Reaction models in nuclear astrophysics

P. Descouvemont
Université Libre de Bruxelles, Brussels, Belgium

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3. Reaction models
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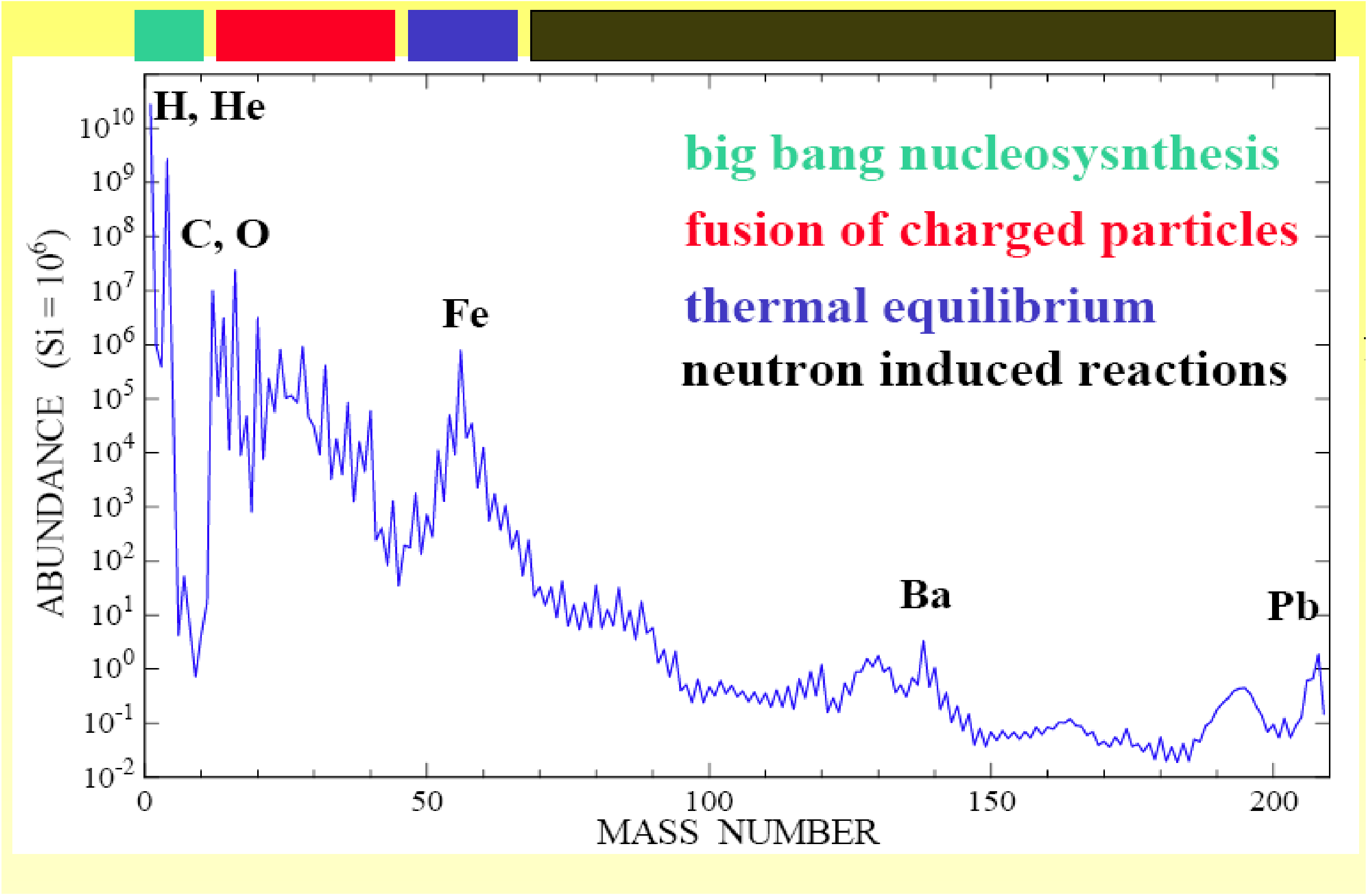
1. Introduction

Goal of nuclear astrophysics: understand the abundances of the elements



- H, ^{4}He most abundant (~75%, ~25%)
- « Gap » between A=4 and A=12: no stable element with A=5 and 8
- Even-odd effects: nuclei with A even are more bound
- Iron peak (very stable)

2. Reactions in astrophysics: general properties

Types of reactions: general definitions valid for all models

Type	Example	Origin
Transfer	$^{3}He(^{3}He,2p)\alpha$	Strong
Radiative capture	$^{2}H(p,\gamma)^{3}He$	Electromagnetic
Weak capture	$p+p \rightarrow d + e^{+} + \nu$	Weak

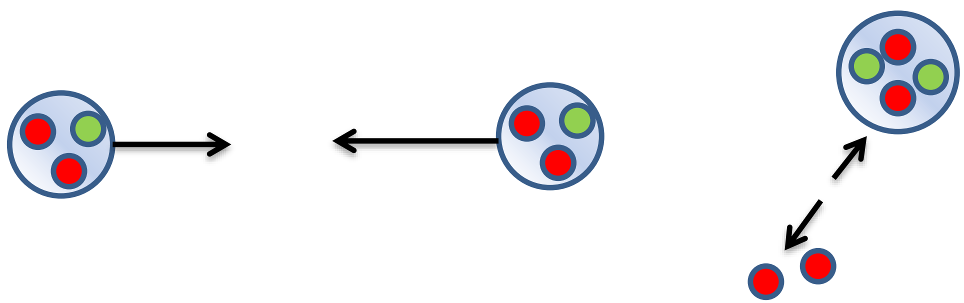
cross section decreases

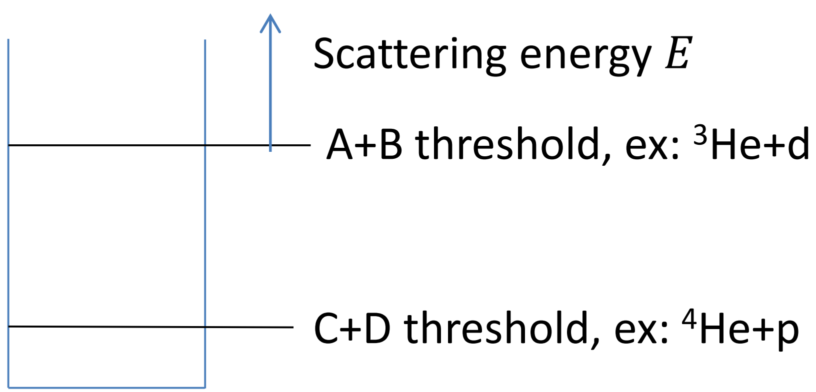
- Transfer: A+B → C+D (σ_t, strong interaction, example: ^{3}He(d,p)^{4}He)

$$\sigma_{t,c\to c'}(E) = \frac{\pi}{k^2}\sum_{J\pi}\frac{2J+1}{(2I_1+1)(2I_2+1)}\left|U^{J\pi}_{cc'}(E)\right|^2$$

$U^{J\pi}_{cc'}(E)$ =collision matrix (obtained from scattering theory → various models)
c, c' =entrance and exit channels

Transfer reaction:
Nucleons are transfered





Compound nucleus, ex: ^{5}Li

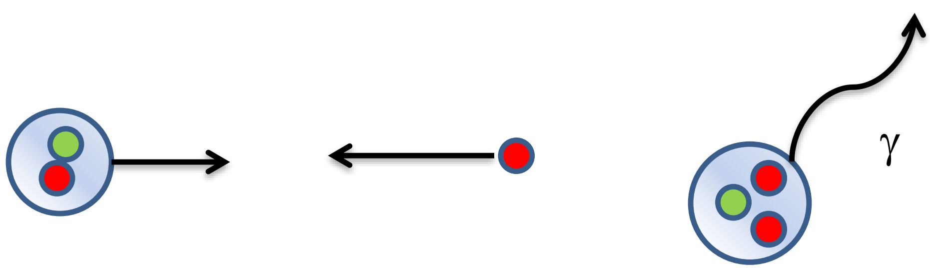
2. Reactions in astrophysics: general properties

- Radiative capture : A+B → C+γ (σ_C, electromagnetic interaction, example: $^{12}C(p,\gamma)^{13}N$)

$$\sigma_C^{J_f\pi_f}(E) \sim \sum_{\lambda}\sum_{J_i\pi_i} k_\gamma^{2\lambda+1} \left| < \Psi^{J_f\pi_f} \| \mathcal{M}_\lambda \| \Psi^{J_i\pi_i}(E) \right|^2$$

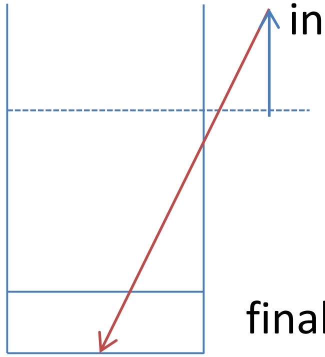
$J_f\pi_f$=final state of the compound nucleus C
$\Psi^{J_i\pi_i}(E)$=initial scattering state of the system (A+B)
$\mathcal{M}_{\lambda\mu}$=electromagnetic operator (electric or magnetic): $\mathcal{M}_{\lambda\mu} \sim e\, r^\lambda Y_\lambda^\mu(\Omega_r)$

