SMS NLO/N²LO interaction

Selected results (show $\Lambda = 550\,\mathrm{MeV}$, others are very similar in quality)



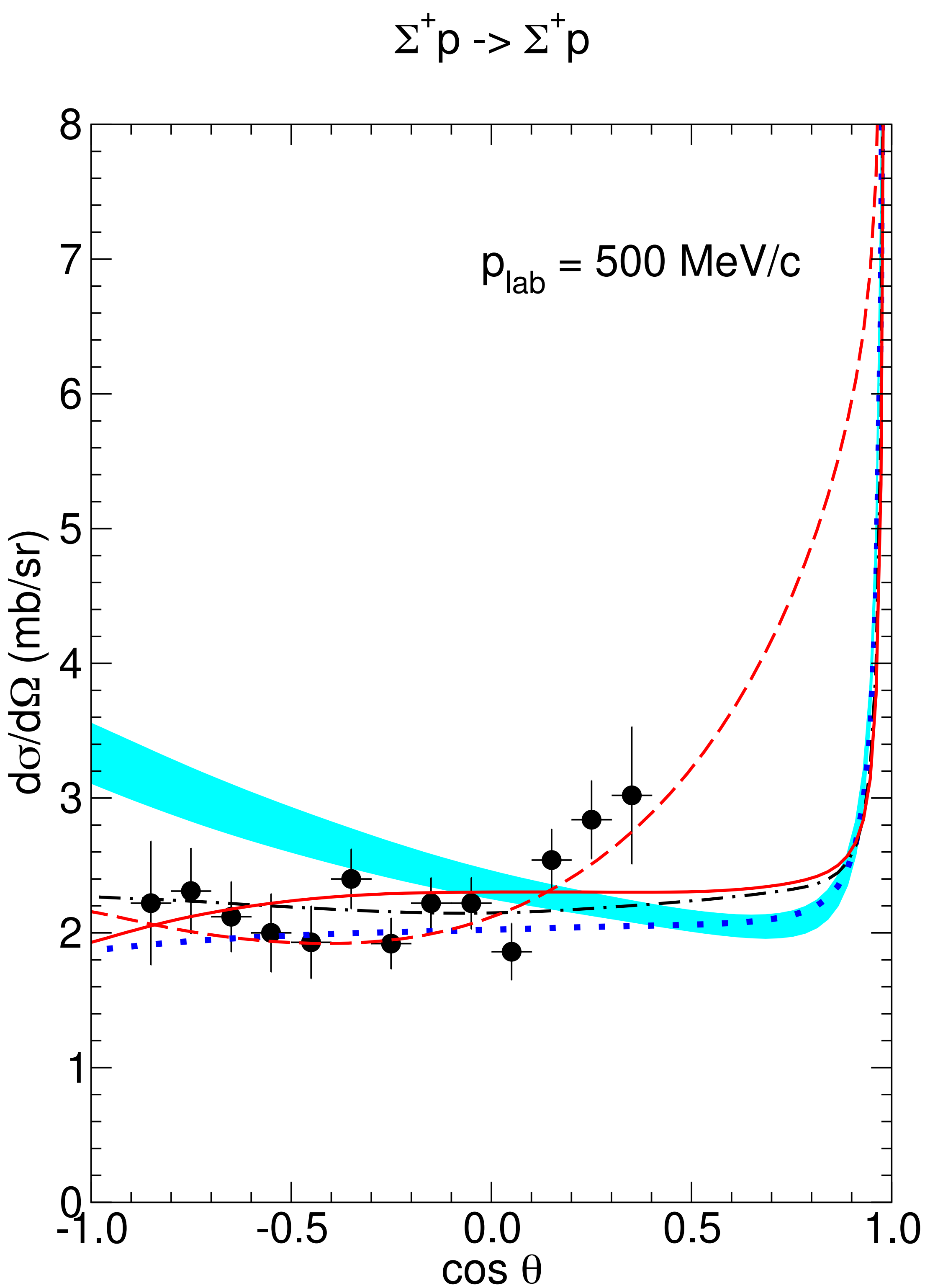
- most relevant cross sections very similar in NLO and N²LO
- similar to NLO19
- alternative fit (see later)

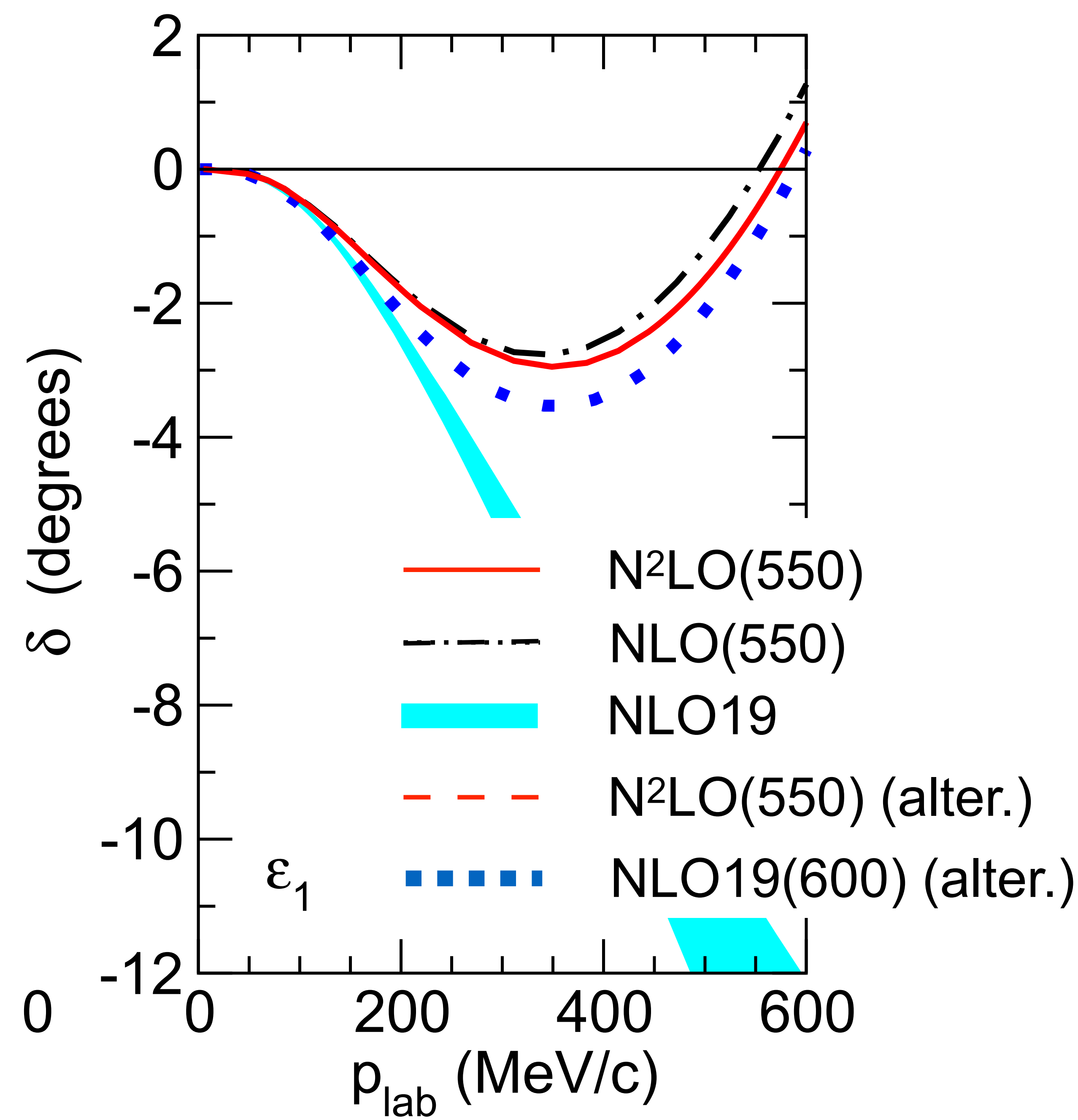


J. Haidenbauer et al. EPJ A 59, 63 (2023)

SMS NLO/N²LO interaction

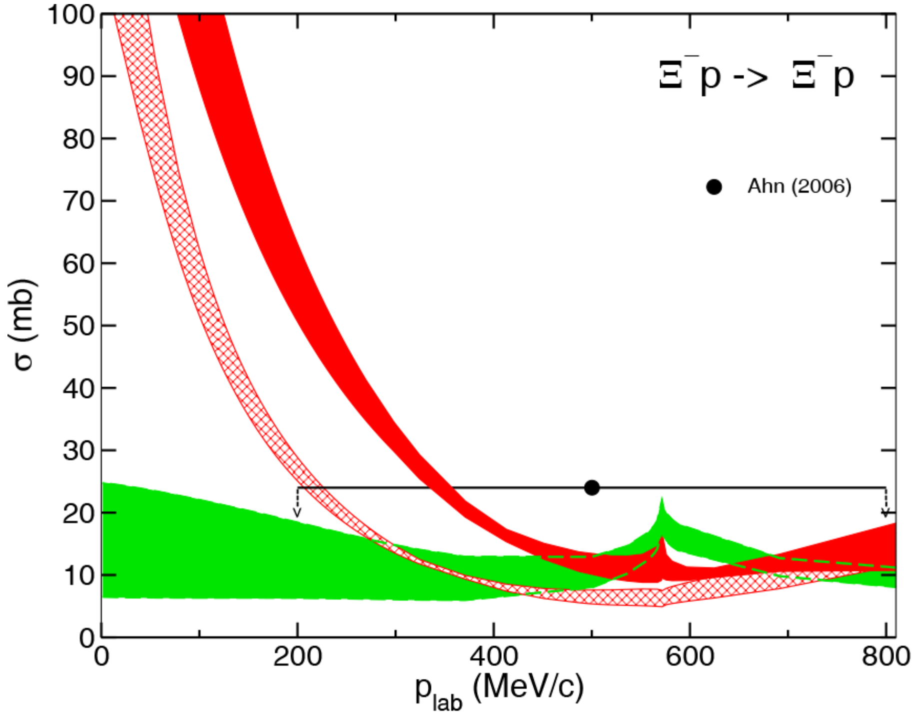
new data (Miwa(2022)) at higher energies provides **new constraints**!

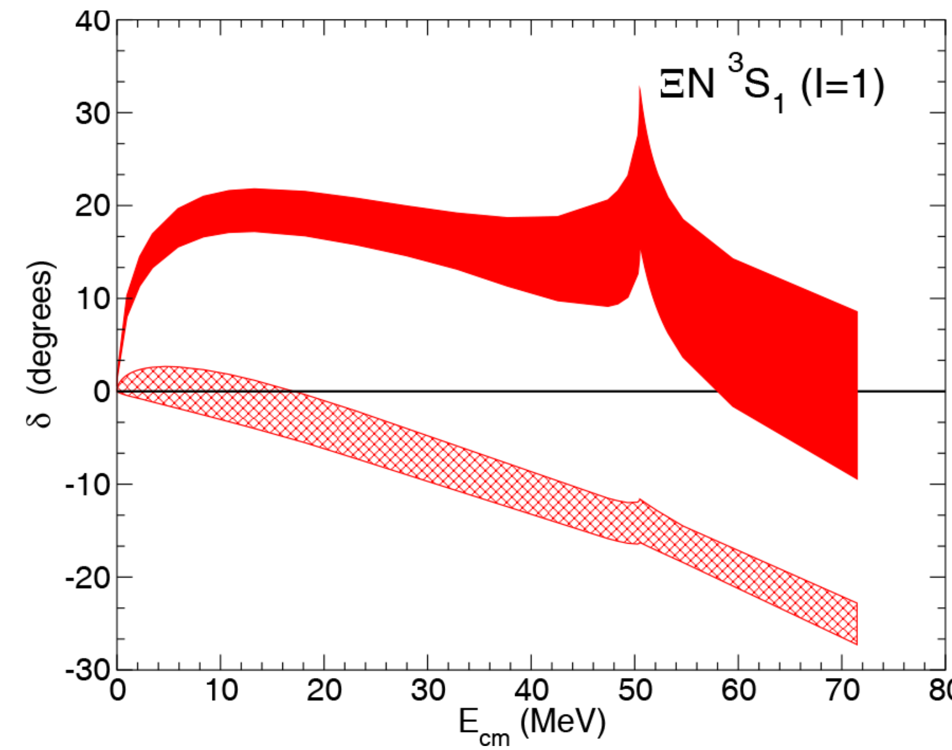


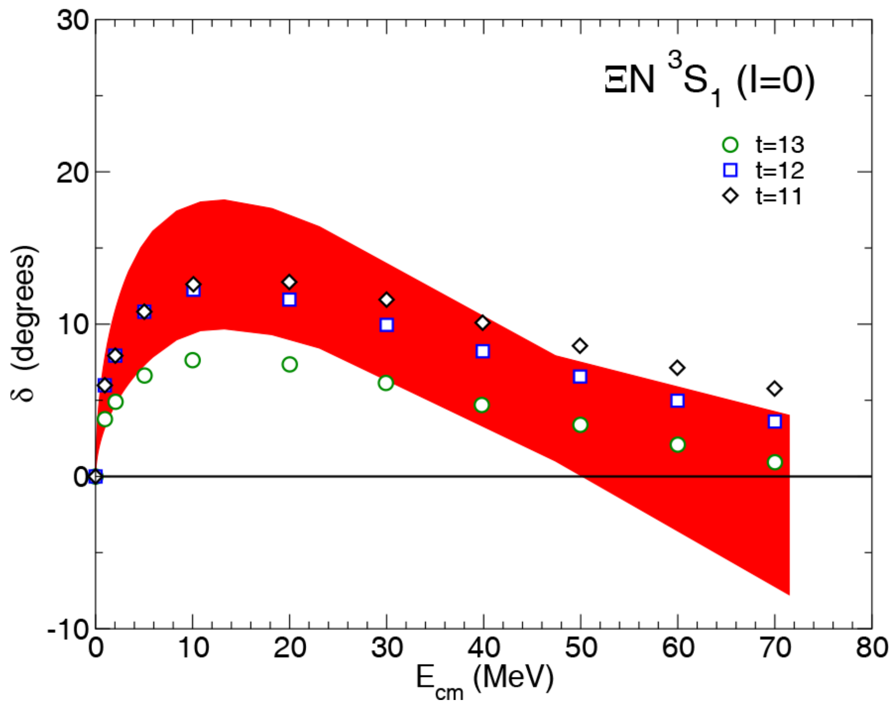


J. Haidenbauer et al. EPJ A 59, 63 (2023)

YY interaction







LO

NLO

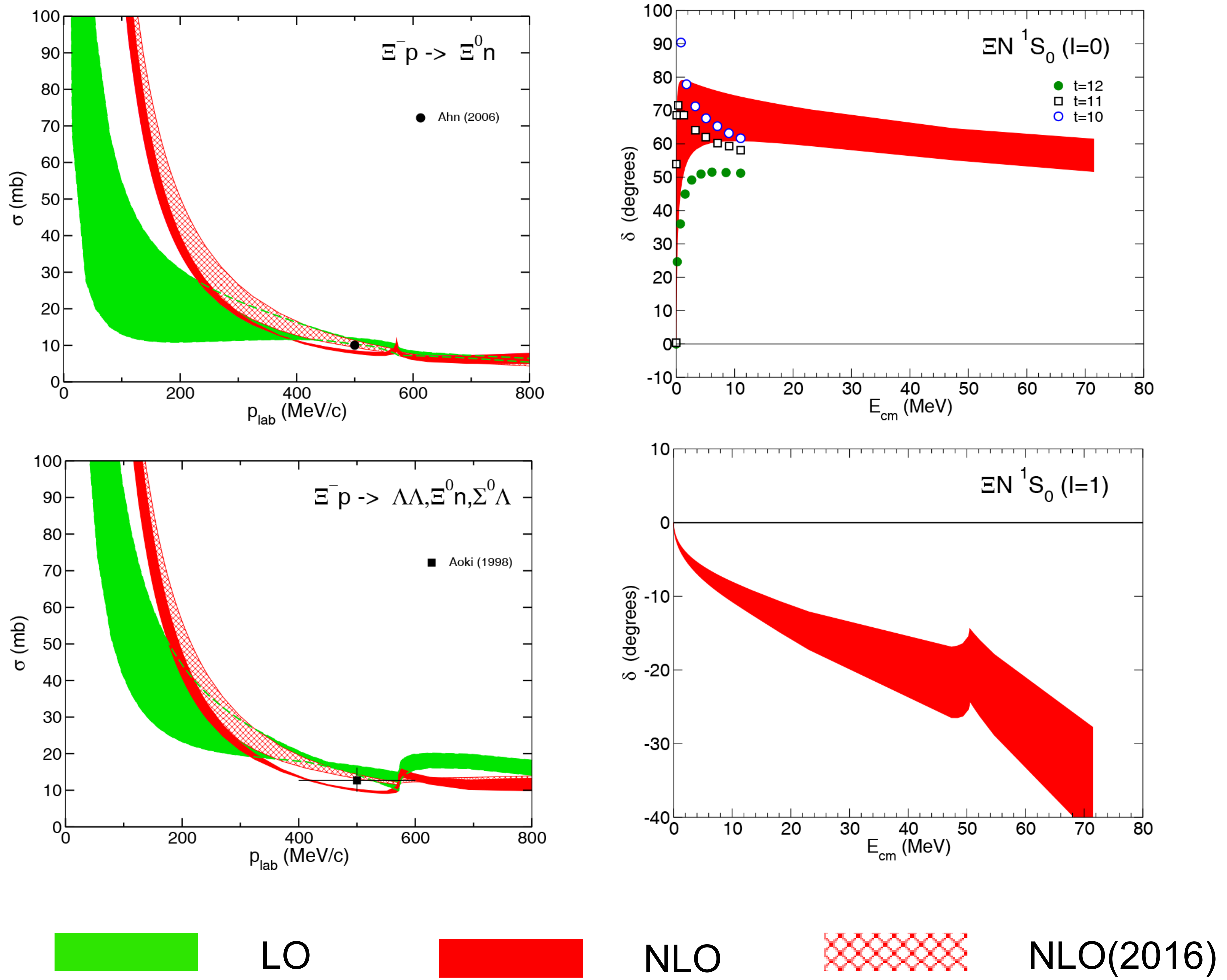
NLO(2016)

