More is different: Particle-number-dependence in EFT

Joint seminar in Central China Normal University

Chieh-Jen (Jerry) Yang

Dec. 2, 2021

Akademie věd České republiky

Collaborators: A. Ekström, C. Forssén, G. Hagen, U. van Kolck, G. Rupak

A story

Continue driving? (change oil/fuses, add additives, perform minor fixes until it doesn't work)

Full (expensive) checks/fixes

Normally you choose…

Chiral EFT approach

to nuclear physics (Baryon-Baryon section)

History

- Weinberg (90's): Proposal of ideas.
- Bira van Kolck (94-96's): Implementations (including the Δ-full version) on NN, NNN, and EM-probe.

(Let's continue using WPC)

- Entem, Machledit, Epelbaum, Meissner, Kaiser,.., (00-present): Refinements ($\chi^2 \approx 1$) and carried out to higher orders.

- Ekstrom et al (13-present): Refinement to fit nuclei (NNLOopt, NNLOsat, NNLOgo,… etc,)

Bayesian error analysis (on-going): Uncertainty quantifications of WPC.

(Let's try something else)

- Kaplan, Savage, Wise (97): m_π dependence is not absorbed under WPC.

- Nogga, Timmerman, van Kolck (05): Renormalization problem (even without varying m_π).

- Birse, Valderrama, Long/Yang (06-13): New, RG-o.k. PCs.

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New, RG-o.k. PCs.

For further information/opinions, see:

arXiv:2107.11675 and arXiv:2111.00930

Nuclear Effective Field Theories: Reverberations of the early days

U. van Kolck

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and
Department of Physics, University of Arizona, Tucson, AZ 85721, USA

July 27, 2021

What Can Possibly Go Wrong?