Capture reaction:
A photon is emitted



Long wavelength approximation:
Wave number $k_\gamma = E_\gamma/\hbar c$, wavelength: $\lambda_\gamma = 2\pi/k_\gamma$
Typical value: $E_\gamma = 1\ MeV, \lambda_\gamma \approx 1200$ fm >> typical dimensions of the system (R)
→ $k_\gamma R \ll 1$= Long wavelength approximation

2. Reactions in astrophysics: general properties



initial state E>0 , contains all $J_i\pi_i$,

A+B threshold, ex: ^{12}C+p

final states E_f <0, specific $J_f\pi_f$

$$\sigma_C^{J_f\pi_f}(E) \sim \sum_{J_i\pi_i}\sum_{\lambda} k_\gamma^{2\lambda+1}\left|< \Psi^{J_f\pi_f}\|\mathcal{M}_\lambda\|\Psi^{J_i\pi_i}(E) >\right|^2$$

- $k_\gamma = \left(E - E_f\right)/\hbar c$ = photon wave number
- In practice
  - Summation over λ limited to 1 term (often E1, or E2/M1 if E1 is forbidden)

    $\frac{E2}{E1} \sim \left(k_\gamma R\right) \ll 1$ (from the long wavelength approximation)
  - Summation over $J_i\pi_i$ limited by selection rules

    $\left|J_i - J_f\right| \leq \lambda \leq J_i + J_f$

    $\pi_i\pi_f = (-1)^\lambda$ for electric, $\pi_i\pi_f = (-1)^{\lambda+1}$ for magnetic

2. Reactions in astrophysics: general properties

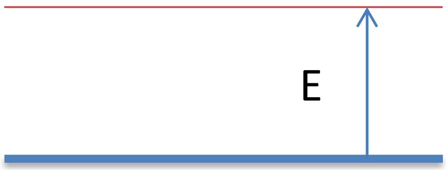
- Weak capture : tiny cross section → no measurement (only calc.)

$$\sigma_W^{J_f\pi_f}(E) \sim \sum_{J_i\pi_i} \left| < \Psi^{J_f\pi_f} \left\| O_\beta \right\| \Psi^{J_i\pi_i}(E) > \right|^2$$

  - Calculations similar to radiative capture
  - O_β= Fermi ($\sum_i t_{i\pm}$) and Gamow-Teller ($\sum_i t_{i\pm}\sigma_i$) operators
  - Examples: p+p→ d+ν+e$^+$: first reaction in H burning (pp chain)
    ^{3}He+p→ ^{4}He+ν+e$^+$: produces high-energy neutrinos

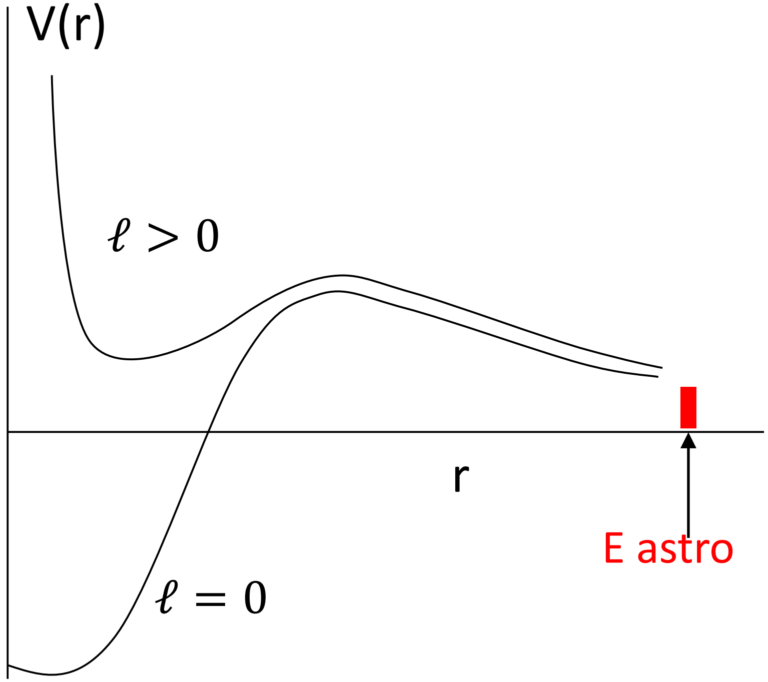
- Fusion: similar to transfer, but with many output channels
  → statistical treatment
  → optical potentials
  Examples: ^{12}C+^{12}C, ^{16}O+^{16}O, etc.

General properties



Scattering energy E: wave function $\Psi_i(E)$ common to all processes

Reaction threshold



- Cross sections dominated by Coulomb effects
  Sommerfeld parameter $\eta = Z_1 Z_2 e^2/\hbar v$
- Coulomb functions at low energies

$$F_\ell(\eta, x) \to \exp(-\pi\eta)\,\mathcal{F}_\ell(x),$$
$$G_\ell(\eta, x) \to \exp(\pi\eta)\,\mathcal{G}_\ell(x),$$

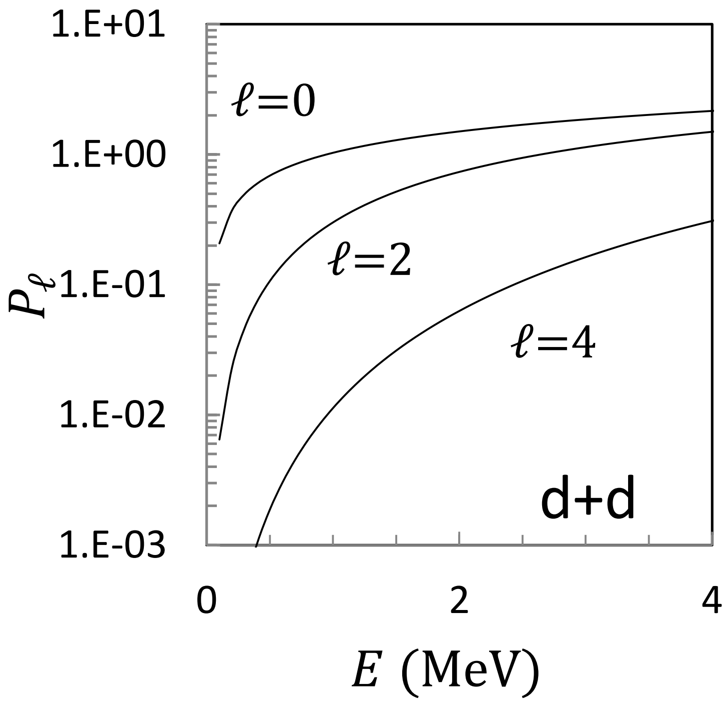
- Coulomb effect: strong E dependence : $\exp(-2\pi\eta)$
  neutrons: $\sigma(E) \sim 1/v$
- Strong ℓ dependence
  Centrifugal term: $\sim \frac{\hbar^2}{2\mu}\frac{\ell(\ell+1)}{r^2}$
  → stronger for nucleons ($\mu \approx 1$) than for α ($\mu \approx 4$)

2. Reactions in astrophysics: general properties

General properties: specificities of the entrance channel → common to all reactions

- All cross sections (capture, transfer) involve a summation over ℓ: $\sigma(E) = \sum_\ell \sigma_\ell(E)$
- The partial cross sections $\sigma_\ell(E)$ are proportional to the penetration factor

$$P_\ell(E) = \frac{ka}{F_\ell(ka)^2 + G_\ell(ka)^2}$$ (a =typical radius)

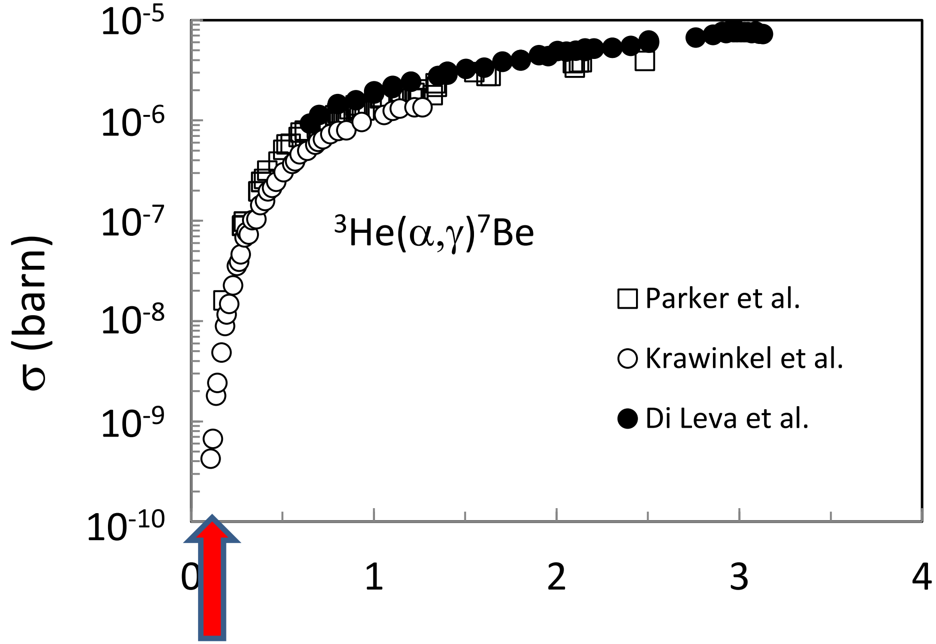


Consequences
- $\ell > 0$ are often negligible at low energies
- $\ell = \ell_{min}$ is dominant (often $\ell_{min} = 0$)
- For $\ell = 0$, $P_0(E) \sim \exp(-2\pi\eta)$

Astrophysical S factor: $S(E) = \sigma(E)E\exp(2\pi\eta)$ (Units: E∗L^2: MeV-barn)