(Haidenbauer at al., 2019)

→ adjusted to data & LQCD (HAL QCD)

updated version consistent with Ξ-nuclei (only change in ΞN $^3S_1$)

YY interaction



(Haidenbauer at al., 2019)

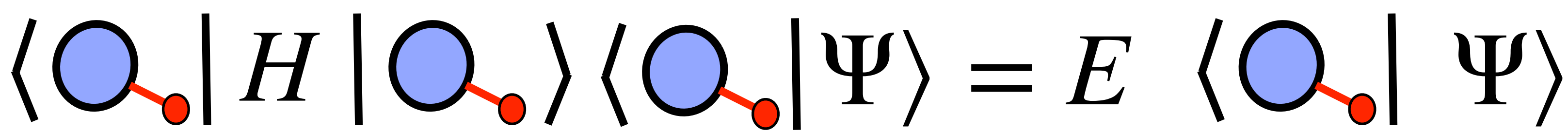
Jacobi-NCSM

Solve the Schrödinger equation using **HO states**

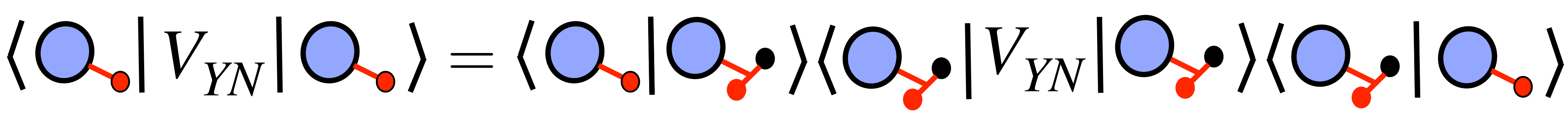
Two ingredients are necessary:

- **cfp** — antisymmetrized states for nucleons
- **transition coefficients** to separate off NN, YN, 3N and YNN

Schrödinger equation



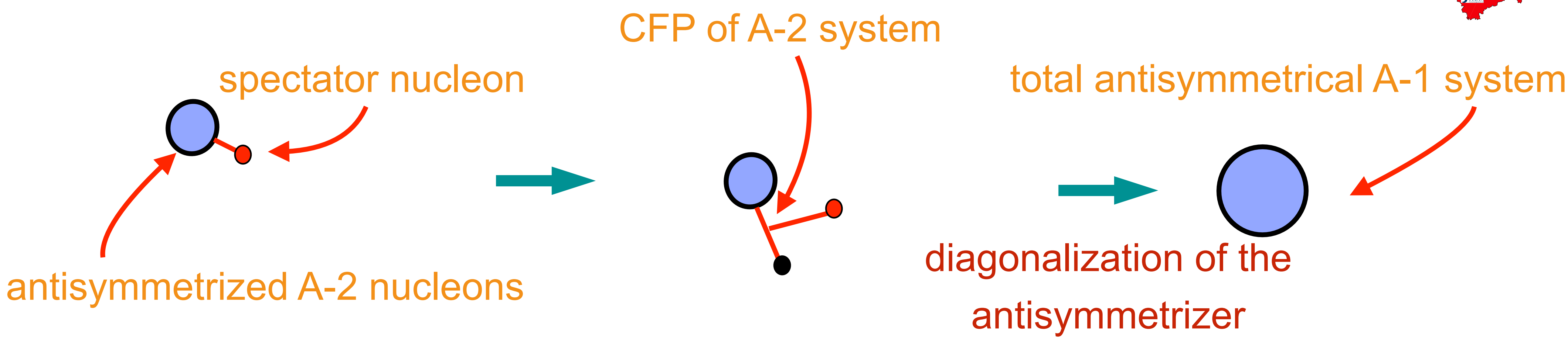
e.g. for YN interaction

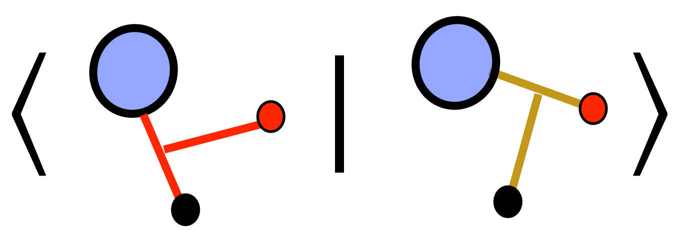


Application of to NN, YN, 3N and YNN interactions require the representation of particle transitions. (see Liebig et al. EPJ A 52,103 (2016), Le et al. EPJ A 56, 301 (2020) for combinatorical factors see Le et al. EPJ A 57, 217 (2021))

Jacobi-NCSM — CFP

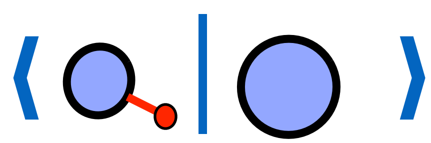
First, generate **antisymmetrized states** for the A-1 nucleon system





antisymmetrizer is equivalent to coordinate trafo
expression in terms of Talmi-Moshinsky brackets

(Navrátil et al. PRC 61,044001(2000))

The CFP coefficients  are obtained by diagonalization of the antisymmetrizer.

HO states guarantee:

- complete separation of antisymmetrized and other states
- **independence** of HO **length/frequency**

These coefficients will be openly accessible as **HDF5** data files

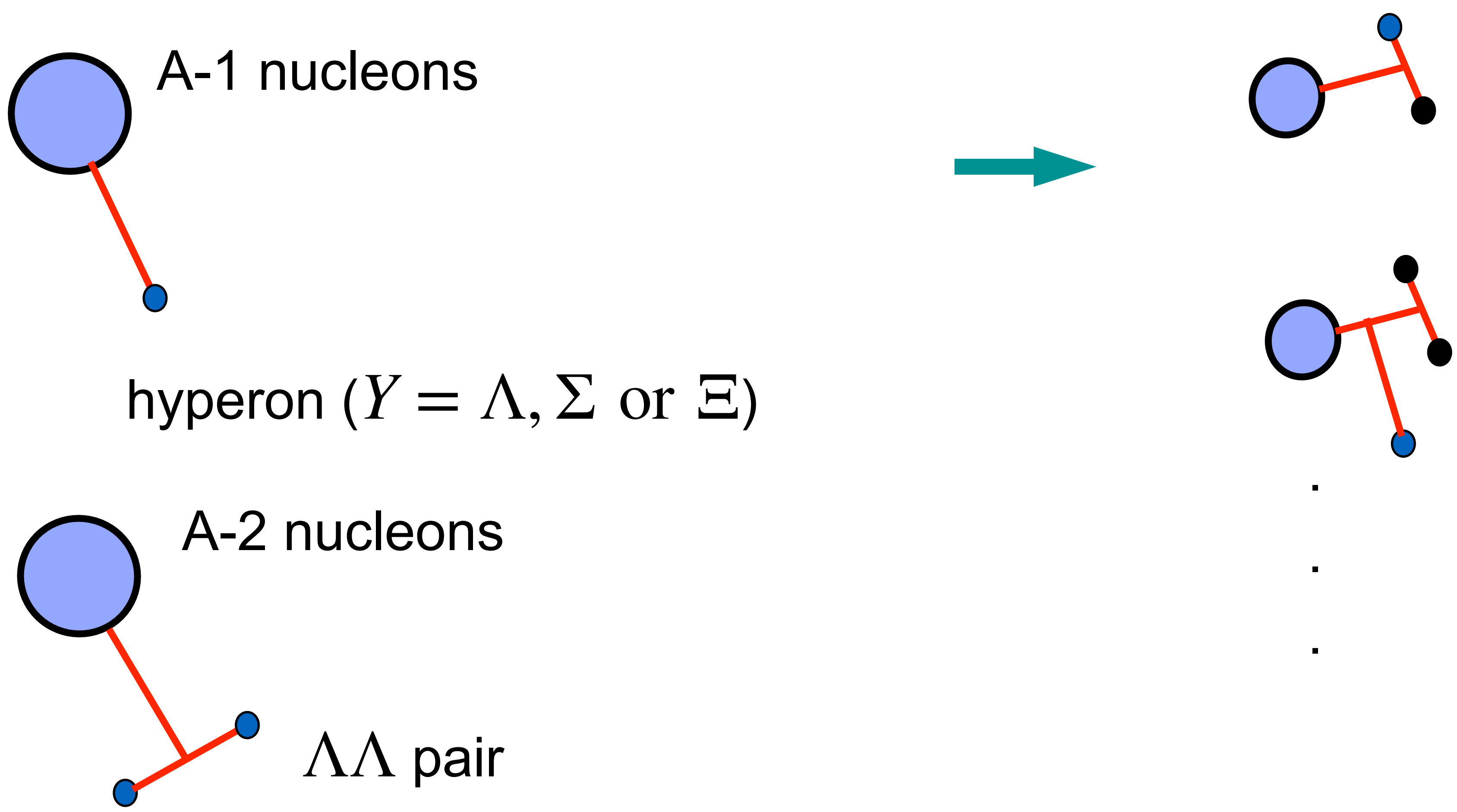
(download server is in preparation *(please contact me when interested!)*)

(Liebig et al. EPJ A 52,103 (2016))

Jacobi-NCSM states for $S = -1$

A-body hypernuclei state (no antisymmetrization with respect to nucleons required)

Third, rearrange baryons for the application of interactions, …



Again HO states guarantee the independence of HO length/frequency.

Transition coefficients are also accessible as **HDF5** data files to anyone interested.

(Le, Haidenbauer, Meißner, AN, 2020 & 2021)

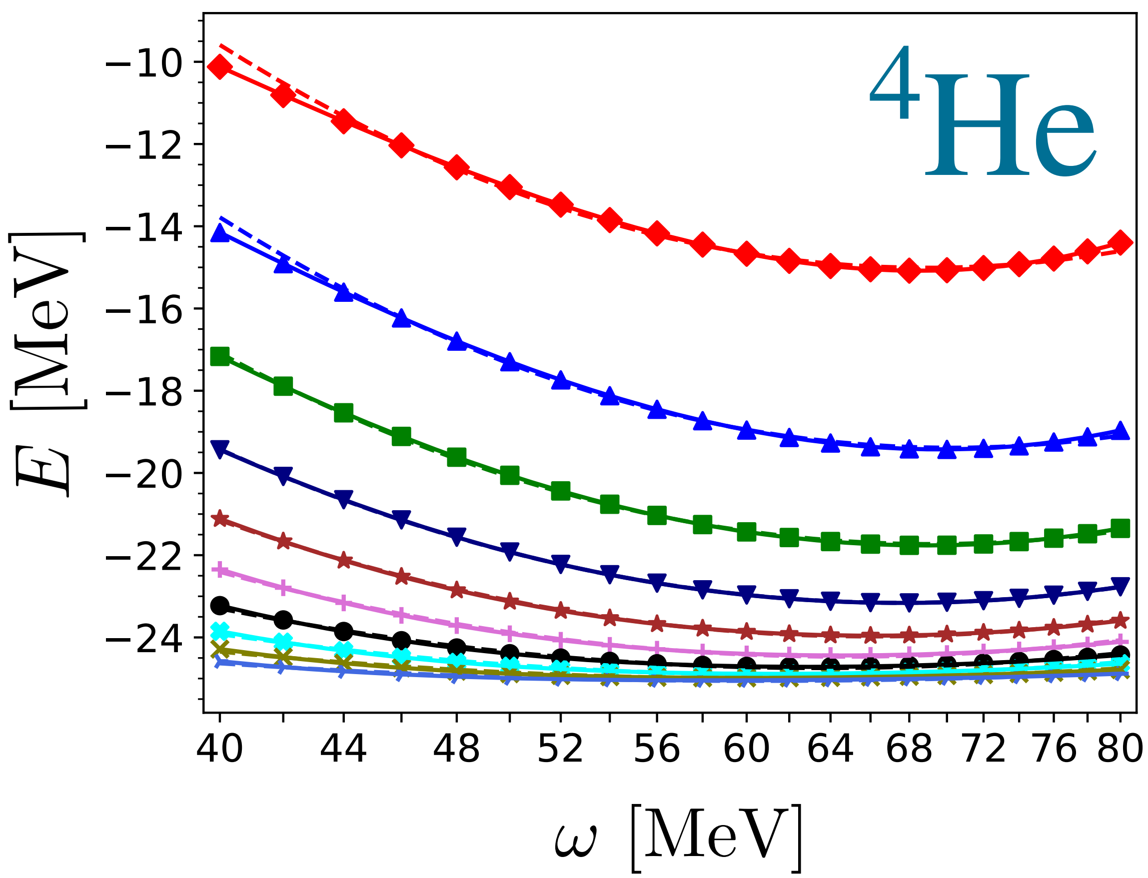
Converged results feasible for **"soft"** interactions.

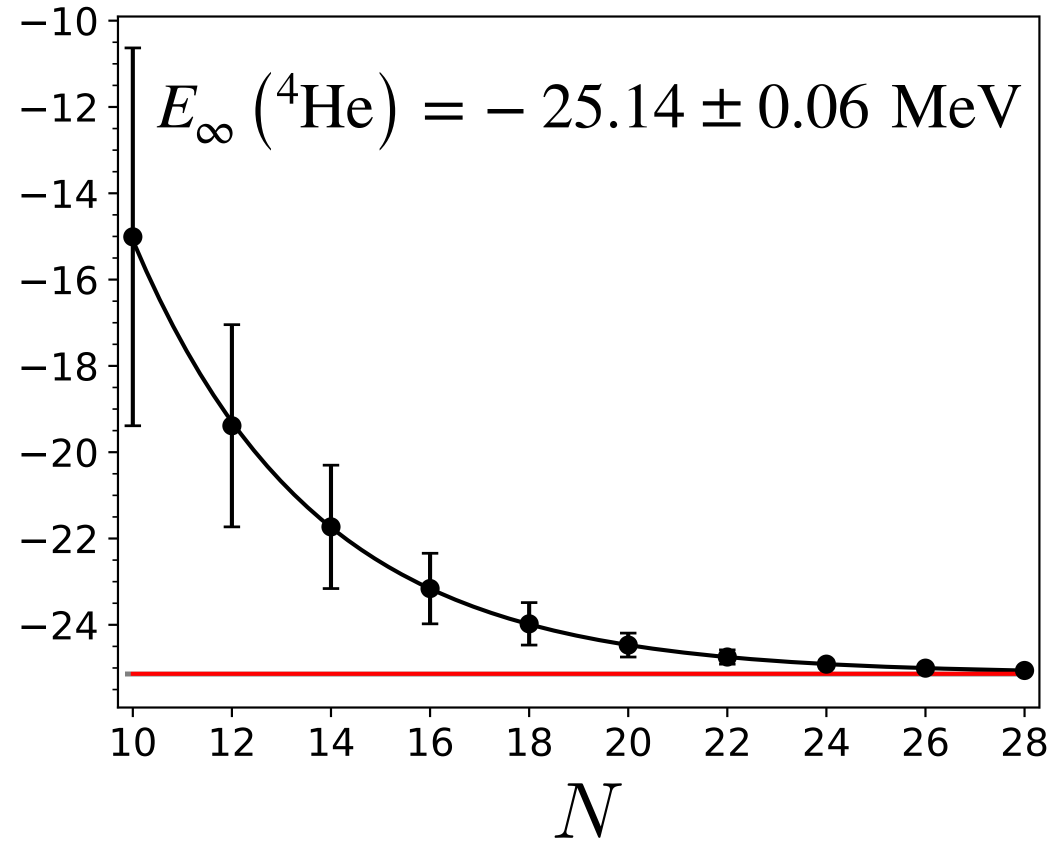
Convergence for Jacobi-NCSM

Simple example: ^{4}He with SMS N^{2}LO(550)

observed dependence on ω and N

$$E(\omega) = E_N + \kappa\left(\log(\omega) - \log(\omega_{opt})\right)^2 \rightarrow E_N = E_\infty + A\,e^{-bN}$$





Conservative uncertainty estimate: difference of $E_{N_{\max}}$ and E_∞

Numerical uncertainties for light nuclei are small.