Harald. W. Grießhammer

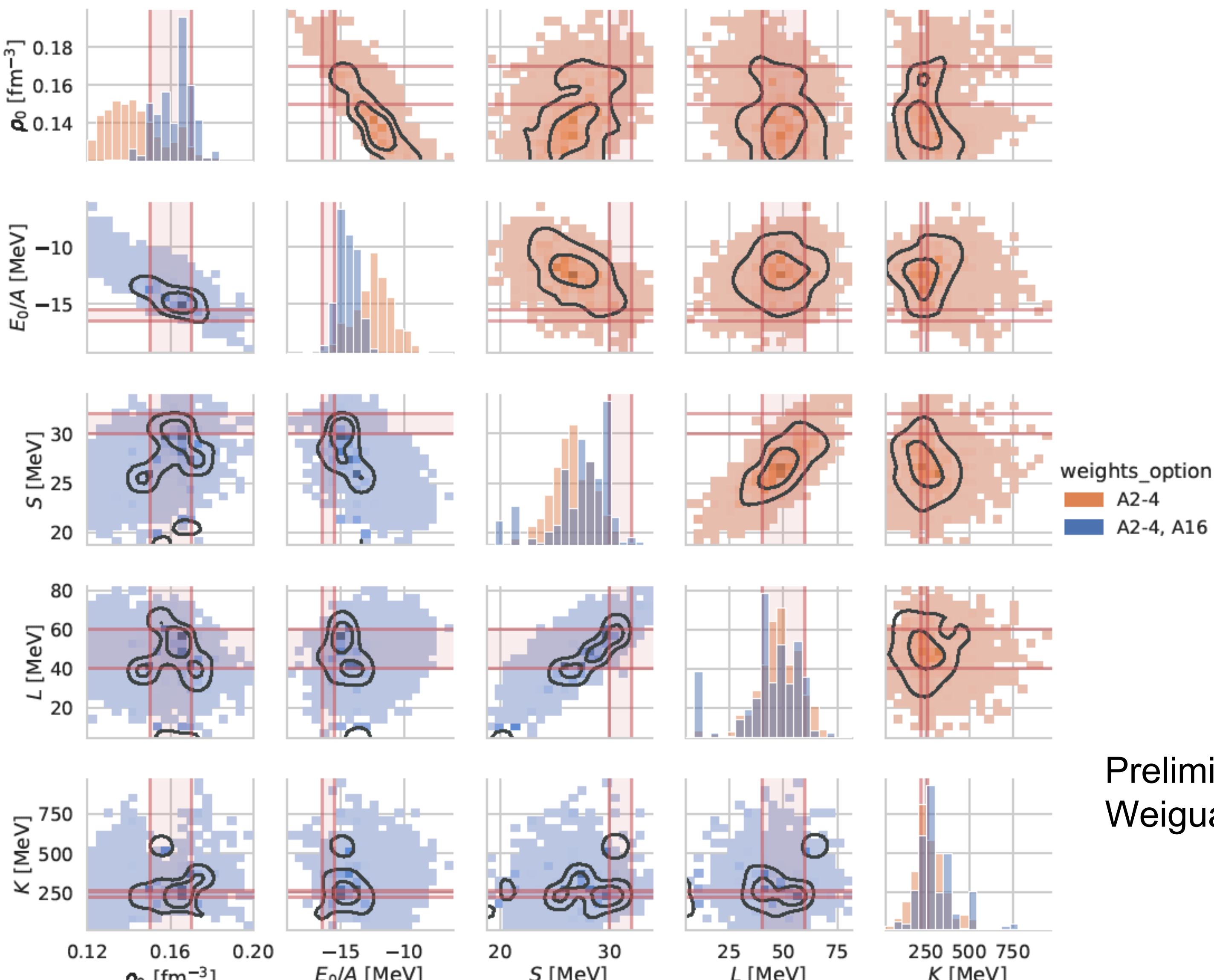
Received: date / Accepted: date

Abstract A lot.

Current status: nuclear structure

- Most ab-initio calculations adopt chiral EFT potential organized under **Weinberg power counting** (WPC). Van Kolck, Epelbaum, Machleidt, and more.
- Good results (w.r.t. exp. data) for light systems, if low-energy constants (LECs) are renormalized at NN/NNN-level.
- But not quite the same for ^{16}O (or heavier)➔ **need to refit** (optimize) the potential and **sacrifice** NN.

Optimizing NNLO with Δ(1232)



Preliminary work
Weiguang Jiang, et al.

Possibilities for improvements

WPC is wrong, one needs to organize things differently.

Current approach is correct, just need to include more higher-order potentials.

Need improvement

Need improvement

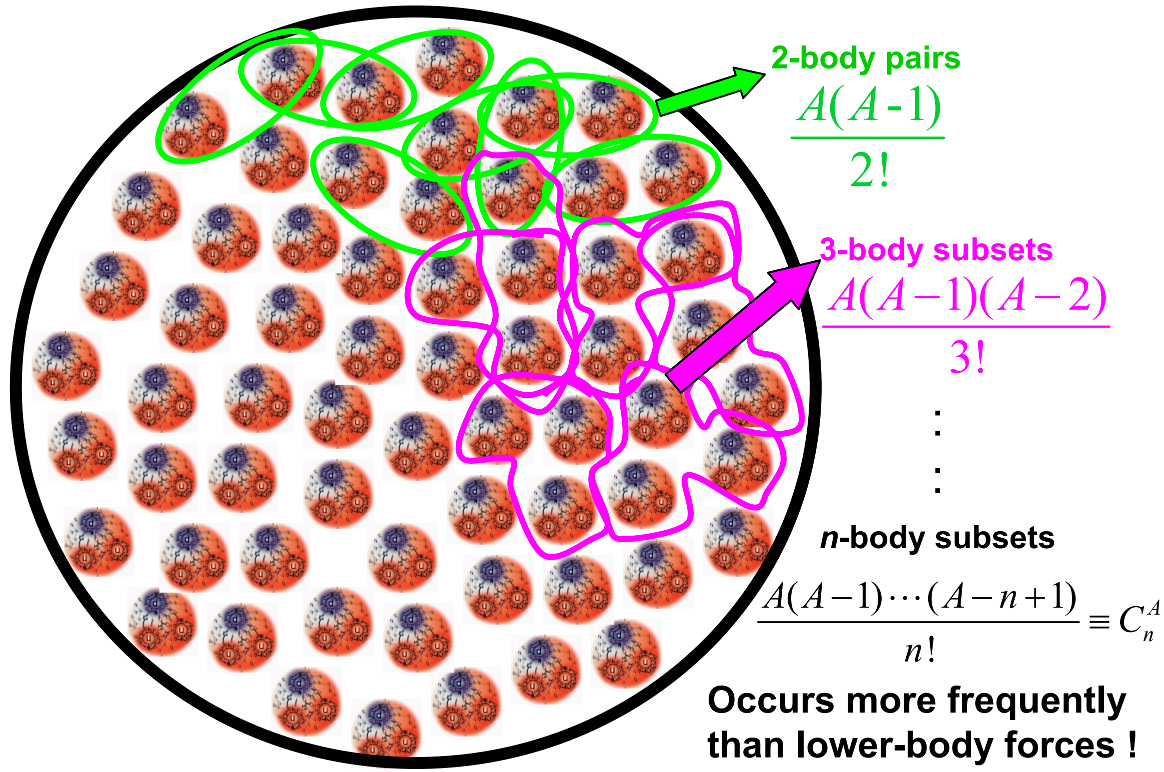
This work:
The importance of higher-body forces (3,4-body forces) need to grow with number of the particles in the system.
C.-J. Yang et al: arXiv:2109.13303