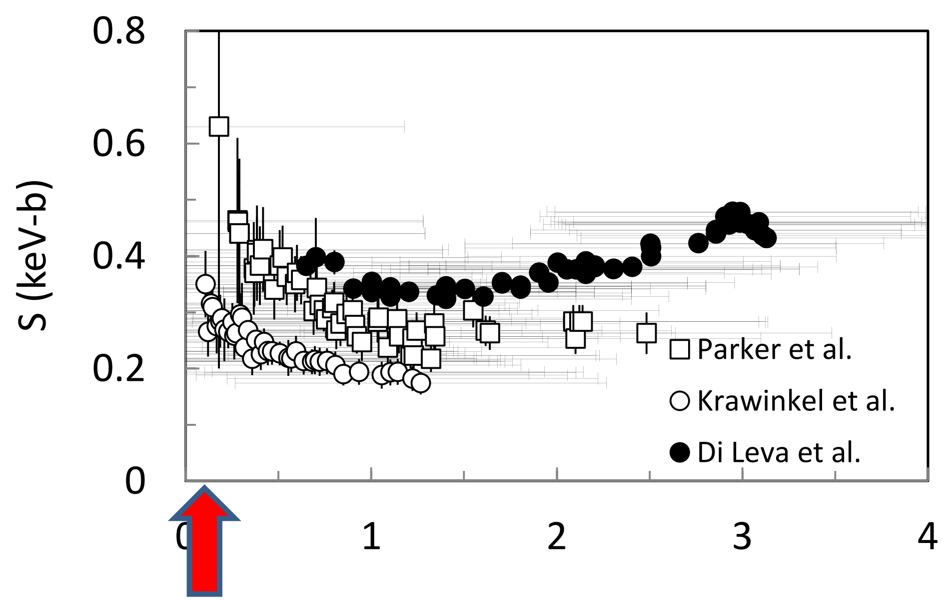
- removes the coulomb dependence → only nuclear effects
- weakly depends on energy → $\sigma(E) \approx S_0 \exp(-2\pi\eta)/E$ (any reaction at low E)

2. Reactions in astrophysics: general properties



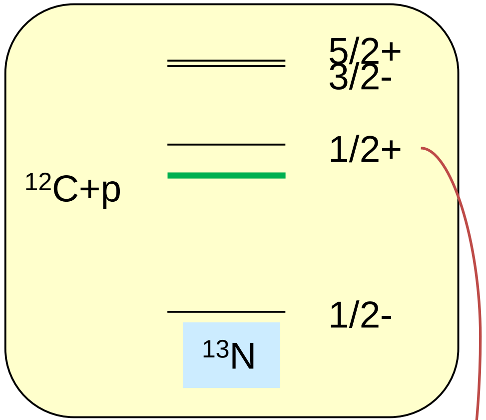
Example 1: $^{3}He(\alpha,\gamma)^{7}Be$ reaction

- Cross section $\sigma(E)$ Strongly depends on energy
- Logarithmic scale



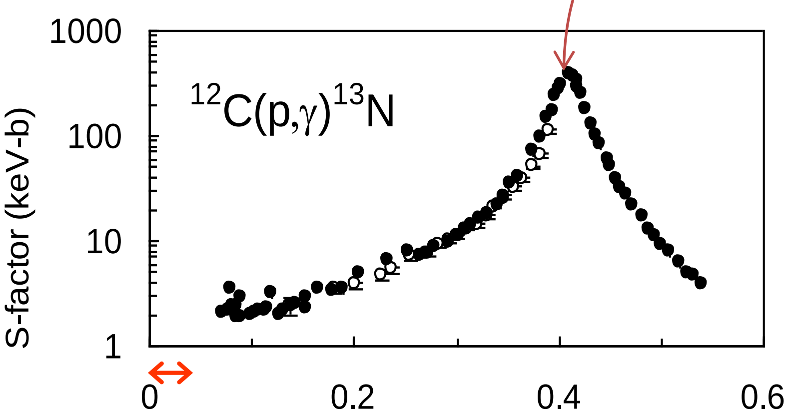
S factor

- Coulomb effects removed
- Weak energy dependence
- Linear scale



Example 2: $^{12}C(p,\gamma)^{13}N$ reaction

- Resonance $1/2^+$: $\ell = 0$
- Resonances $3/2^-$, $5/2^+$ $\ell = 1,2$ → negligible



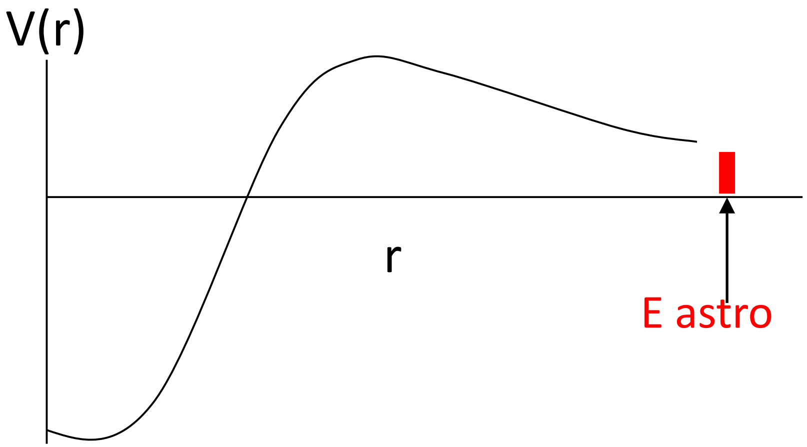
Note: BW is an approximation

- Neglects background, external capture
- Assumes an isolated resonance
- Is more accurate near the resonance energy

2. Reactions in astrophysics: general properties

- Nucleosynthesis:
  - Primordial (Bigbang): 3 first minutes of the Universe
  - Stellar: star evolution, energy production

- Input required: reaction rate <σv>
  - strongly depend on temperatures
  - given by the low-energy part of the cross section σ(E) (Gamow window)



Gamow peak :
$E_0 = 0.122\ \mu^{1/3} (Z_1 Z_2 T_9)^{2/3}$ MeV:
$\Delta E_0 = 0.237\ \mu^{1/6} (Z_1 Z_2)^{1/3} T_9^{5/6}$ MeV
Example: $^{12}C(\alpha,\gamma)^{16}O$ at $T_9=0.2$: $E_0=300$ keV

- Astrophysical energies: much lower than the Coulomb barrier
  → Coulomb effects are dominant
  → Very small cross sections

2. Reactions in astrophysics: general properties

General problems in nuclear astrophysics

- Low energies → very low cross sections (Coulomb barrier)
- For heavy nuclei: high level densities → many resonances must be known
- Need for radioactive beams
- No systematics (many different types of reactions)
  - transfer, capture
  - resonant, non-resonant
  - low or high level densities

→ in most cases a theoretical support is necessary

- data extrapolation (example: R-matrix method)
  Available cross sections are parametrized, and extrapolated down to stellar energies

- determination of cross sections
  The cross sections are determined from the wave functions of the system
  No need for experimental data (in principle!)
  Examples: potential model, microscopic models (low level densities)
  shell model (resonance properties in for high level densities)