For p-shell, numerical uncertainty is more sizable due to smaller $N_{\max}$ and smaller separation energies.

(Liebig et al. EPJ A 52,103 (2016))

In future: neural networks for extrapolation (see Wolfgruber et al. PRC 110,014327 (2024))

SRG interactions

Similarity renormalization group is by now a **standard tool** to obtain soft effective interactions for various many-body approaches (NCSM, coupled-cluster, MBPT, …)

Idea: perform a unitary transformation of the NN (and YN interaction) using a cleverly defined "generator"

(Bogner et al. PRC 75,061001 (2007))

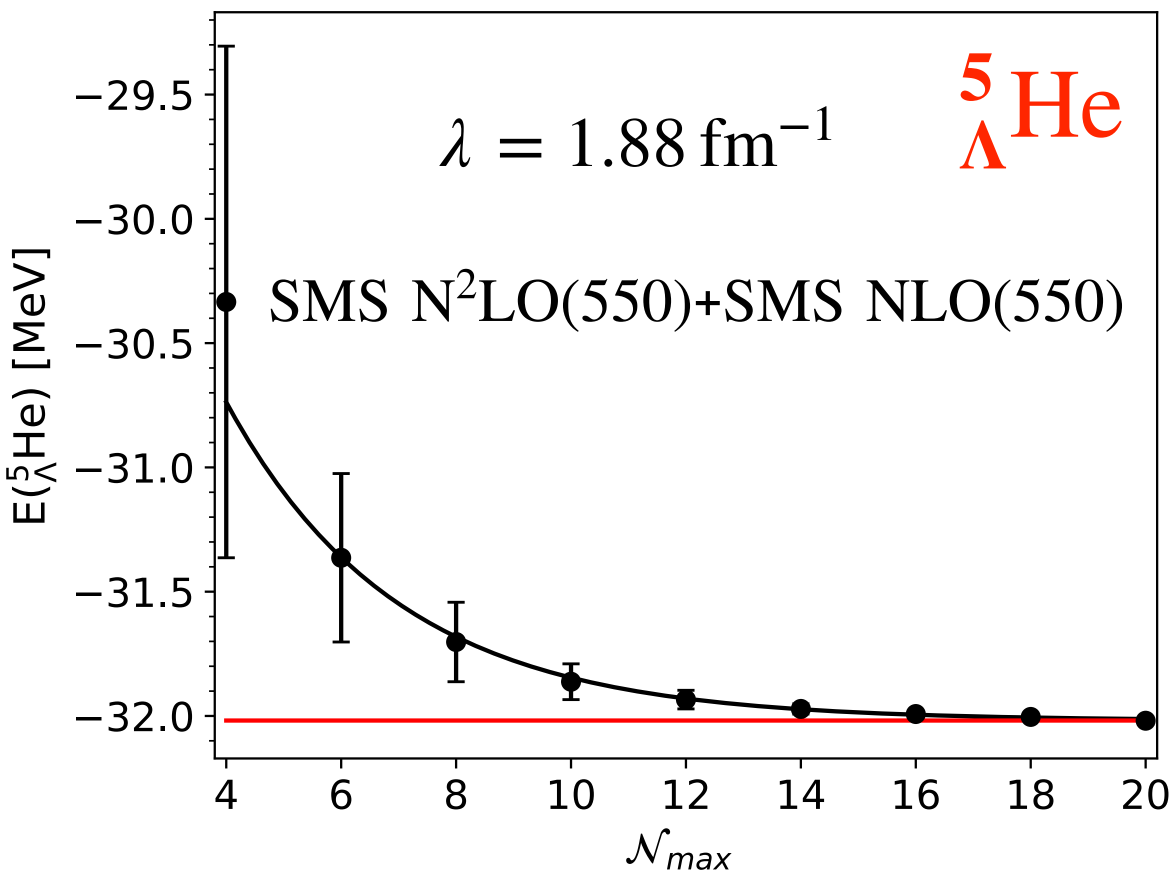
$$\frac{dH_s}{ds} = \Big[\underbrace{[T\,, H(s)]}_{\equiv\eta(s)}, H(s)\Big] \qquad H(s) = T + V(s)$$

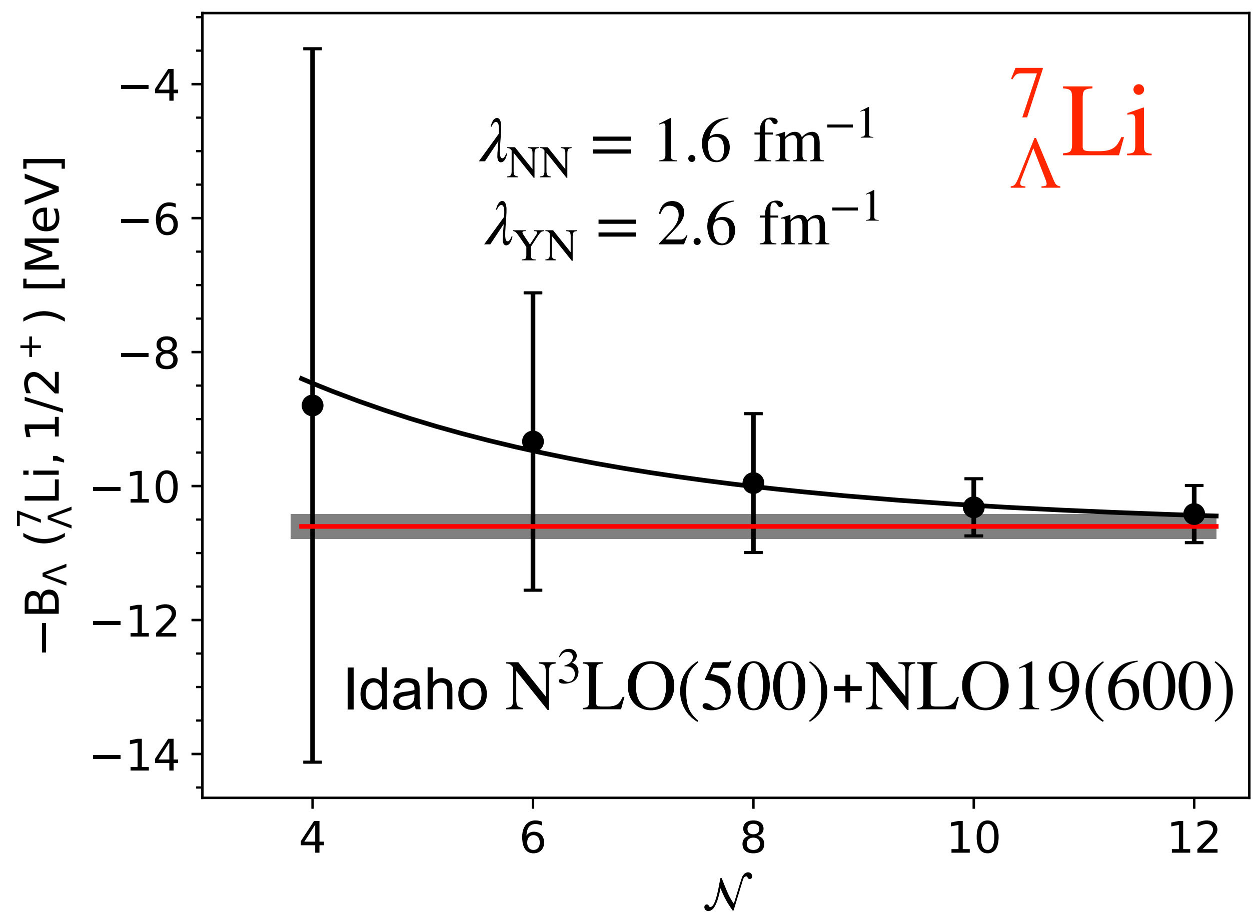
this choice of generator drives *V(s)* into a diagonal form in momentum space

- *V(s)* will be **phase equivalent** to original interaction
- short range *V(s)* will change towards **softer interactions**
- Evolution can be restricted to **2-,3-, … body level** (approximation)
- $\lambda = \left(\frac{4\mu_{BN}^2}{s}\right)^{1/4}$ is a measure of the width of the interaction in momentum space
- **dependence** of results on λ or s is a measure for **missing terms**

J-NCSM convergence

SRG evolution improves convergence





$$E\left(^{5}_{\Lambda}\mathrm{He}\right) = -\,32.018 \pm 0.001\ \mathrm{MeV} \qquad E_{\Lambda}\left(^{7}_{\Lambda}\mathrm{Li}\right) = 10.6 \pm 0.2\ \mathrm{MeV}$$

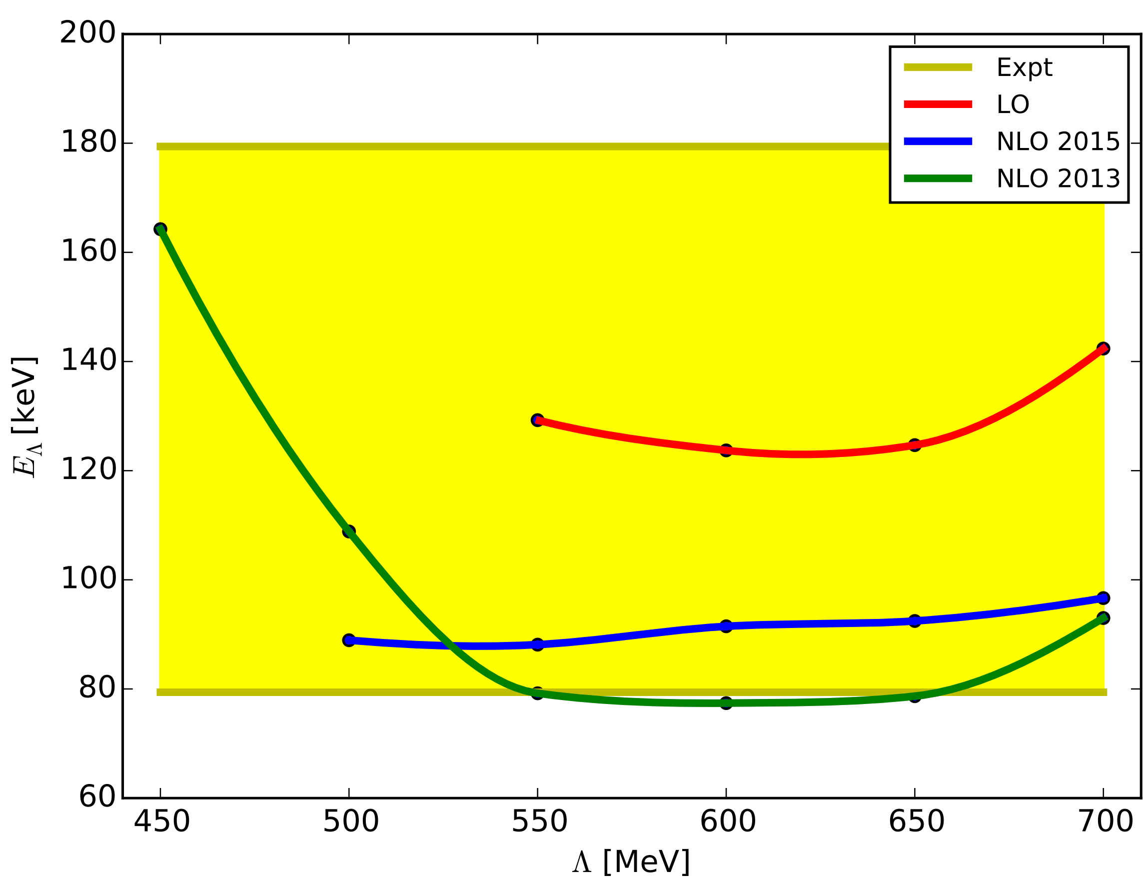
- for light nuclei and hypernuclei, the numerical uncertainty is negligible.
- for p-shell nuclei/hypernuclei, the uncertainty is visible
- extrapolation of separation energy can reduce uncertainty of this quantity