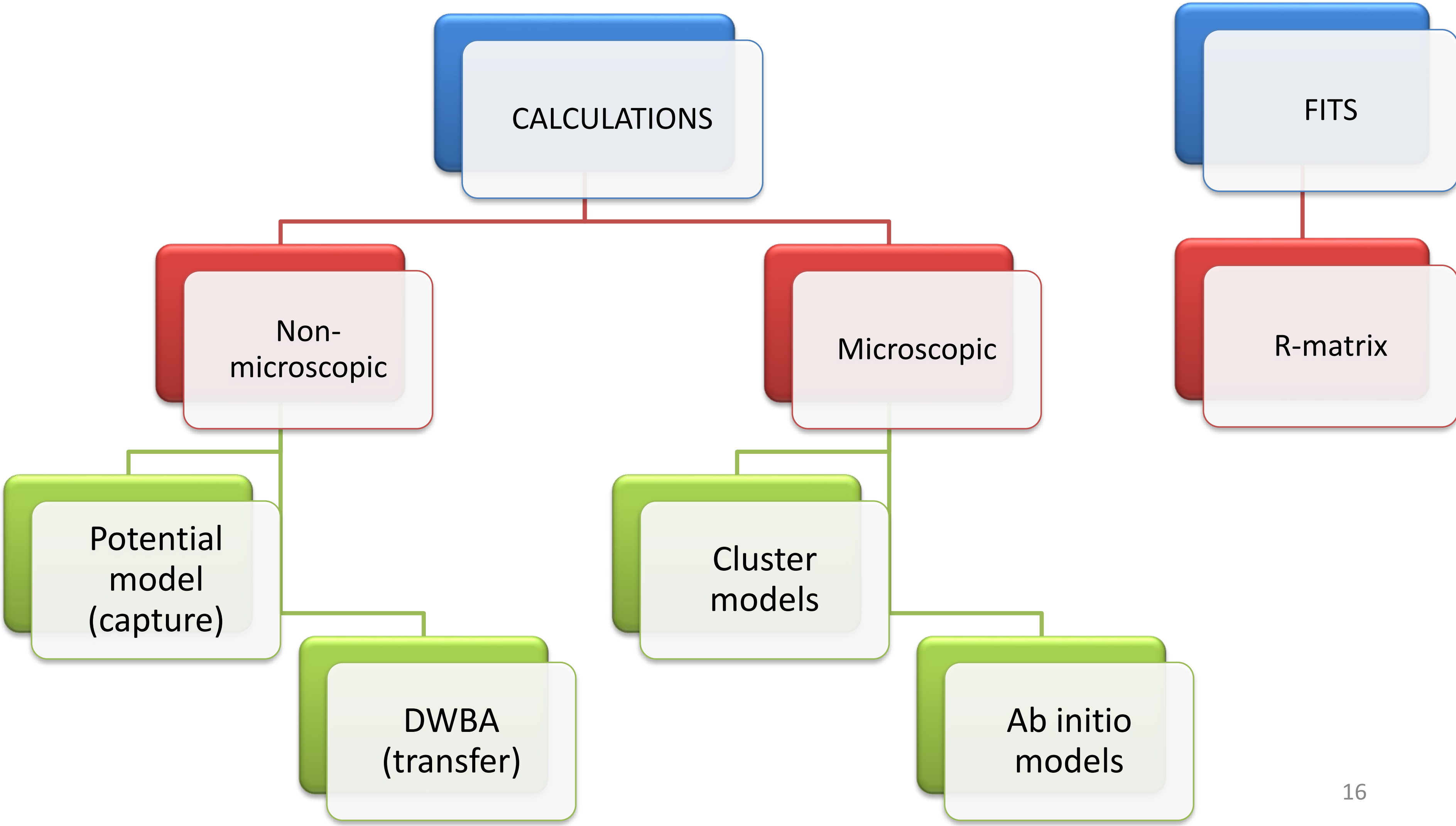
3. Reaction models

3. Reaction models

Applications: standard techniques applied to nucleus-nucleus scattering

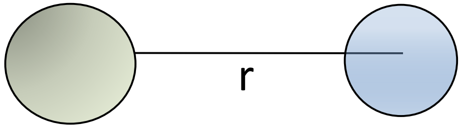
Theoretical point of view: compute the cross sections

Experimental point of view: fit the data and extrapolate them to low energies



3. Reaction models

Potential model



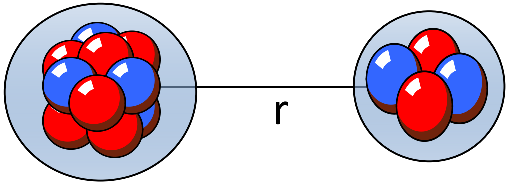
$H = T_r + V(r)$

$\Psi = g_L(\boldsymbol{r})$

V(r)=nucleus-nucleus potential

Microscopic cluster model

RGM, GCM



$H = \sum_i T_i + \sum_{j>i} V_{ij}$

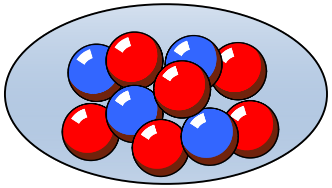
$\Psi = \mathcal{A}\phi_1\phi_2 g(\boldsymbol{r})$

V_{ij}=effective nucleon-nucleon interaction

$\phi_1\phi_2$=shell-model wave functions for clusters 1 and 2 → not solution of the Hamiltonian

Microscopic « ab intio » models

AMD, FMD, NCSM

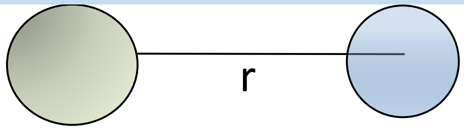


$H = \sum_i T_i + \sum_{j>i} V_{ij} + \cdots$

V_{ij}=realistic nucleon-nucleon interaction

3. Reaction models

Potential model



Internal structure is neglected

Advantage:

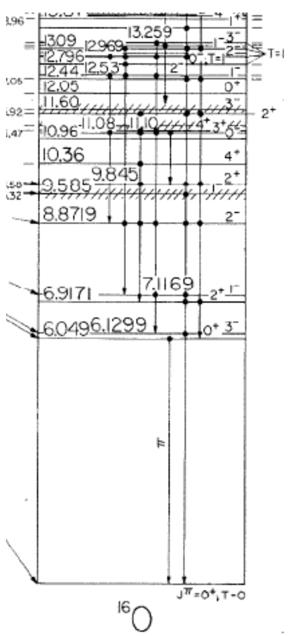
☺ Simple

Limitations:

☹ Not applicable to transfer reactions

☹ Choice of the potential?

☹ Not applicable if reaction channels are open



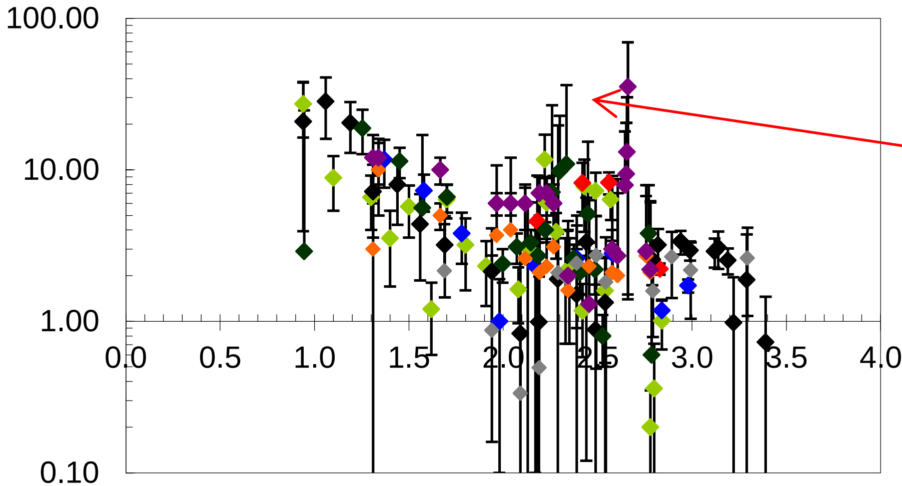
^{15}N+p threshold: ^{15}N(p,α)^{12}C is open
→ PM not applicable to ^{15}N(p,γ)^{16}O

^{12}C+α threshold

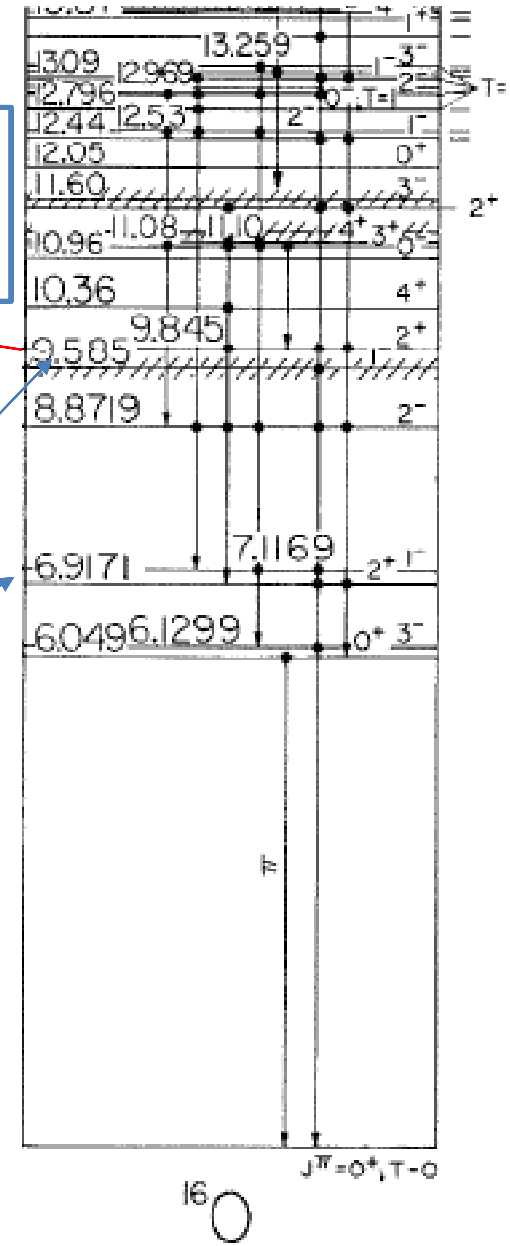
3. Reaction models

☹ Resonances may not be described by the PM

example: $^{12}C(\alpha,\gamma)^{16}O$ E2



2_2^+ resonance with a $^{12}C(2^+)+\alpha$ structure



☹ 2 resonances in the same partial wave

$^{12}C(\alpha,\gamma)^{16}O$ E1: two 1- resonances

➔ Low predictive power

4. Microscopic models

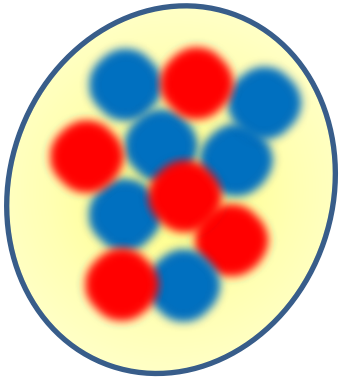
Microscopic models

- Pauli principle taken into account
- Depend on a nucleon-nucleon (NN) interaction → more predictive power

$$H_0(r_1, \dots r_A) = \sum_i T_i + \sum_{ij} V_{ij}$$

- Two approaches: *« ab initio »,* cluster models

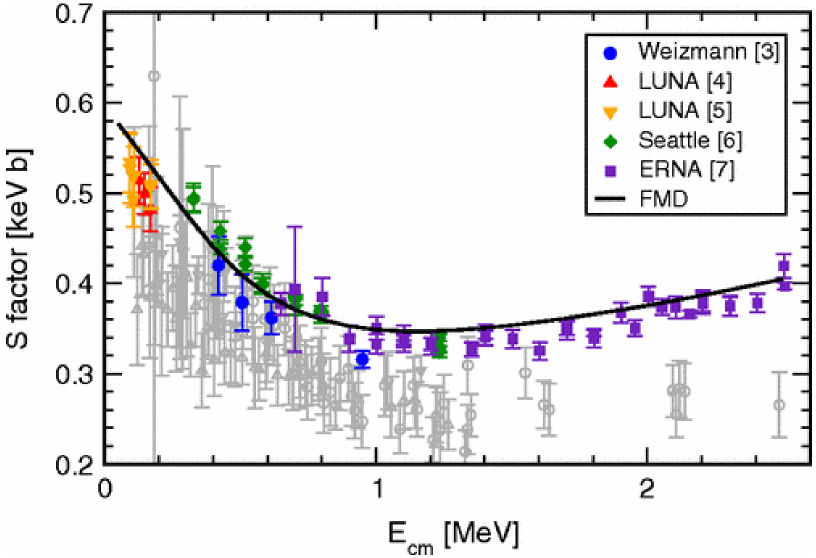
« Ab initio » (No-cluster approximation)



- Try to find an exact solution of the (A-body) Schrödinger equation
- Use realistic NN interactions (fitted on NN properties)
- In general:
  - $A \leq 12$
  - Scattering states difficult/impossible to obtain
  - Not well adapted to halo structure, resonant states

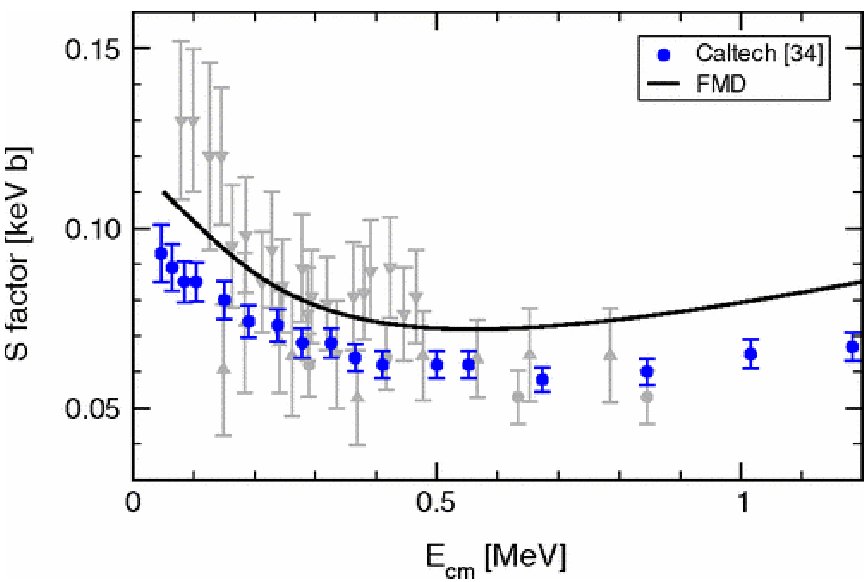
4. Microscopic models

Example 1: T. Neff, Phys. Rev. Lett. **106**, 042502



$^3He(\alpha,\gamma)^7Be$

- Many experiment, many calculations
- First RGM calculation (1981) Liu et al.
- Low energies: external capture
- ERNA data (2007): different for E>1.5 MeV



$^3H(\alpha,\gamma)^7Li$

- Mirror reaction
- Overestimates recent data

4. Microscopic models

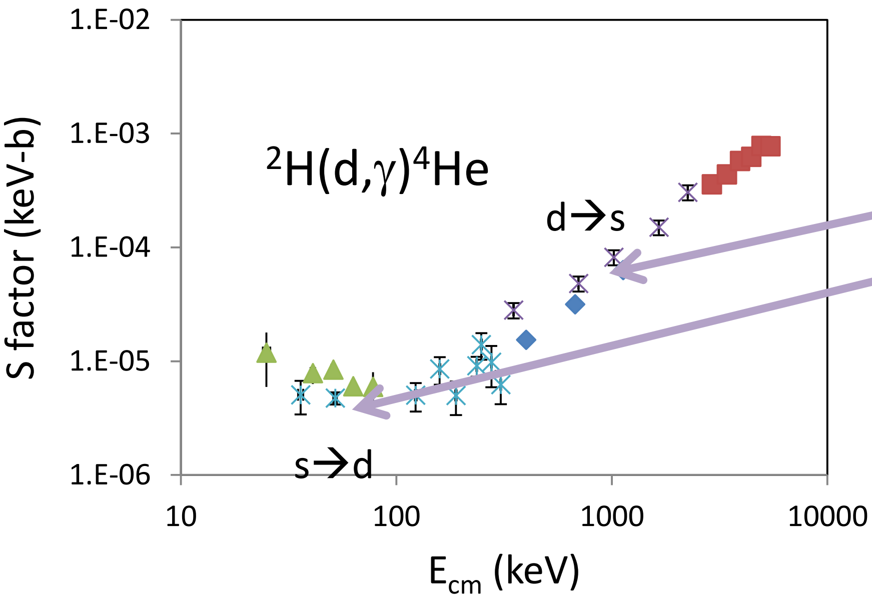
Example 2: d+d systems ^{2}H(d,γ)^{4}He, ^{2}H(d,p)^{3}H, ^{2}H(d,n)^{3}He

two physics issues

- Analysis of the d+d S factors (Big-Bang nucleosynthesis)
- Role of the tensor force in ^{2}H(d,γ)^{4}He

^{2}H(d,γ)^{4}He S factor

- Ground state of ^{4}He=0^+
- E1 forbidden→ main multipole is E2 → 2^+ to 0^+ transition→ d wave as initial state
- Experiment shows a plateau below 0.1 MeV: typical of an s wave



E2 matrix element $< \boldsymbol{\Psi^{0+}} \mid \boldsymbol{E2} \mid \boldsymbol{\Psi^{2+}} >$

$\approx < 0^+, 0 \mid \mathrm{E2} \mid 2^+, 2 >$: d→s, dominant E>100 keV

$+ < 0^+, 2 \mid \mathrm{E2} \mid 2^+, 0 >$: s→d, tensor (E<100 keV)

→ direct effect of the tensor force

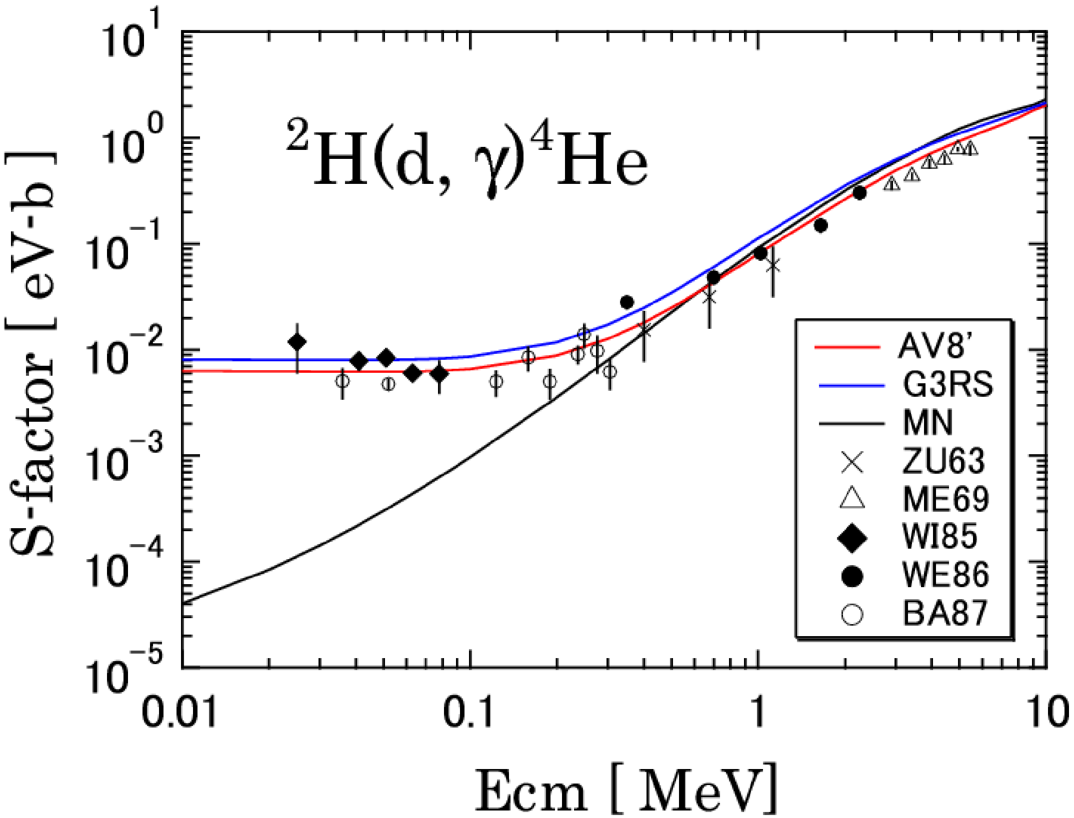
4. Microscopic models

Collaboration Niigata (K. Arai, S. Aoyama, Y. Suzuki)-Brussels (D. Baye, P.D.)