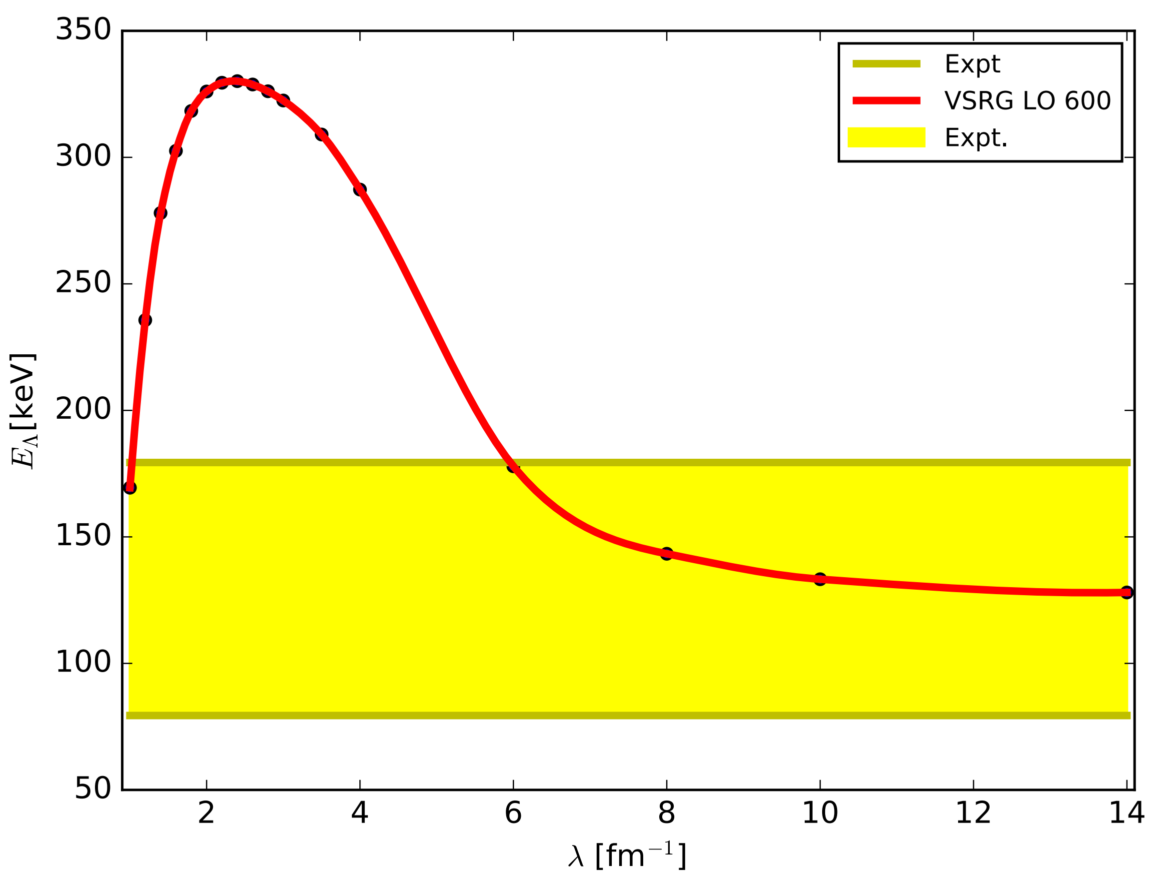
Induced 3BF …

SRG parameter dependence is significant when NN and YN interactions are evolved

→ missing 3N and YNN interactions

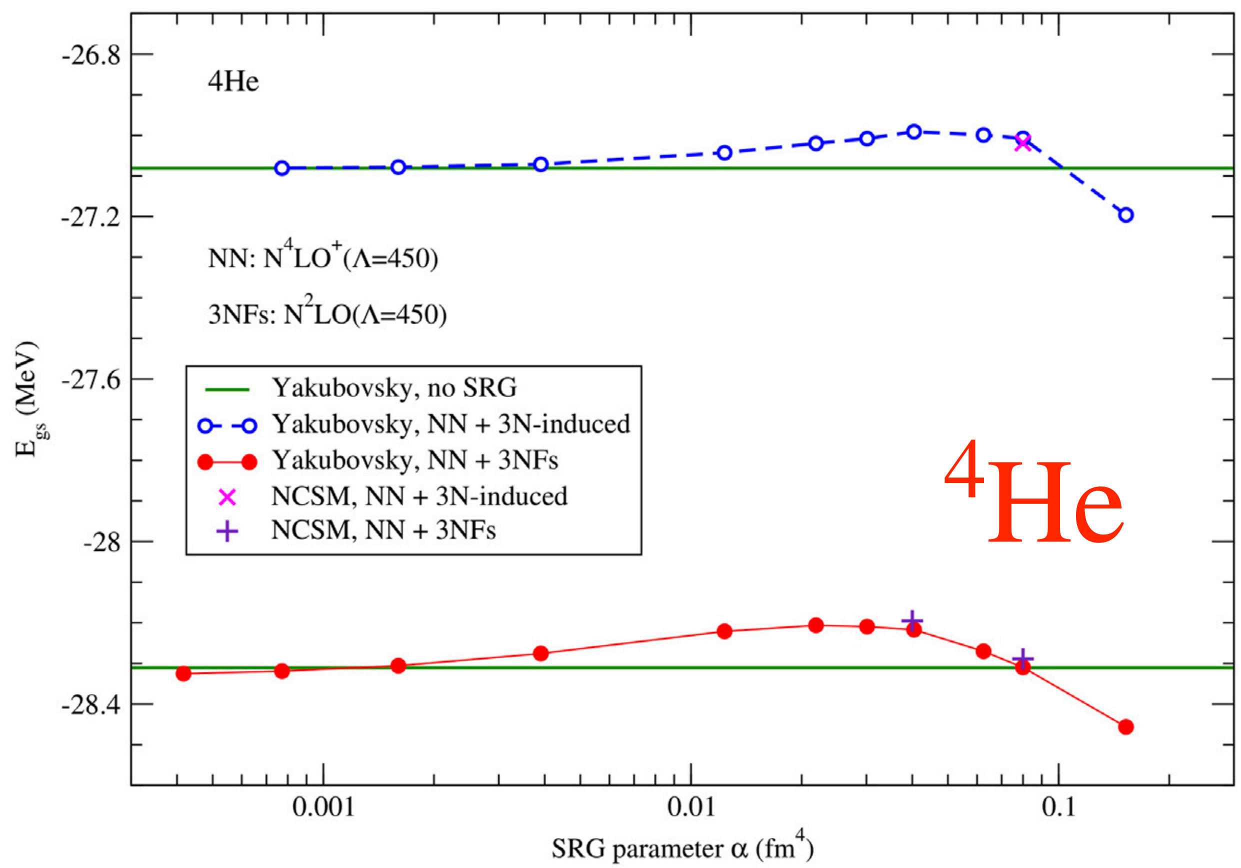
- 3NF is comparable to chiral 3NF
- YNN is larger than chiral YNN





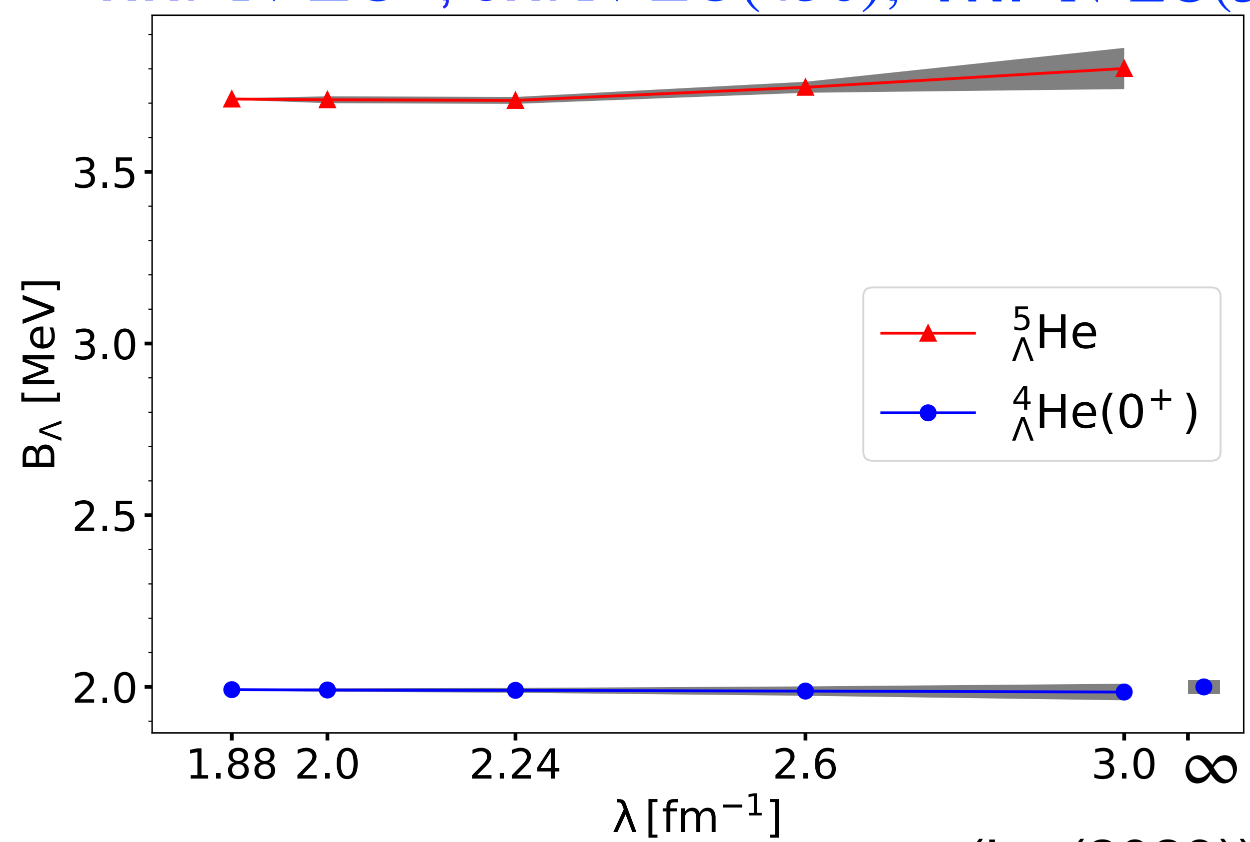
SRG dependence of results

- SRG-induced 3N and YNN interactions
- ^{4}He binding energies varies by $\approx 100 - 200$ keV (relevant in the future?)
- separation energies are even less dependent (YNNN forces small)



(Maris, Le, Nogga, Roth, Vary (2023))

NN: N^{4}LO$^+$, **3N**: N^{2}LO(450); **YN**: N^{2}LO(550)

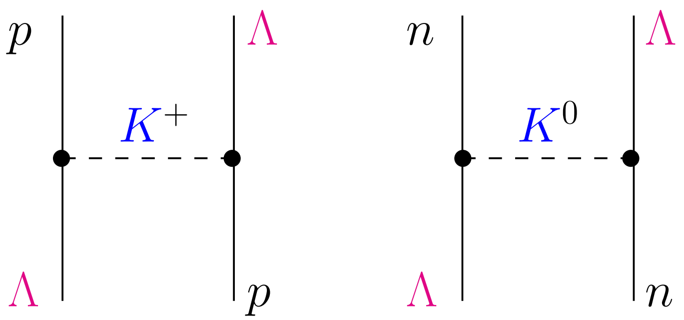


(Le (2023))

For **hypernuclei**, calculations based on SRG induced BB and 3B interactions are sufficiently accurate!

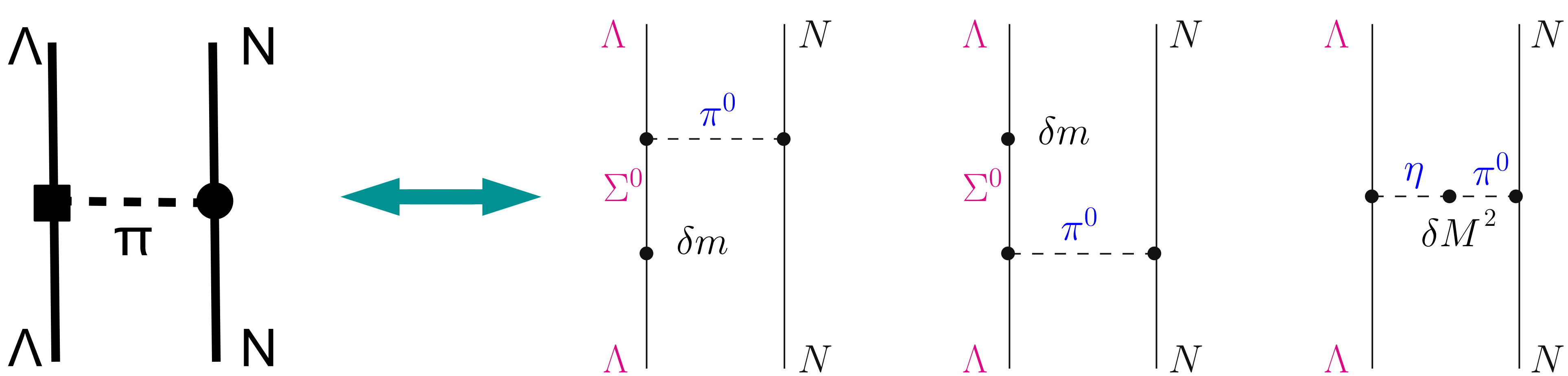
CSB contributions to YN interactions

- **formally leading** contributions:
  Goldstone boson mass difference
  - — very small due to the small relative difference of kaon masses

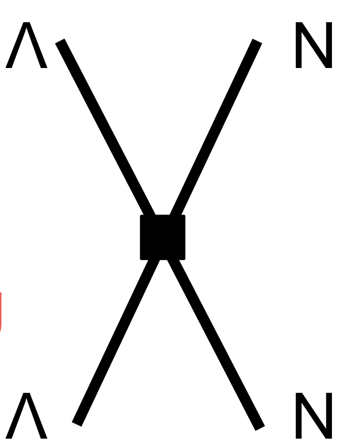


- **subleading but most important**
  - — effective CSB ΛΛπ coupling constant (Dalitz, von Hippel, 1964)

$$f_{\Lambda\Lambda\pi} = \left[-2\frac{\langle \Sigma^0 | \delta m | \Lambda \rangle}{m_{\Sigma^0} - m_\Lambda} + \frac{\langle \pi^0 | \delta M^2 | \eta \rangle}{M_\eta^2 - M_{\pi^0}^2} \right] f_{\Lambda\Sigma\pi} \approx (-0.0297 - 0.0106)\, f_{\Lambda\Sigma\pi}$$

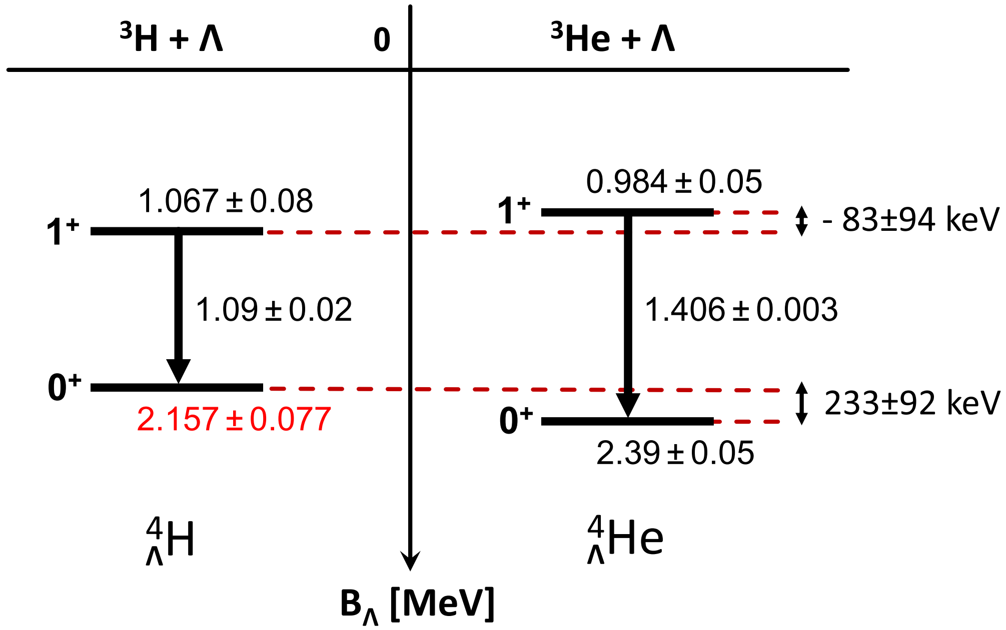


- **so far less considered but necessary for proper renormalization**
  - — CSB contact interactions (for singlet and triplet)



Aim: determine the two unknown CSB LECs and predict Λn scattering

Fit of contact interactions



(Schulz et al.,2016; Yamamoto, 2015)

- Adjust the two CSB contact interactions to one main scenario (**CSB1)**
- update: Mainz average of CSB including new star data: 178 ± 55 keV/ -139 ± 58 keV

  This was not used here.

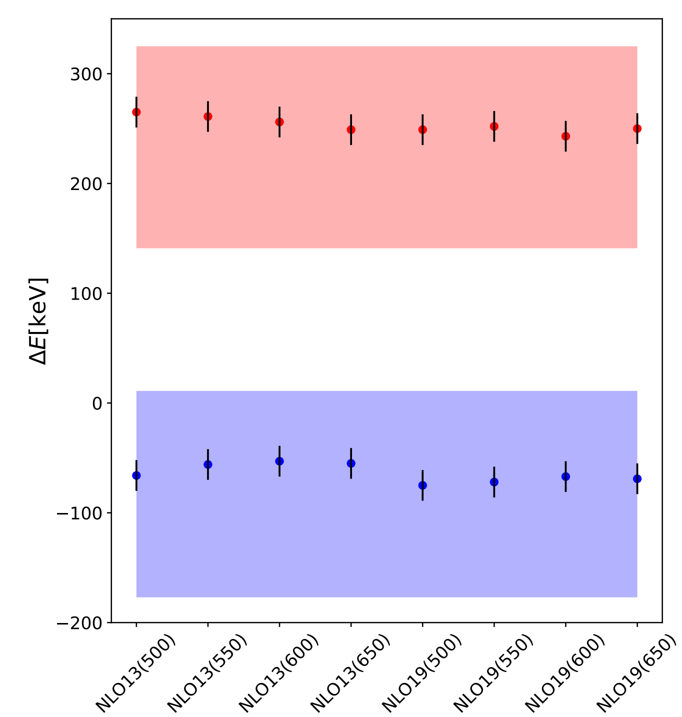
- Fit of counter terms to data: size of LECs as expected by power counting

$$\frac{m_d - m_u}{m_u + m_d} \left(\frac{M_\pi}{\Lambda} \right)^2 C_{S,T} \approx 0.3 \cdot 0.04 \cdot 0.5 \cdot 10^4 \, \mathrm{GeV}^{-2} \propto 6 \cdot 10^{-3} \cdot 10^4 \, \mathrm{GeV}^{-2}$$

- Problem: large experimental uncertainty of experiment later adjust of CSB predictions is likely

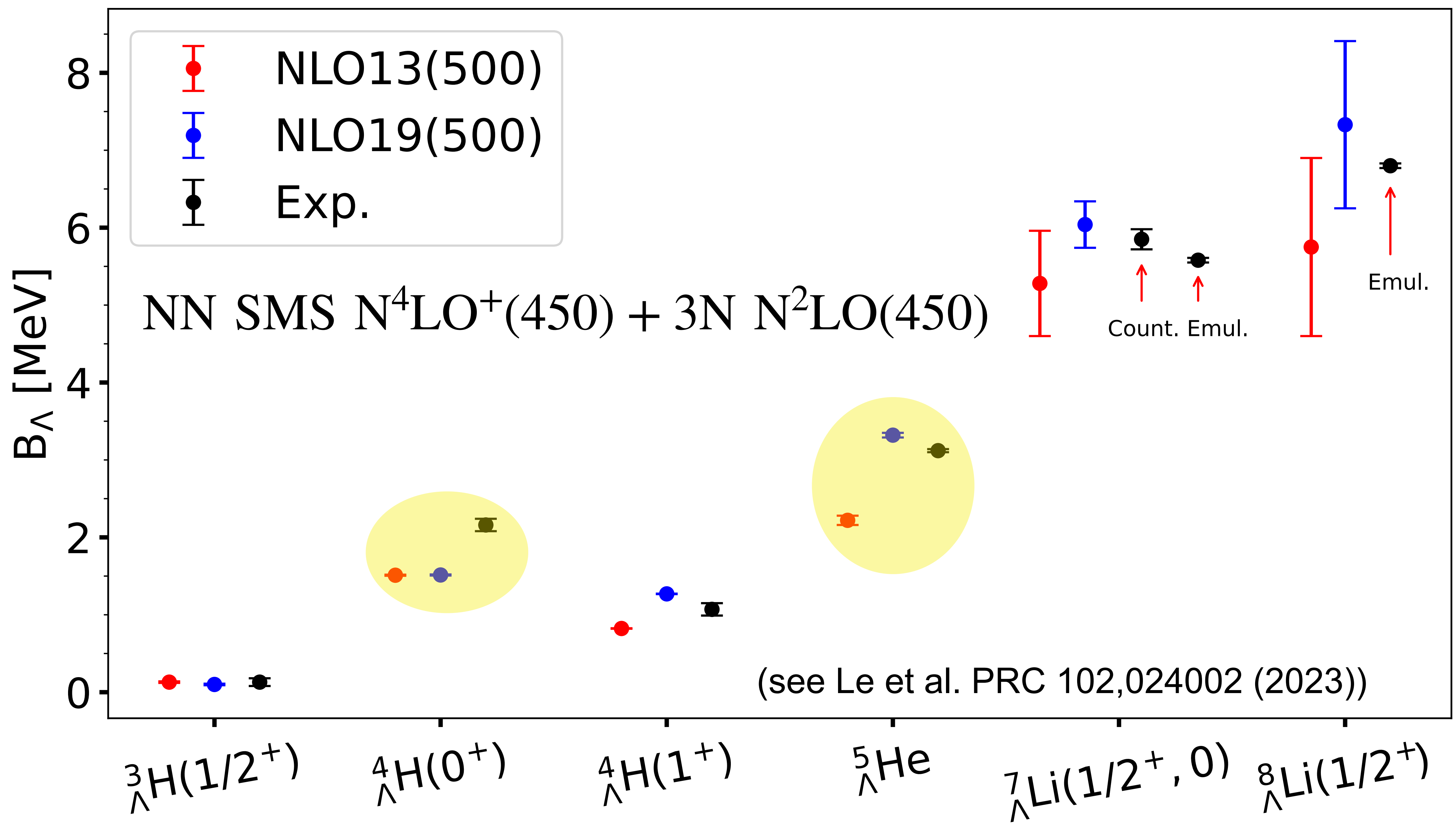
- here only **fit to central values** to test theoretical uncertainties

(see Haidenbauer et al. FBS 62,105 (2021))



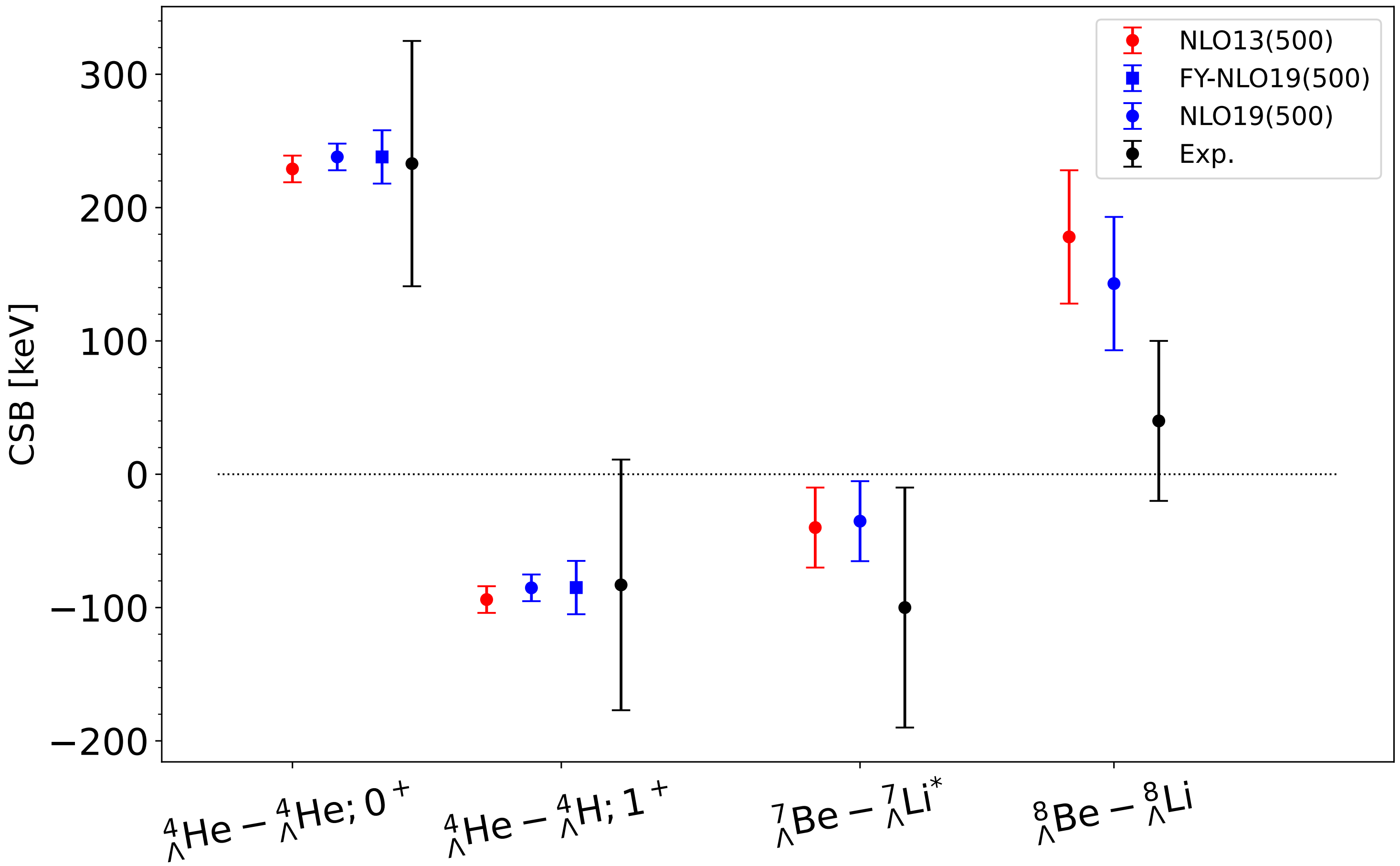
Application to $A = 7$ and 8

- YN interaction adjusted to the hypertriton — YNN is small
- based only on YN interactions: splitting for ${}^{4}_{\Lambda}\mathrm{H}$ is not well reproduced — YNN(?)
- NLO19 gives better results for ${}^{5}_{\Lambda}\mathrm{He}$ and heavier hypernuclei
  — accidentally small YNN interaction?
- uncertainties are numerical — no estimate of chiral uncertainties yet



Predictions for $A = 2,7$ and 8

- CSB scattering length predicted **independent of the realization**
- keep in mind: CSB still not fixed — experimental uncertainty is large
- scenario studied here is only **marginally consistent** with CSB in $A = 8$



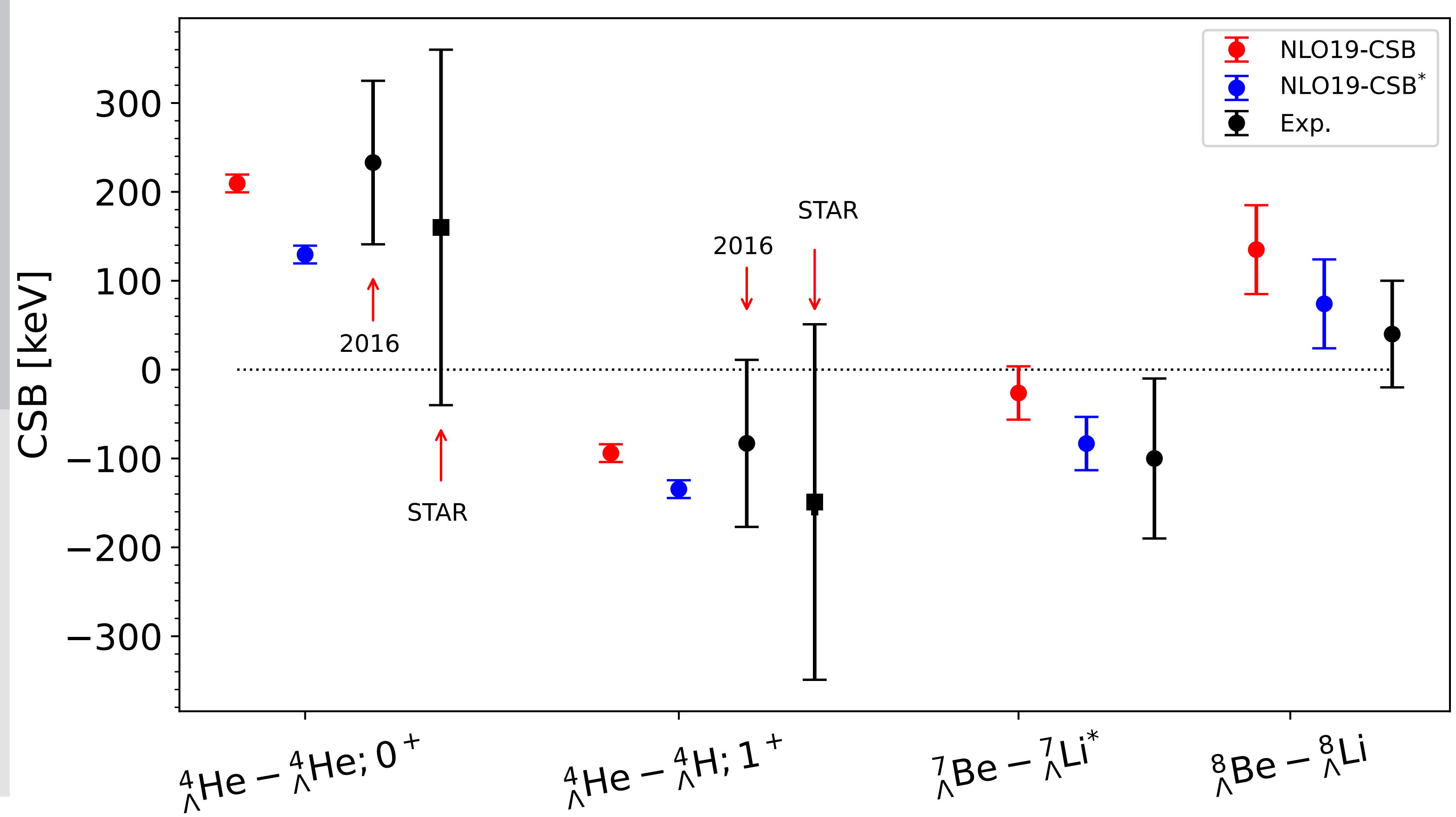
(Le et al. PRC 102,024002 (2023))

	$a_s^{\Lambda p}$	$a_t^{\Lambda p}$	$a_s^{\Lambda n}$	$a_t^{\Lambda n}$
NLO13(500)	-2.604	-1.647	-3.267	-1.561
NLO13(550)	-2.586	-1.551	-3.291	-1.469
NLO13(600)	-2.588	-1.573	-3.291	-1.487
NLO13(650)	-2.592	-1.538	-3.271	-1.452
NLO19(500)	-2.649	-1.580	-3.202	-1.467
NLO19(550)	-2.640	-1.524	-3.205	-1.407
NLO19(600)	-2.632	-1.473	-3.227	-1.362
NLO19(650)	-2.620	-1.464	-3.225	-1.365

(Haidenbauer et al. FBS 62,105 (2021))

New STAR data for $A = 4$ CSB

- fit to STAR data only
- only slight adjustment required
- improves description to p-shell CSB
- higher experimental accuracy is desirable
- good example of using hypernuclei to determine YN interactions



	NLO19(500)	CSB	CSB*
$a_s^{\Lambda p}$	-2.91	-2.65	-2.58
$a_s^{\Lambda n}$	-2.91	-3.20	-3.29
$\boldsymbol{\delta a_s}$	**0**	**0.55**	**0.71**
$a_t^{\Lambda p}$	-1.42	-1.57	-1.52
$a_t^{\Lambda n}$	-1.41	-1.45	-1.49
$\boldsymbol{\delta a_t}$	**-0.01**	**-0.12**	**-0.03**

(see Le et al. PRC 102,024002 (2023))

Uncertainty analysis to $A = 3$ to 5

Order N²LO requires combination of chiral NN, YN, 3N and YNN interaction

Results for **different orders** enable uncertainty estimate:

Ansatz for the order by order convergence:

$$X_K = X_{ref} \sum_{k=0}^{K} c_k \, Q^k \quad \text{where} \quad Q = M_\pi^{eff} / \Lambda_b \quad (X_{ref} \text{ LO, exp., max, …})$$

Bayesian analysis of the uncertainty following Melendez et al. 2017,2019

Extracting c_k for $k \leq K$ **from calculations**

→ **probability distributions** for c_k

→ $$\delta X_K = X_{ref} \sum_{k=K+1}^{\infty} c_k \, Q^k$$

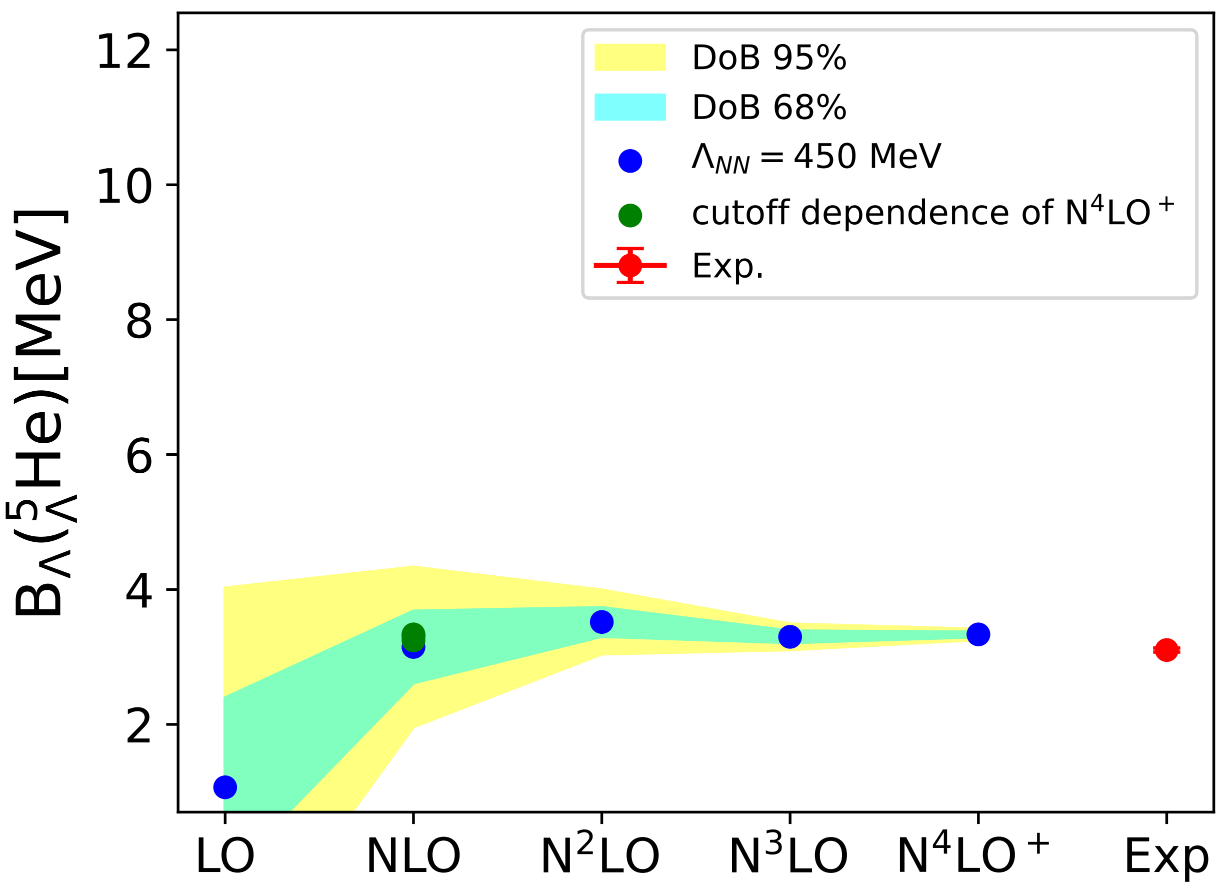
Uncertainty due to missing higher orders is more relevant than numerical uncertainty! (for light nuclei)