K. Arai et al., *Phys. Rev. Lett. 107 (2011) 132502*

3 nucleon-nucleon interactions:

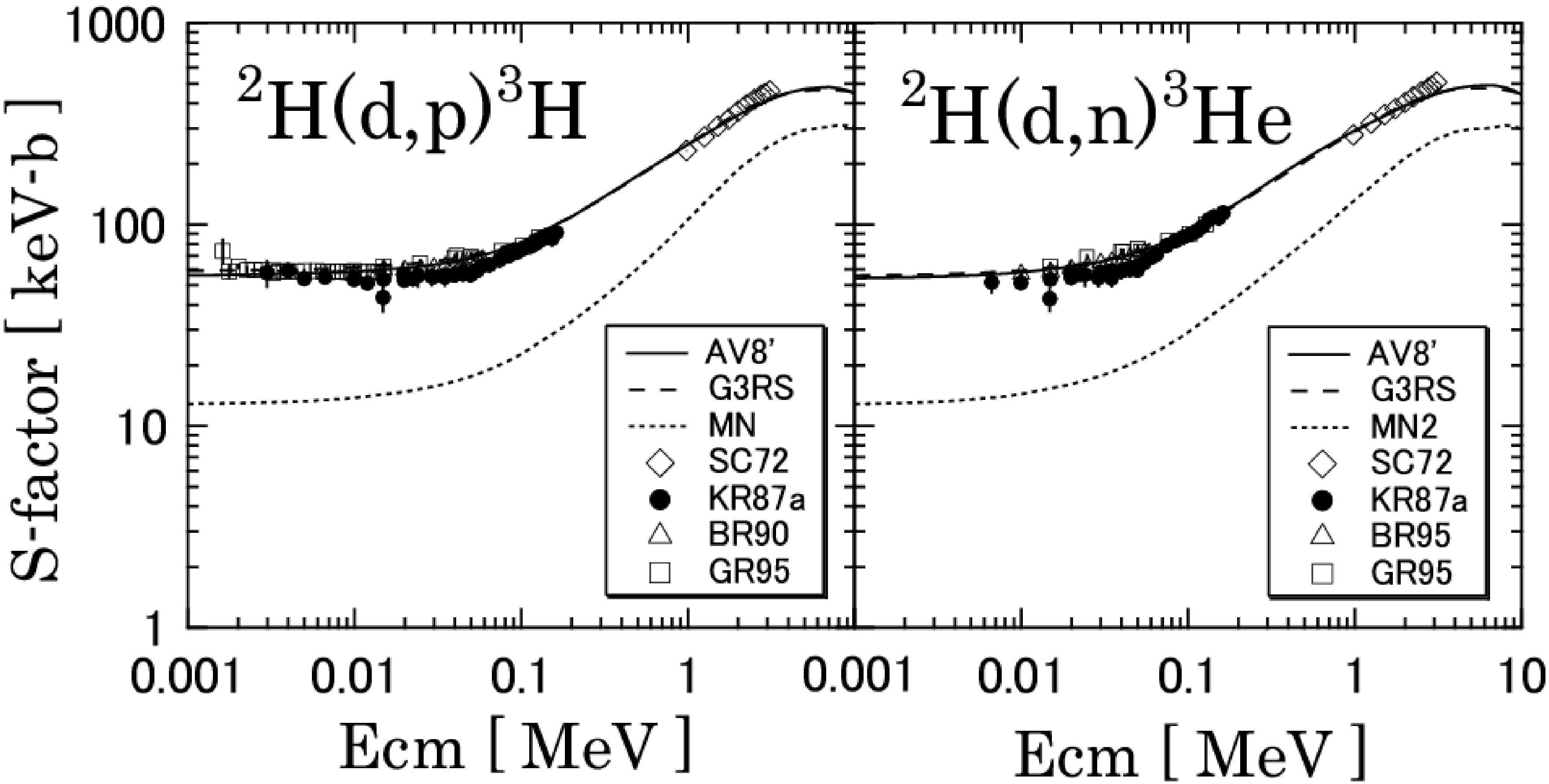
- Realistic: Argonne AV8’, G3RS
- Effective: Minnesota MN



- No parameter
- MN does not reproduce the plateau (no tensor force)
- D wave component in ^4He:
  13.8% (AV8’)
  11.2% (G3RS)

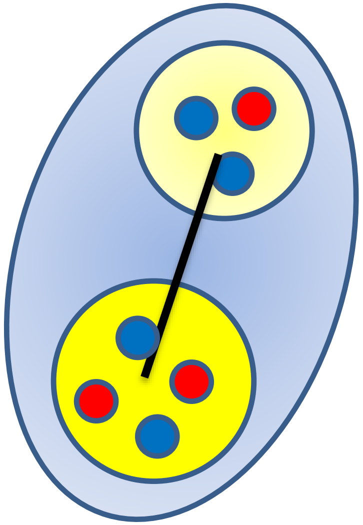
4. Microscopic models

Transfer reactions $^2H(d,p)^3H$, $^2H(d,n)^3He$



4. Microscopic models

Cluster approximation



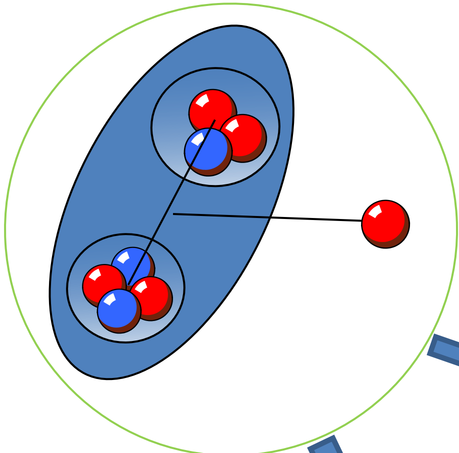
- Wave function defined by

$\Psi = \mathcal{A}\Phi_1\Phi_2 g(r)$ (Φ_1, Φ_2=internal wave functions (shell-model))
=Resonating Group Method (RGM)

- Effective NN interactions (Minnesota, Volkov)
- Extensions to 3 clusters, 4 clusters, etc.
- Core excitations can be easily included
- Scattering states possible
- Calculations easier than in ab initio theories
→Many applications (up to Ne isotopes) in spectroscopy and scattering
- Textbook example: $\alpha+\alpha$
- First application in astrophysics: $^3He(\alpha,\gamma)^7Be$

4. Microscopic models

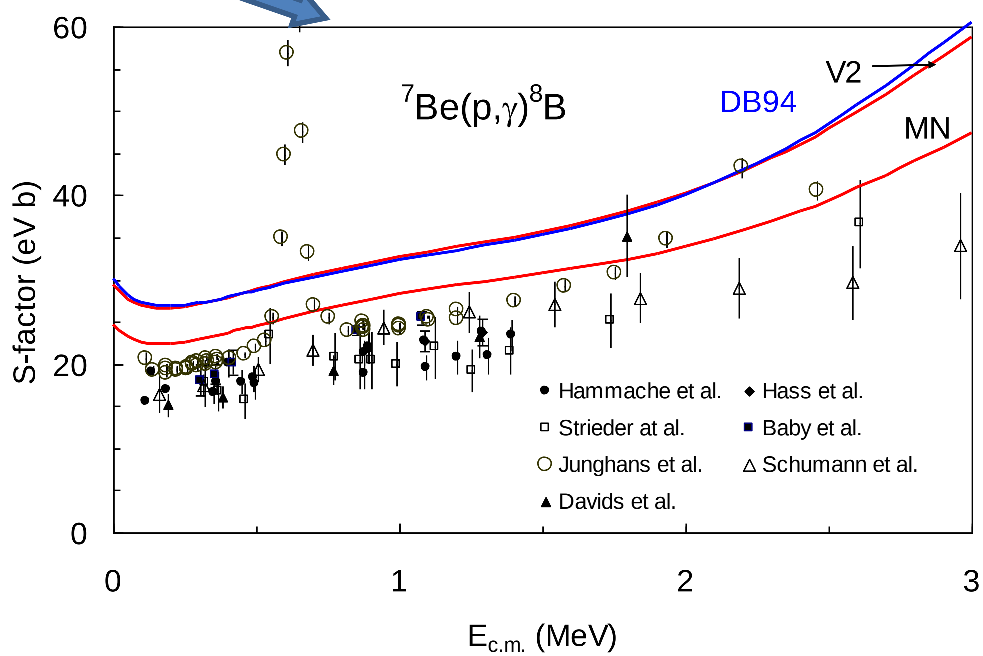
Application to $^7Be(p,\gamma)^8B$



- 2 generator coordinates
- ^7Be ($3/2^-$,$1/2^-$,$5/2^-$,$7/2^-$)+p
- Double angular-momentum projection
    1) ^7Be
    2) ^7Be+p

	experiment	theory
μ (2^+) (μ_N)	1.03	1.52
Q(2^+) (e.fm^2)	6.83 ± 0.21	6.0
B(M1,$1^+\rightarrow2^+$) (W.u.)	5.1 ± 2.5	3.8

P.D., Phys. Rev. C70, 065802 (2004)



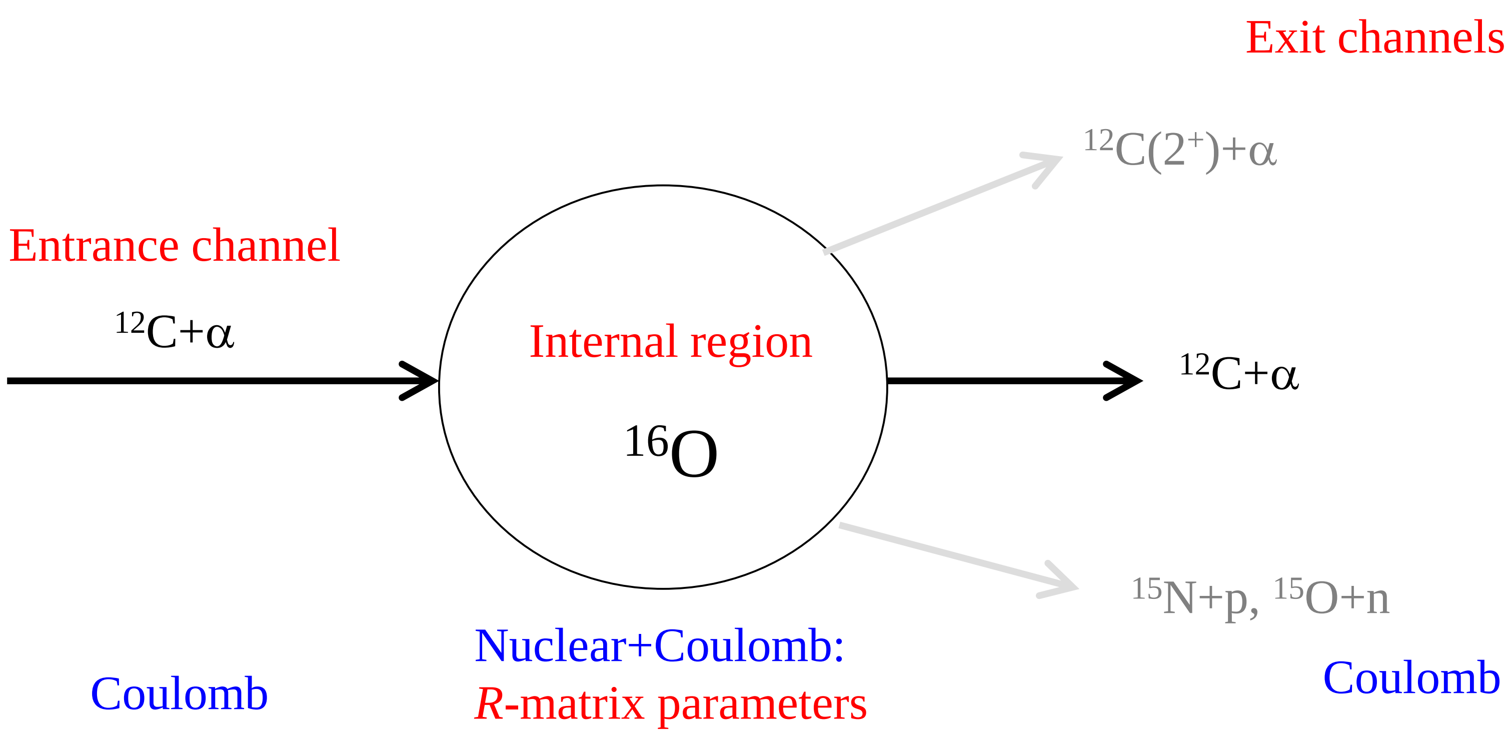
5. The R-matrix method

5. The R-matrix method

- Introduced by Wigner (1937) to parametrize resonances (nuclear physics)
  In nuclear astrophysics: used to fit data

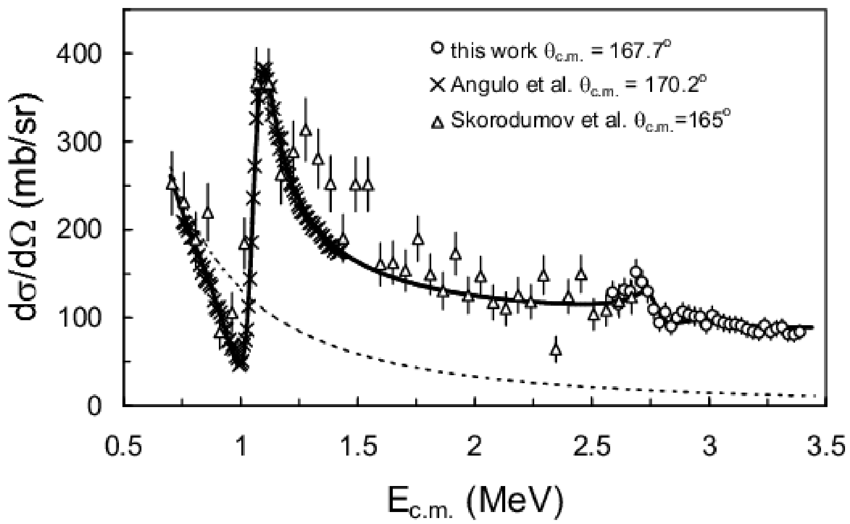
- Provides scattering properties at all energies (not only at resonances)

- Based on the existence of 2 regions (radius a):
  - Internal: coulomb+nuclear
  - external: coulomb

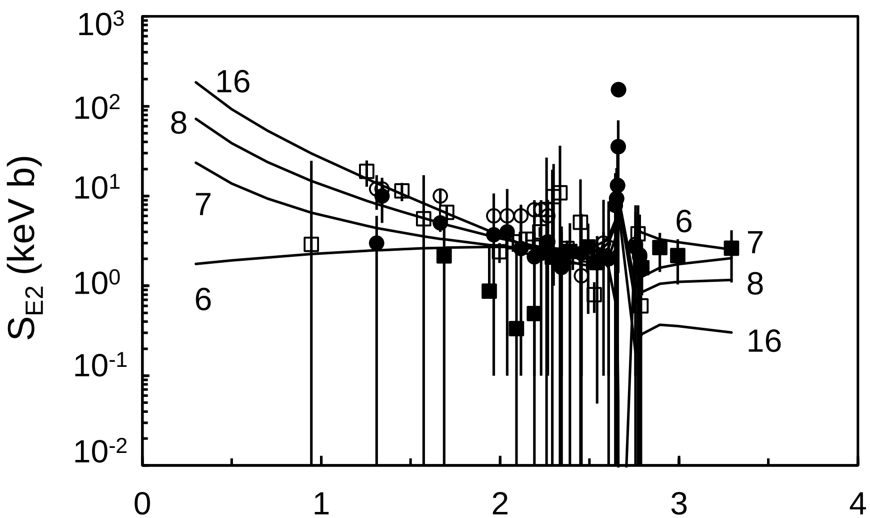


5. The R-matrix method

Main Goal: fit of experimental data



^{18}Ne+p elastic scattering
→ resonance properties

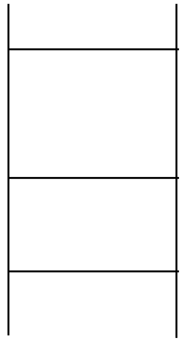


Nuclear astrophysics: $^{12}C(\alpha,\gamma)^{16}O$ (E2)
→Extrapolation to low energies

5. The R-matrix method

- Internal region: The R matrix is given by a set of resonance parameters **(=poles)** E_i, γ_i^2

$$R(E) = \sum_i \frac{\gamma_i^2}{E_i - E} = a\frac{\Psi'(a)}{\Psi(a)}$$



- External region: Coulomb behaviour of the wave function

$$\Psi(r) = I(r) - UO(r)$$

→the collision matrix U is deduced from the R-matrix (repeated for each spin/parity $J\pi$)

- Two types of applications:
    - **phenomenological R matrix**: γ_i^2 and E_i are fitted to the data (astrophysics)
    - **calculable R matrix**: γ_i^2 and E_i are computed from basis functions (scattering theory)

- R-matrix radius a is not a parameter: the cross sections must be insensitive to a

- Can be extended to multichannel calculations (transfer), capture, etc.

- Well adapted to nuclear astrophysics: low energies, low level densities

5. The R-matrix method

Different processes with common parameters → constraints

- Phase shifts related to $\sum_i \frac{\gamma_i^2}{E_i - E}$
- Capture cross section related to $\sum_i \frac{\gamma_i \sqrt{\Gamma_{\gamma,i}}}{E_i - E}$
- Transfer cross section related to $\sum_i \frac{\gamma_{1i}\gamma_{2i}}{E_i - E}$
- Beta decay to the continuum related to $\sum_i \frac{\gamma_i A_i}{E_i - E}$

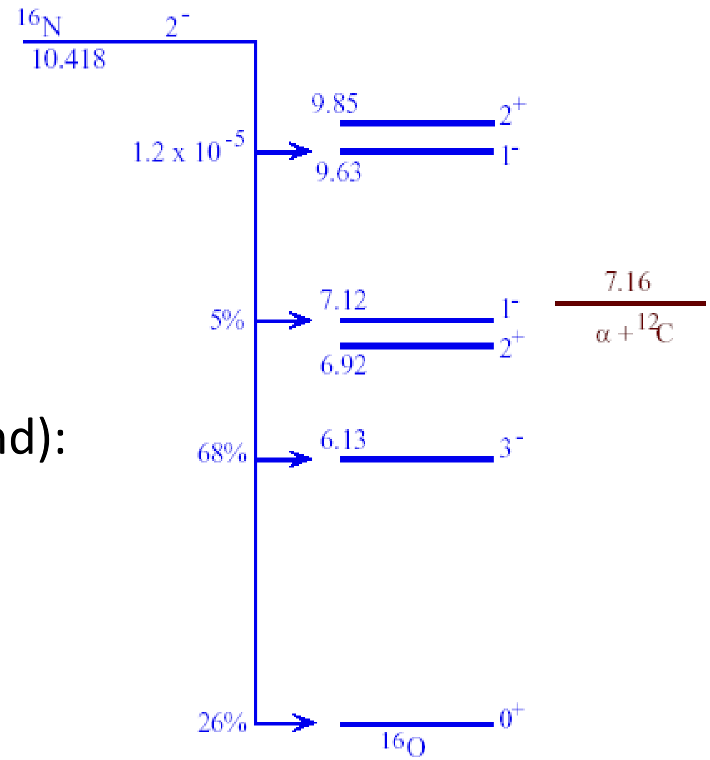
E_i, γ_i: energies and reduced widths: common to all processes
$\Gamma_{\gamma,i}, \gamma_{2i}, A_i$: specific to the individual processes