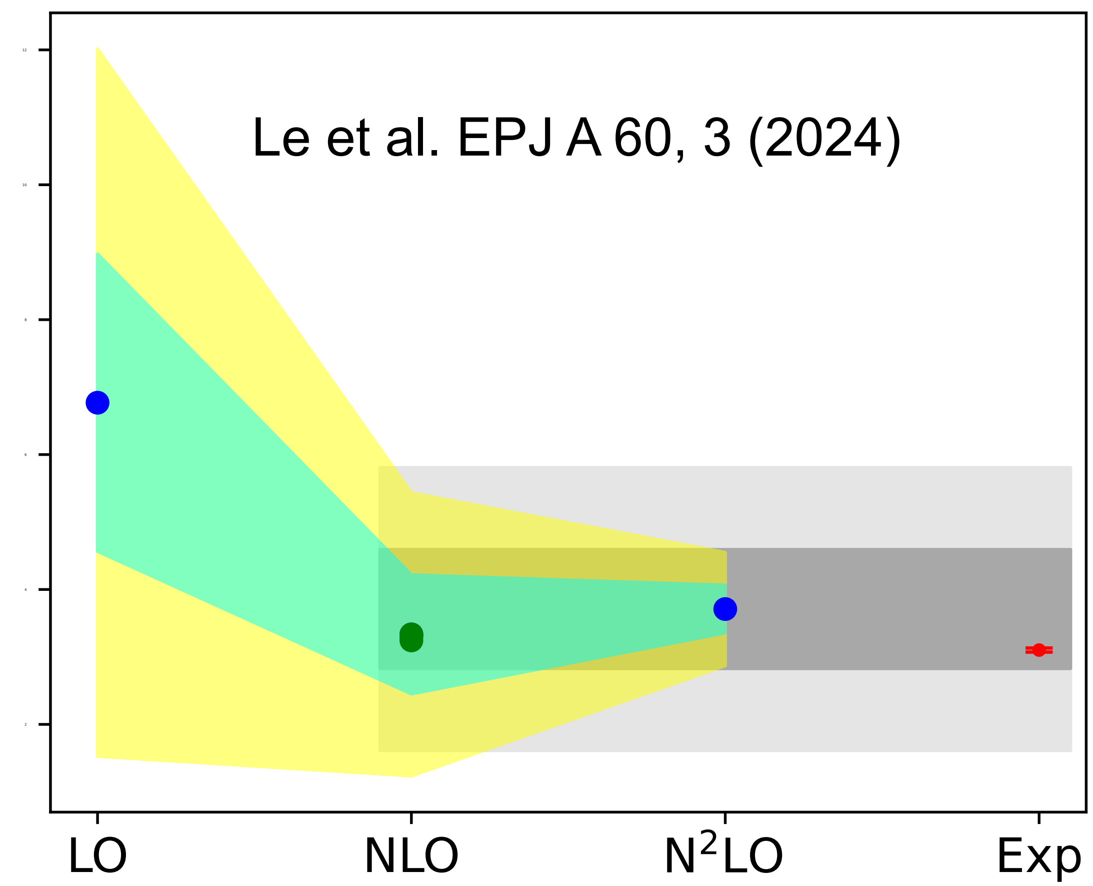
Application to ${}^{5}_{\Lambda}\mathrm{He}$ and summary

- **without YNN**: sizable uncertainties at $A = 4$ and 5
- $A = 3$ sufficiently accurate
- NN/YN dependence small at least for $A = 3$

nucleus	$\Delta_{68}(NN)$	$\Delta_{68}(YN)$
${}^{3}_{\Lambda}\mathrm{H}$	0.011	0.015
${}^{4}_{\Lambda}\mathrm{He}\,(0^+)$	0.157	0.239
${}^{4}_{\Lambda}\mathrm{He}\,(1^+)$	0.114	0.214
${}^{5}_{\Lambda}\mathrm{He}$	0.529	0.881

→ at the same time: estimate of YNN !

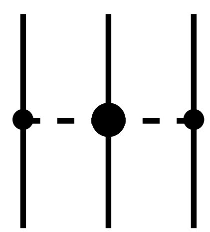




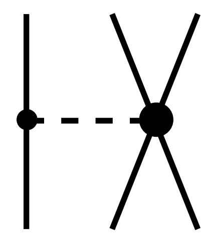
YNN (ΛNN) interactions

Leading 3BF with the usual topologies (Petschauer et al. PRC 93, 014001 (2016))

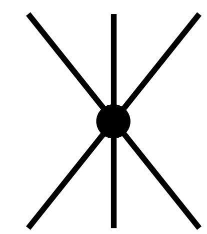
ChPT → all octet mesons contribute → **only take π** explicitly into account



2 LECs in ΛNN
(up to 10)



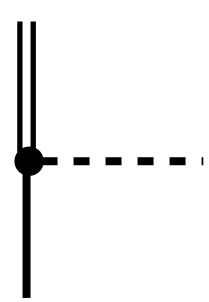
2 LECs in ΛNN
(up to 14)



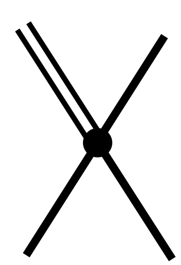
3 LECs in ΛNN
5 LECs in ΣNN + 1 Λ-Σ transition

only few data → need to keep the **# of LECs** small
Decuplet baryons ($\Sigma^*\ldots$) might enhance YNN partly to NLO
(Petschauer et al., NPA 957, 347 (2017))

By decuplet saturation all LECs can be related to the following
leading octet-decuplet transitions (Petschauer et al. Front. Phys. 8,12 (2020))



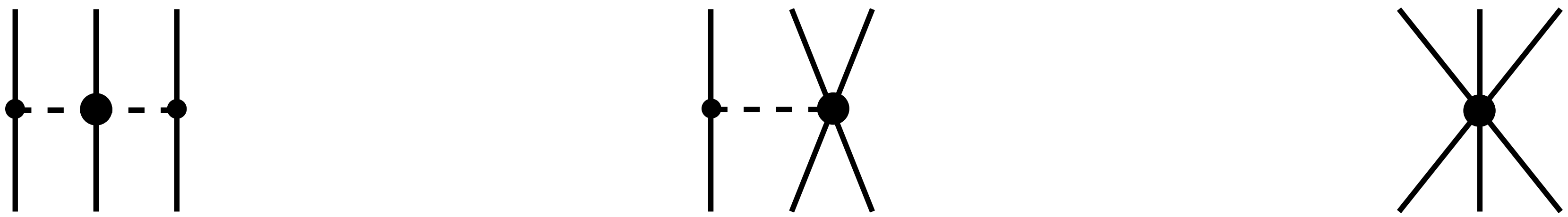
$\propto C = \frac{3}{4} g_A$



$\propto G_1, G_2$ → **reduction to 2 LECs**

YNN (ΛNN) interactions

Decuplet saturation relates all LECs to G_1 and G_2



	$\propto C^2$	$\propto CG_1, CG_2$	$\propto (G_1)^2, (G_2)^2, G_1G_2$
For ΛNN:	$\propto C^2$	$\propto C(G_1 + 3G_2)$	$\propto (G_1 + 3G_2)^2$ 1 LEC

→ density dependent BB interactions (Petschauer et al., NPA 957, 347 (2017))

→ application to nuclear matter (Haidenbauer et al., EPJ A 53, 121 (2017))

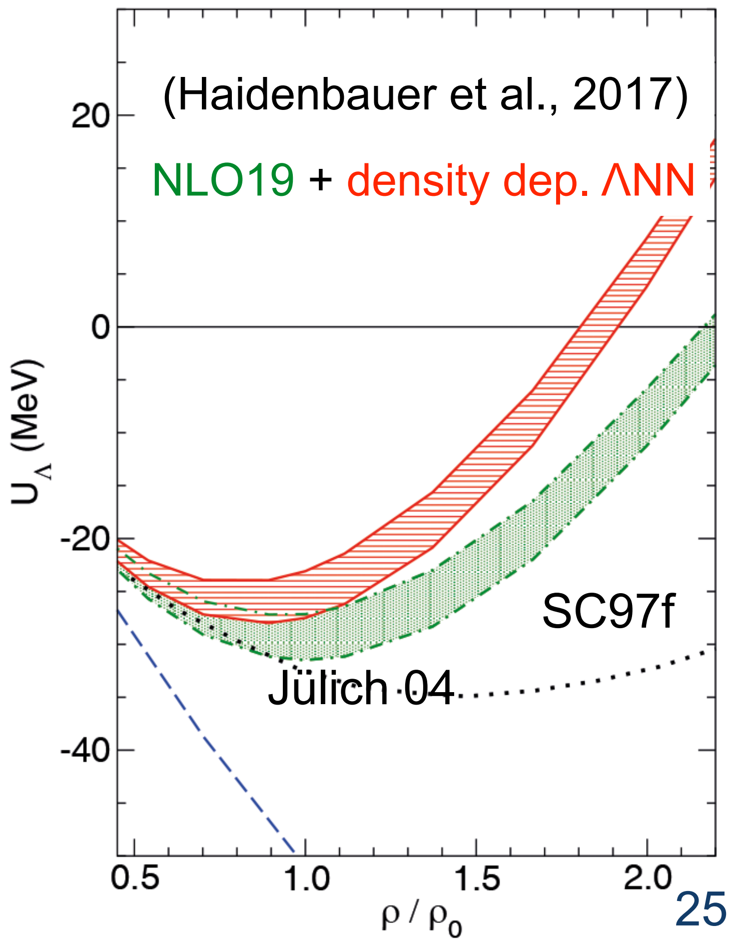
neutron stars (Logoteta et al., EJA 55, 207 (2019))

- contribution on the single particle potentials can be large
- realistic results seem to require partly cancelations of 2π and 1π exchange

(fixes sign of $G_1 + 3G_2$!)

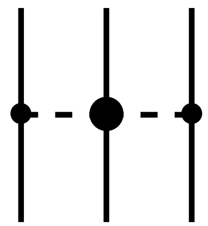
Recently: successful benchmark of matrix elements (Hoai Le et al. EPJ A 61,21 (2025))

and first direct application to light hypernuclei including Σ's (Hoai Le et al. PRL 134, 072502 (2025))

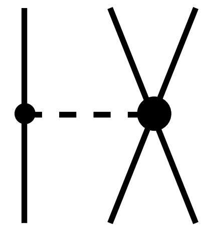


YNN (ΛNN) interactions in practice

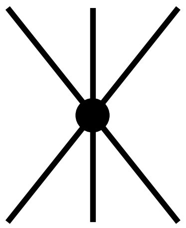
Decuplet approximation in YNN



$\propto C^2$



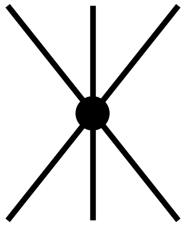
$\propto CG_1, CG_2$



$\propto (G_1)^2, (G_2)^2, G_1 G_2$

is not sufficient to fix spin dependence

→ + ΛNN contact terms **without decuplet constraints**



ΛNN $\propto C'_1, C'_2, C'_3$

***ad hoc* choice:** alter C_2:

$$C'_1 = C'_3 = \frac{(G_1 + 3G_2)^2}{72\Delta}$$

$$C'_2 = 0$$

→

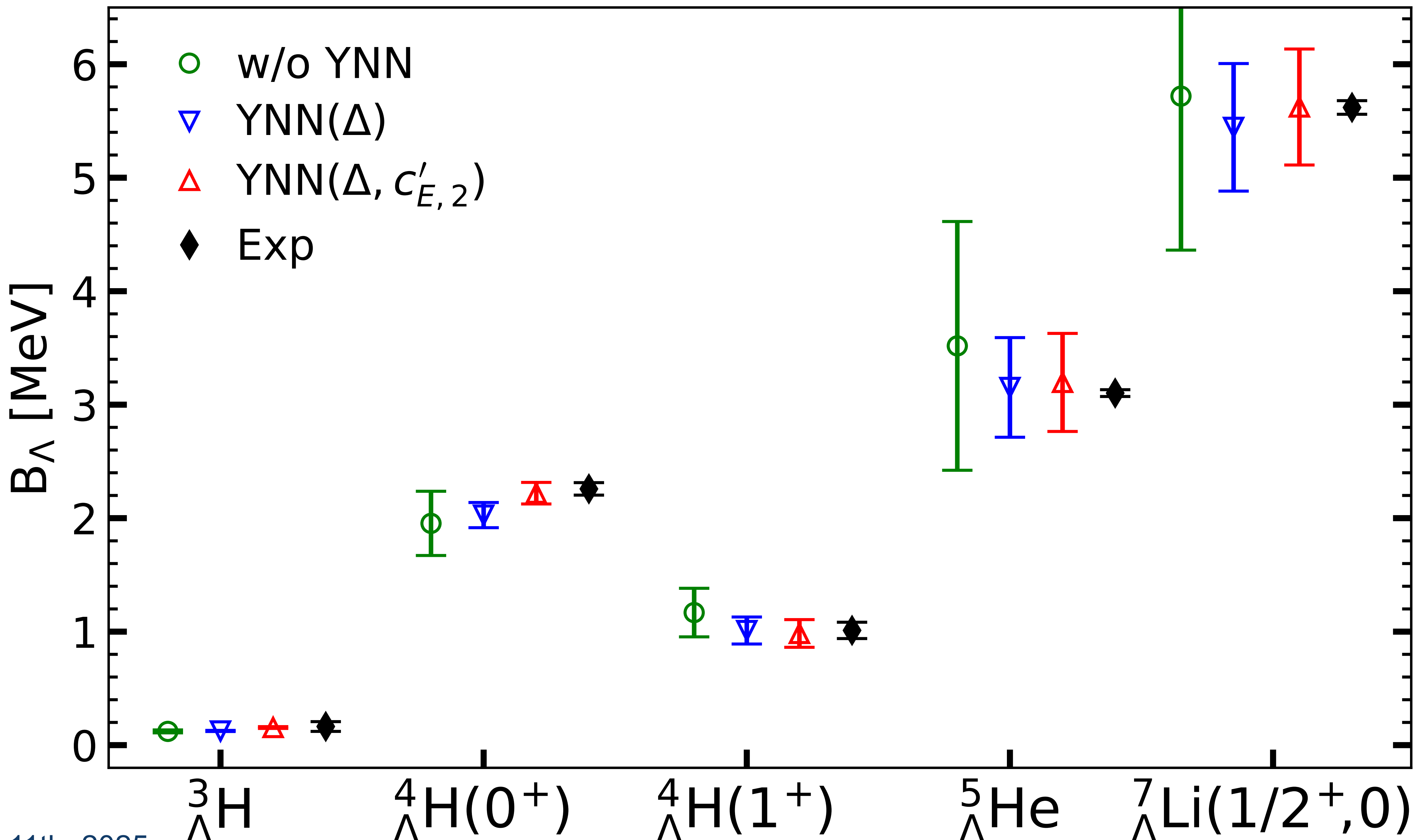
$$V_{\Lambda\mathrm{NN}} = C_2{}' \, \vec{\sigma}_1 \cdot \left(\vec{\sigma}_2 + \vec{\sigma}_3\right) \left(1 - \vec{\tau}_2 \cdot \vec{\tau}_3\right)$$

$$C'_2 = G_3$$

C'_2 introduces a spin dependent interaction in the most relevant particle channel

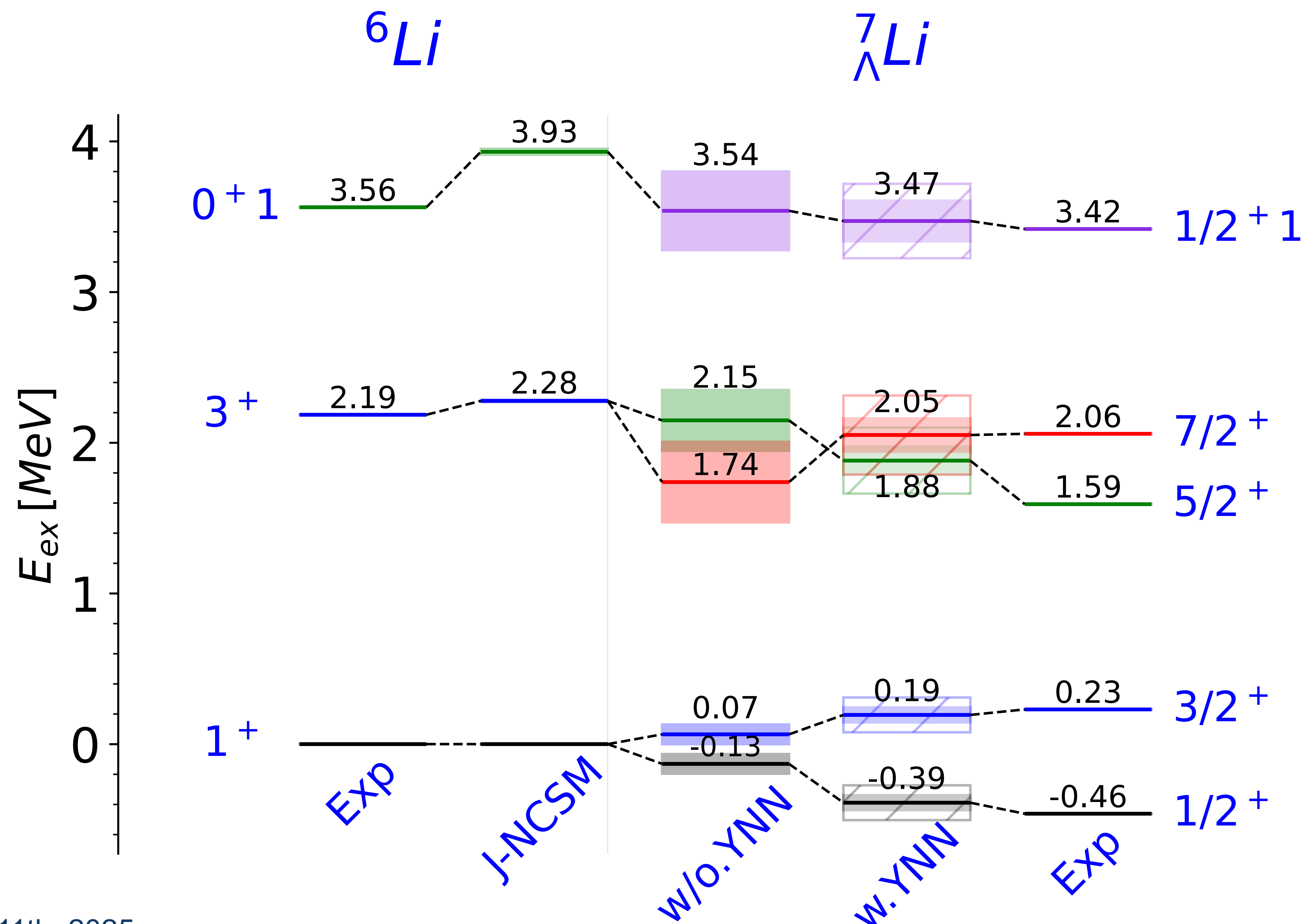
YNN fit

- Fit to 0^+ and 1^+ state of ${}^4_\Lambda\mathrm{He}$ and/or ${}^5_\Lambda\mathrm{He}$
- spin-dependence in A=4 not well explained by decuplet saturation
- C_2' term improves 0^+ of ${}^4_\Lambda\mathrm{He}$ and $1/2^+$ of ${}^7_\Lambda\mathrm{Li}$
- agreement generally much better than $\mathrm{N^2LO}$ uncertainty



YNN prediction for $^{7}_{\Lambda}\mathrm{Li}$

- good agreement
- C_2' term included, but not very important (not shown)
- higher states have significant uncertainty



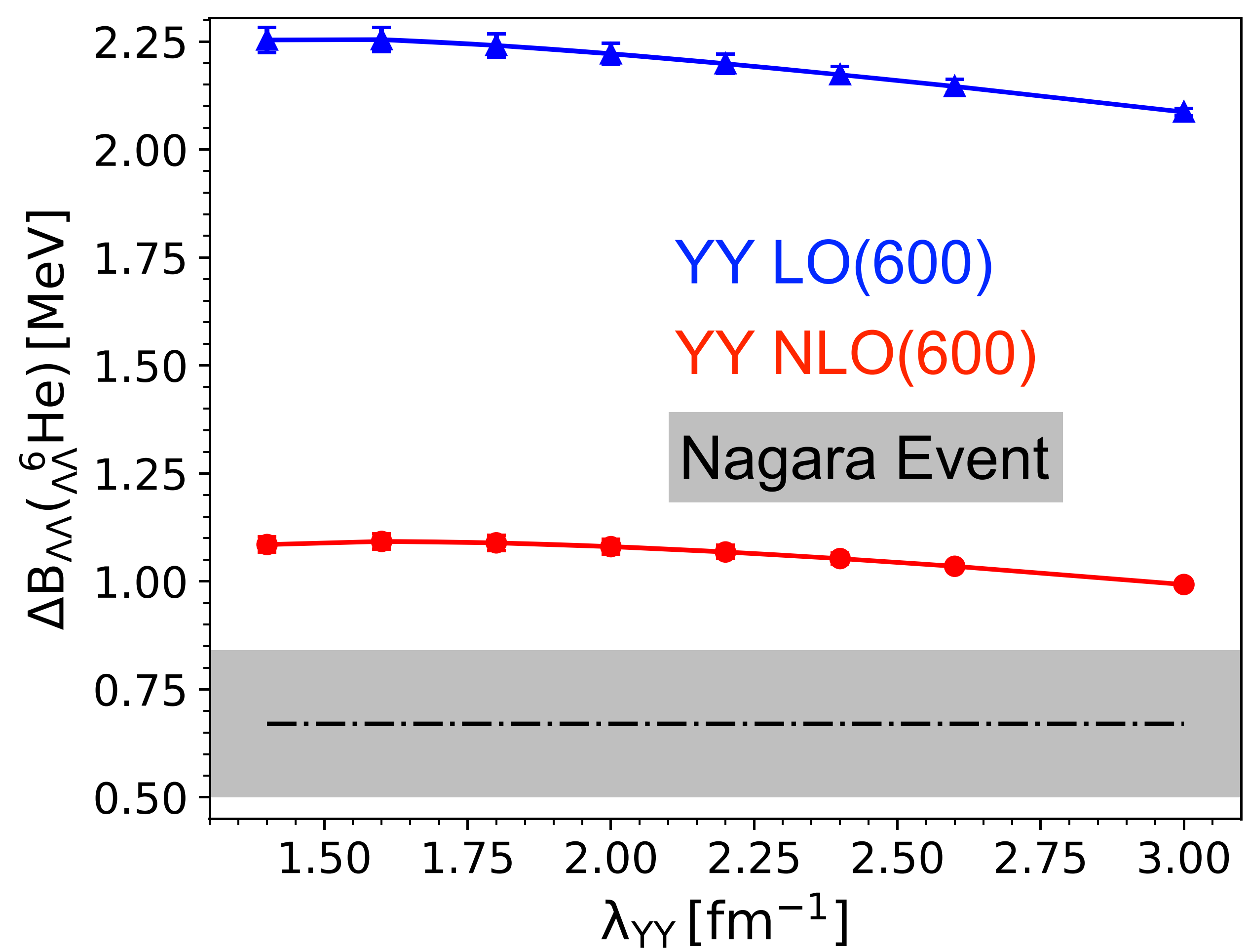
$S = -2$ hypernuclei — $^{6}_{\Lambda\Lambda}\mathrm{He}$

- $\Lambda\Lambda$ excess binding energy

$$\Delta B_{\Lambda\Lambda} = B_{\Lambda\Lambda} - 2B_{\Lambda}$$
$$= 2E\left({}^{A-1}_{\Lambda}X\right) - E\left({}^{A}_{\Lambda\Lambda}X\right) - E\left({}^{A-2}X\right)$$

- NN, YN and YY interactions contribute
- use NN and YN that describe nuclei and single Λ hypernuclei
- small λ_{YY} dependence
- LO overbinds YY
- NLO predicts binding fairly well

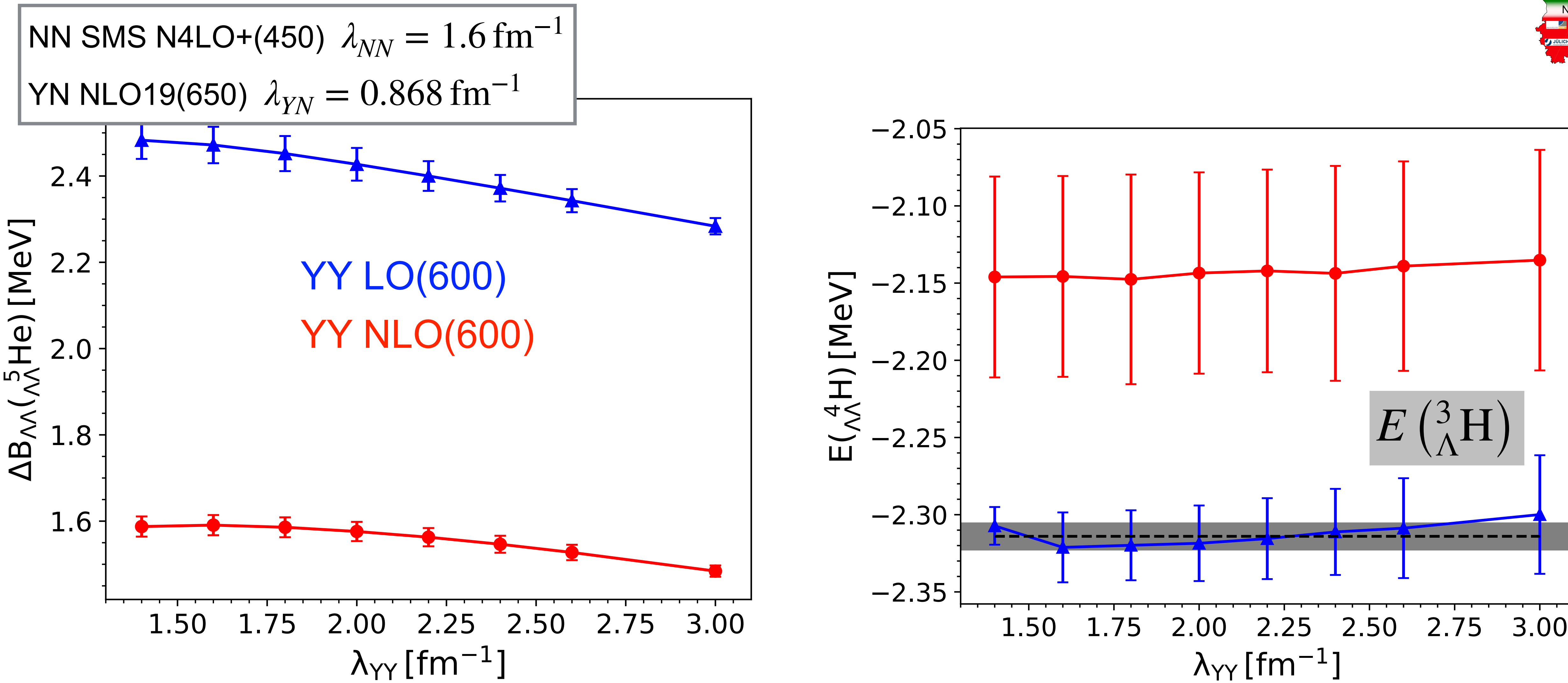
(Le et al., 2021)



NN SMS N4LO+(450) $\lambda_{NN} = 1.6\,\mathrm{fm}^{-1}$
YN NLO19(650) $\lambda_{YN} = 0.868\,\mathrm{fm}^{-1}$

Can an $S = -2$ bound state for $A = 4{,}5$ be expected?

$S = -2$ hypernuclei — ${}^{5}_{\Lambda\Lambda}\mathrm{He}$ & ${}^{4}_{\Lambda\Lambda}\mathrm{H}$



- $A = 5$: $\Lambda\Lambda$ excess binding energy & $A = 4$: binding energy
- $A = 5$: LO & NLO predicts bound state
- $A = 4$: NLO unbound, LO at threshold to binding (see also Contessi et al., 2019)
- excess energy larger for $A = 5$ than for $A = 6$ (in contrast to Filikhin et al., 2002!)

$S = -2$ bound state for $A = 5$ can be expected,

for $A = 4$ less likely but not ruled out!

Two-nucleon densities

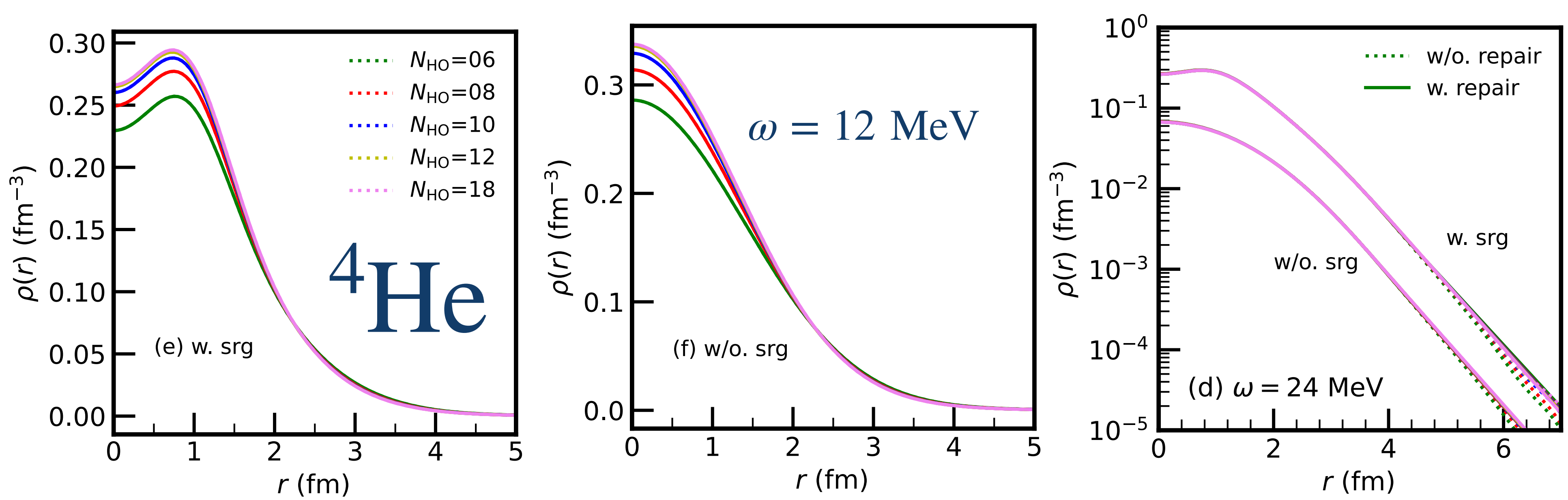
- SRG evolution affects wave functions
- short-range, medium and high momentum observables affected
- unitary transformation of operators

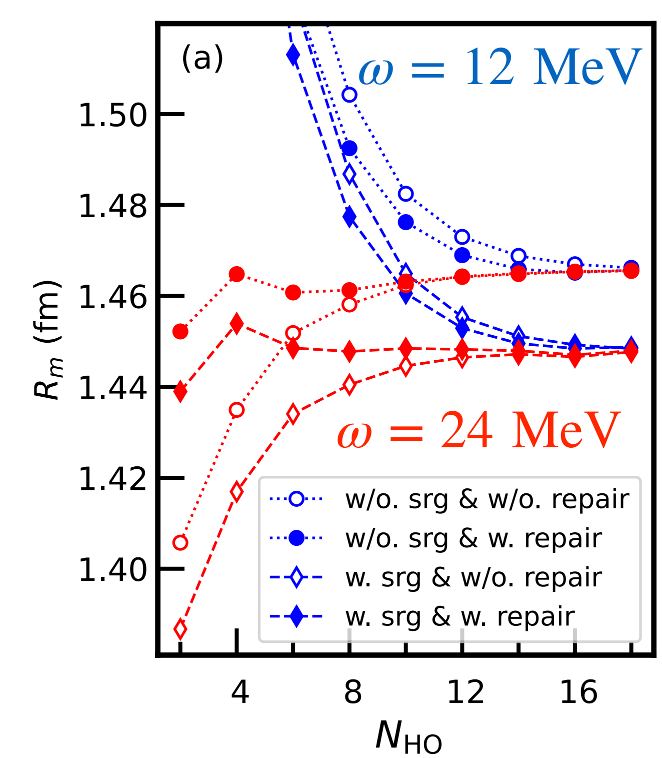
- HO basis inefficient for describing exponential tail of wave functions
- HO frequency usually optimized for describing wave functions in range of interactions

→

- define densities (in p- and r-space, 1-nucleon and 2-nucleon)
- apply SRG on densities (2-nucleon only!)
- correct long-range tail for long-ranged observables (2-nucleon only)
- calculations of matter and charge radii of light nuclei

Xiang-Xiang Sun et al. arXiv:2502.03989 [nucl-th]