5. The R-matrix method

Example: simultaneous fit of

- ^{12}C+α phase shift
- ^{12}C(α,γ)^{16}O *S*-factor (E1)
- ^{16}N β-decay

(Azuma et al, Phys. Rev. C50 (1994) 1194)



parameters of the 1^-_1 and 1^-_2 states (+background):

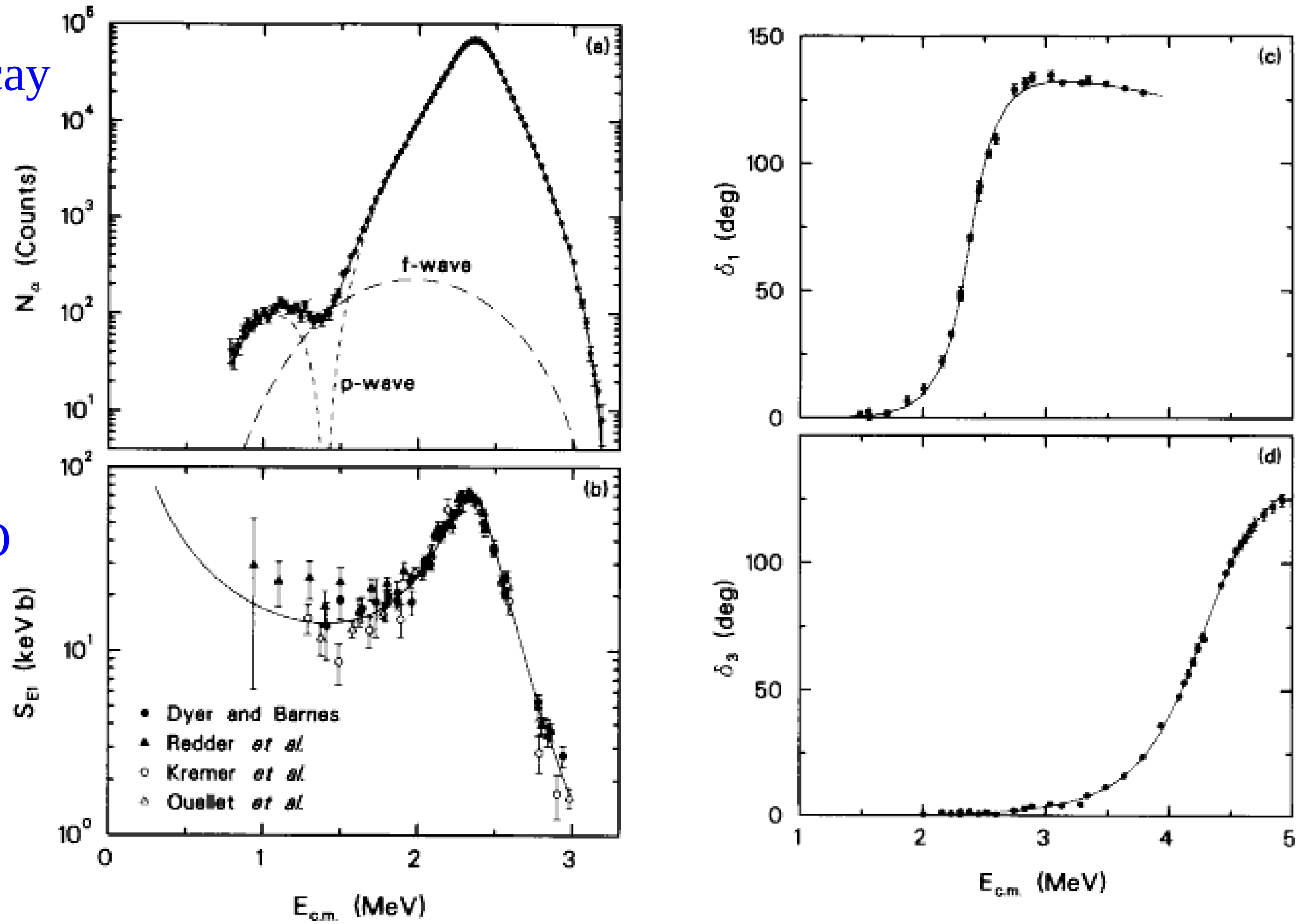
- ^{12}C+α: E_λ, γ_λ
- ^{12}C(α,γ)^{16}O : E_λ, γ_λ, $\Gamma_{\gamma,\lambda}$ (radiative width)
- ^{16}N β decay : E_λ, γ_λ, A_λ (β probabilities)

$\Rightarrow$ Constraints on common parameters E_λ, γ_λ

5. The R-matrix method

^{16}N β decay

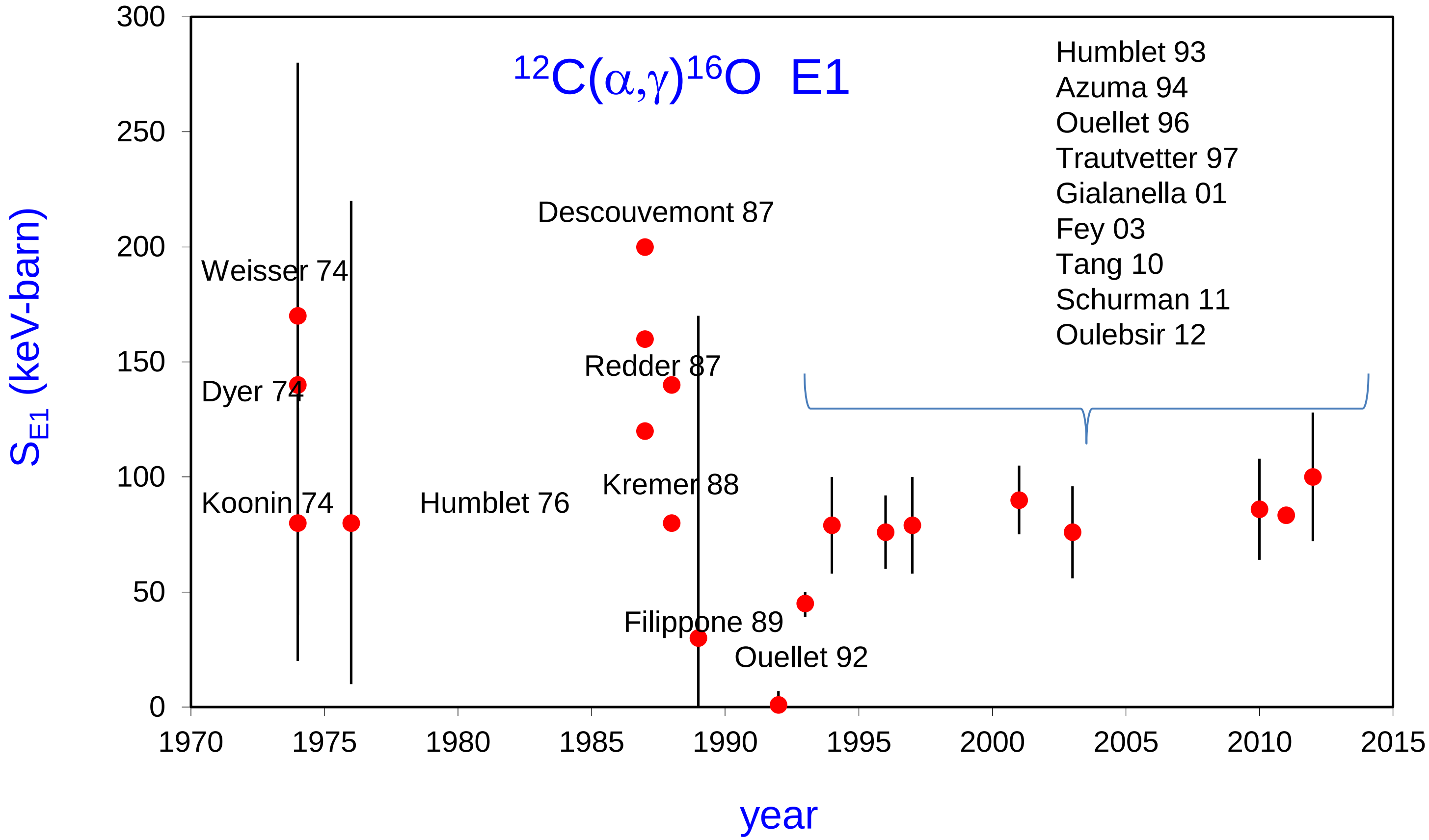
$^{12}C(\alpha,\gamma)^{16}O$



1^- phase shift

3^- phase shift

S(300 keV): extrapolations for E1



^{16}N data available

Review paper: R. deBoer et al., Rev. Mod. Phys. 89 (2017) 035007

6. Conclusion

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Needs for nuclear astrophysics:

- low energy cross sections
- resonance parameters

Theory: various techniques

- fitting procedures (R matrix)→ extrapolation: importance of external constraints
- non-microscopic models: potential, DWBA, etc.
- microscopic models:
  - cluster: developed since 1960's, applied to NA since 1980's
  - ab initio: problems with scattering states, resonances→ limited at the moment
- Indirect methods (resonance and bound-state) properties: many experiments
- (Some) current challenges: triple α process, $^{12}C(\alpha,\gamma)^{16}O$, $^{12}C+^{12}C$, etc.
  s-process: many reactions