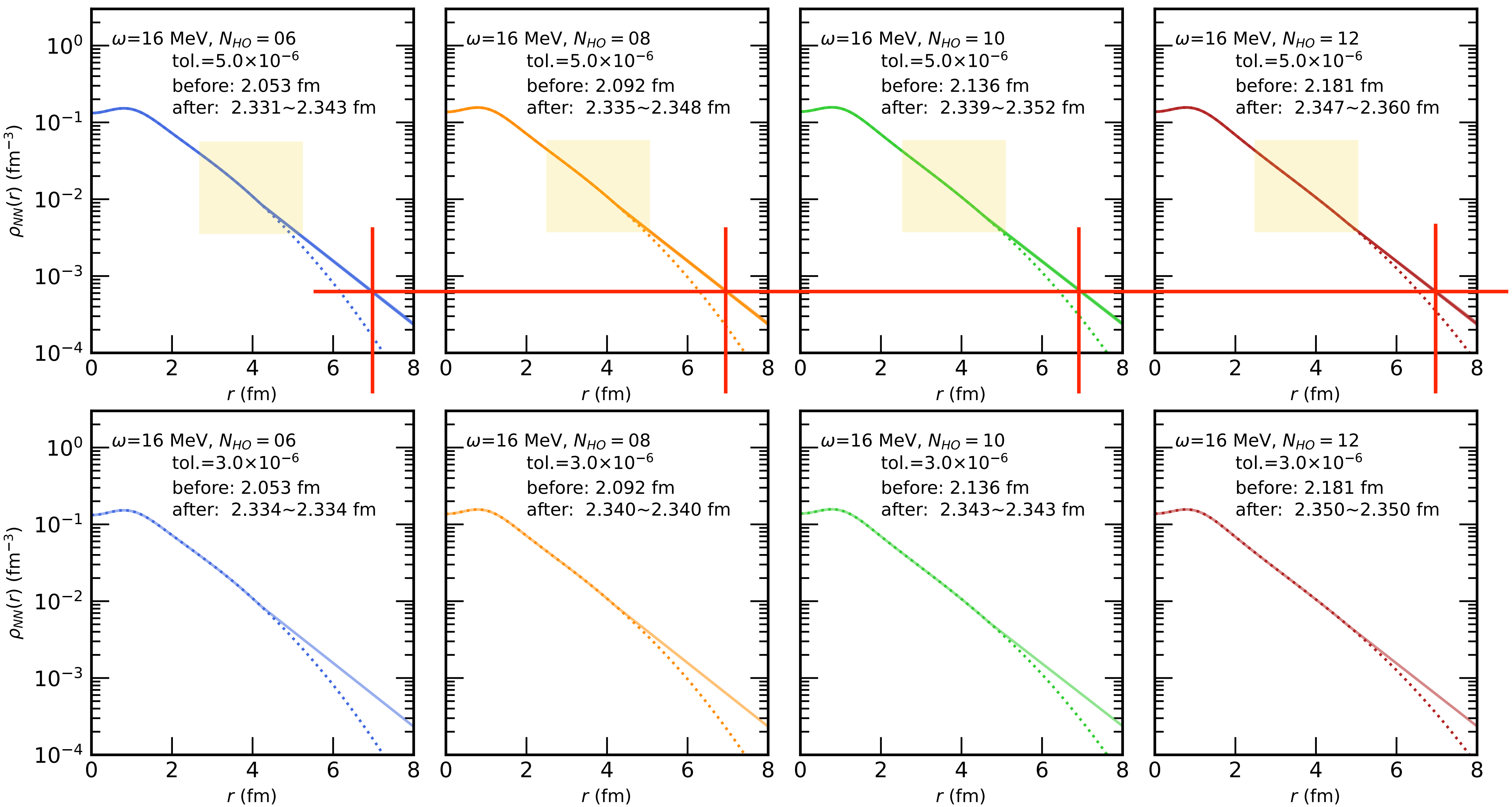


densities available: https://jugit.fz-juelich.de/a.nogga/nucdensity

Two-nucleon densities

^{6}Li: fitting the correction

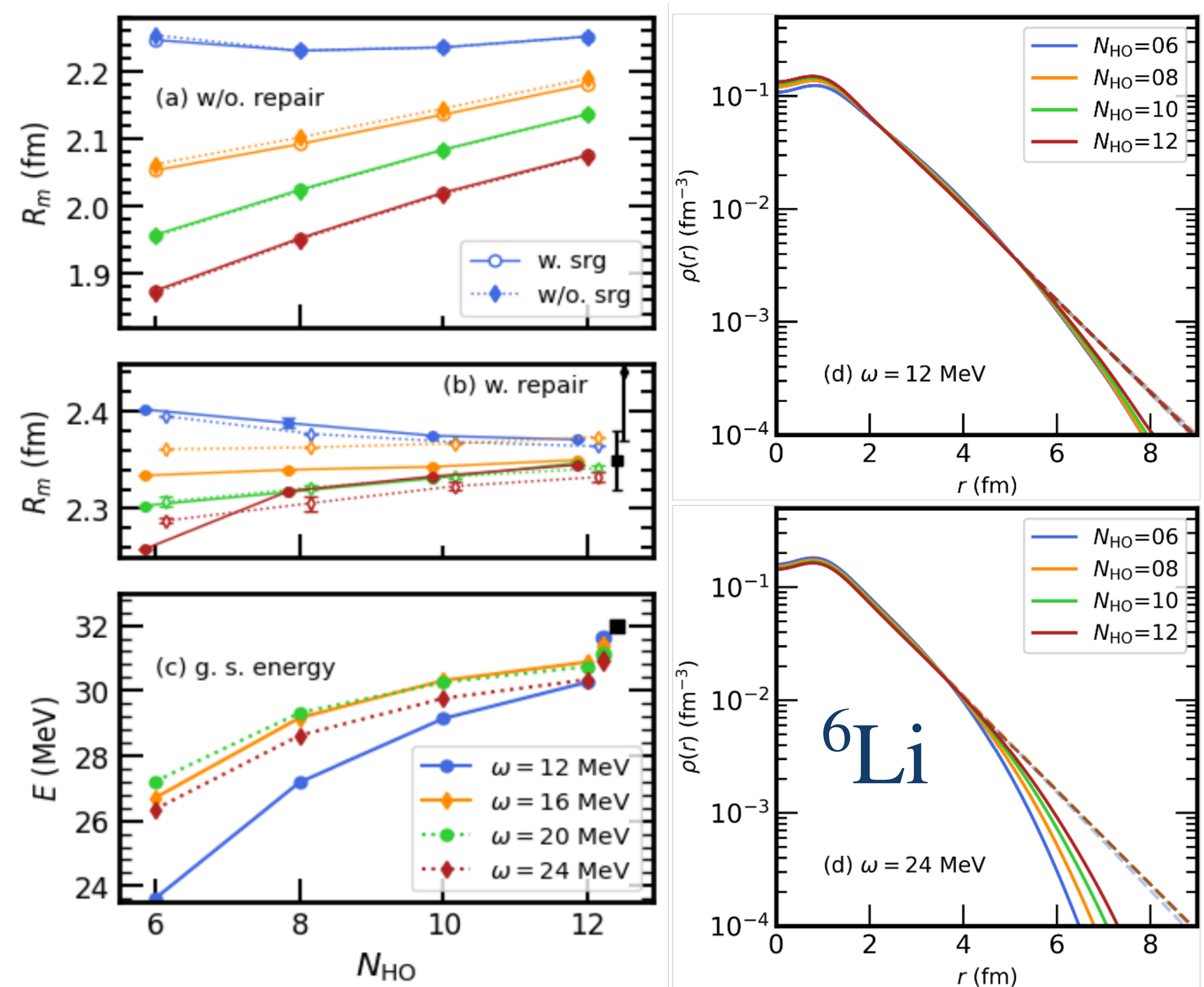
- fit between 77 ranges between 2.5 and 4.8 fm for different N_{HO} and selected ω
- choose densities that give same value at 7 fm for each N_{HO} within tolerance
- lower tolerance until radii are the same



Xiang-Xiang Sun et al. arXiv:2502.03989 [nucl-th]

Two-nucleon densities

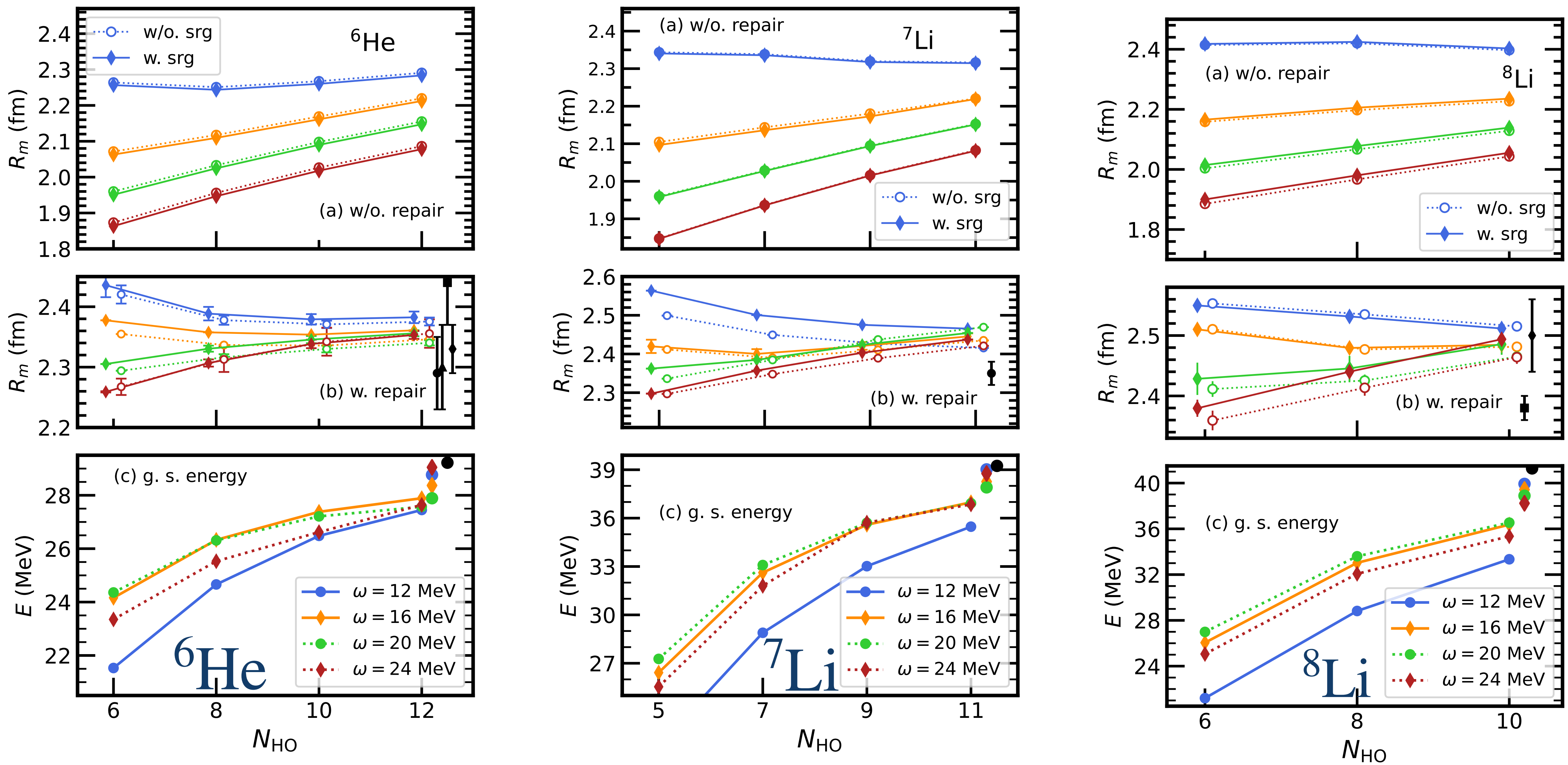
- SRG correction for radius small
- tail correction important to obtain convergence
- matter radius consistent with experiment



Xiang-Xiang Sun et al. arXiv:2502.03989 [nucl-th]

Two-nucleon densities

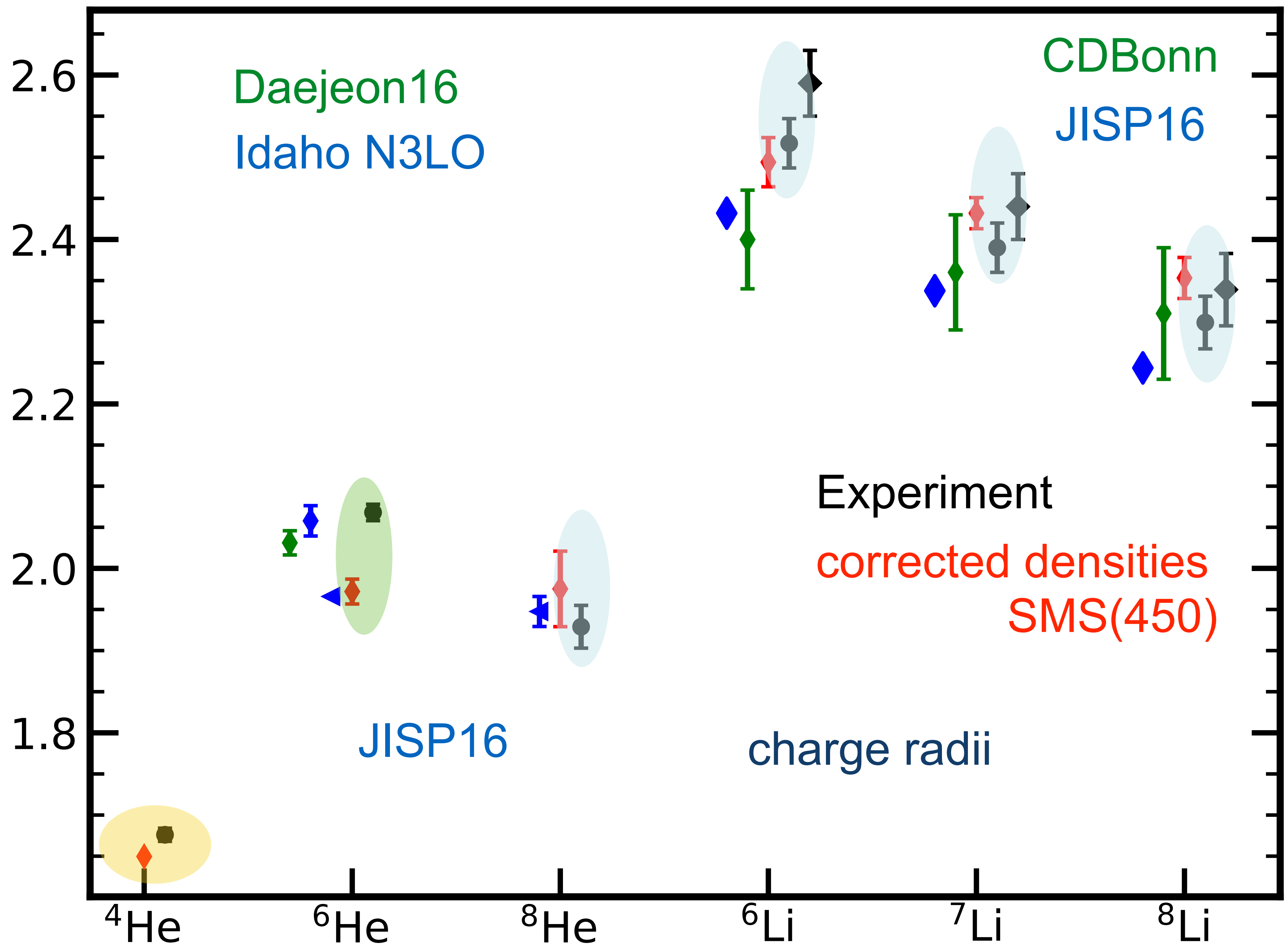
- without correction no convergence and difficult extrapolations
- correction leads to ω **independent** result
- **radii increase** due to correction
- generally agreement with experiment with large uncertainties



Xiang-Xiang Sun et al. arXiv:2502.03989 [nucl-th]

Two-nucleon densities

- does not include 2N corrections (see Filin et al. PRL 2020)
- also **charge radii generally increase** due to correction
- mostly agreement with experiment with large uncertainties



Xiang-Xiang Sun et al. arXiv:2502.03989 [nucl-th]

Conclusions & Outlook

- **YN interactions not well understood**
  - *scarce YN data*
  - *more information necessary to solve "hyperon puzzle"*
- **Hypernuclei provide important constraints**
  - $^1S_0$ ΛN *scattering length &* ${}^{3}_{\Lambda}\mathrm{H}$
  - $^1S_0$ $\Lambda\Lambda$ *scattering length &* ${}^{\;\;6}_{\Lambda\Lambda}\mathrm{He}$ *& predictions for A=4,5*
  - *CSB of* ΛN *scattering &* ${}^{4}_{\Lambda}\mathrm{He}$ / ${}^{4}_{\Lambda}\mathrm{H}$
- **New SMS YN interactions**
  - *order LO, NLO and N²LO allow uncertainty quantification*
  - *have a **non-unique** determination of contact interactions (data necessary)*
- **Chiral 3BF**
  - *decuplet saturation alone does not improve spin dependence*
  - *spin-dependent* $\Lambda\mathrm{NN}$ *leads to further improvement*
  - *study cutoff dependence / application to more p-shell hypernuclei*
- ***SRG & long-range correction to densities***
  - *increased accuracy of densities*
  - *new applications of NCSM wave functions possible*
  - *form factor calculations in progress (including 2N charge densities)*