Many-body forces in complex systems

- Some of many-body couplings are genuine and unknown, i.e., cannot be derived from NN couplings.
- They are estimated to be weaker by naïve dimension analysis (NDA).
- However, their importance can grow in a large system.

Explained in:

C.-J. Yang, Eur.Phys.J.A 56 (2020) 3, 96



A	number of doublets $\frac{A(A-1)}{2}$	number of triplets $\frac{A(A-1)(A-2)}{6}$
3	3	1
4	6	4
5	10	10
6	15	20

“A choose n” enhancements

$$C_n^A = \frac{A(A-1)(A-2)\ldots(A-n+1)}{n!}$$

- In a self-bound system, the above enhancement won’t be fully counted. For example, an n-body subset will have nearly zero contribution if its constituents span a distance much larger than the range of the n-body forces.→ density saturates, not $\to\infty$.

- On the other hand, those small contributions could still add up to become sizable, due to the fact that there are many of them.

- Thus, the growth of n-body forces in large systems depends on multiple factors such as the **range** and the **form** of interactions, the mass of particles, etc., ➔ **Require ab-initio calculations to know the PC.**

N-body forces in NDA

$$F_n = f_\pi^2 M_{hi}^2 \left(\frac{\nabla^2}{M_{hi}^2}\right)^s \left(\frac{N^+ N}{f_\pi^2 M_{hi}}\right)^n$$

$$f_\pi \approx 93 MeV$$

$$\text{3NFs/2NFs} \approx \frac{N^+ N}{f_\pi^2 M_{hi}}$$

$$M_{hi} \approx 500 - 1000 MeV$$

$N^{(+)} : nucleon$ field, $N^+ N \approx \rho.$

s: number of derivatives

*Version: Follows J. L. Friar, Few-Body Systems, 22(4):161–193 (1997) to include the 4 π factor in the infrared enhancement. Otherwise n-body forces further suppressed by one chiral order ~1/3 w.r.t. 2-body forces.

Estimations

- Combine NDA and "A choose n":

Combine both:

Approx. with nuclear saturation density

$$\frac{C_n^A F_n}{C_m^A F_m} = \frac{A-m}{n}\left(\frac{\rho_0}{f_\pi^2 M_{\rm hi}}\right)^{n-m} = \frac{A-m}{n}\left(\frac{142(\rm MeV)}{M_{\rm hi}}\right)^{n-m}$$

~1

NN and NNN becomes the same important starting from A=13-26 (M_{hi}=500-1000 MeV)

*NNN and NNNN becomes the same important starting from A=17-34.

*5^+-body force is more suppressed (s≥1), only equal to NNNN after A>500.

***Might be weaker due to the previous page's argument.**

Let’s start from light systems: ignore 3NFs first

For ^{3}H, ^{3}He, ^{4}He, use no-core shell model (NCSM).
For ^{16}O and nuclear matter, use coupled-cluster method (CC).

We want to test RG also, so adopt modified Weinberg counting (MWPC)

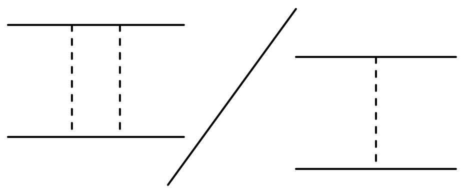
Modified Power counting

(Decided by RG)

Long & Yang, (2010-2013).

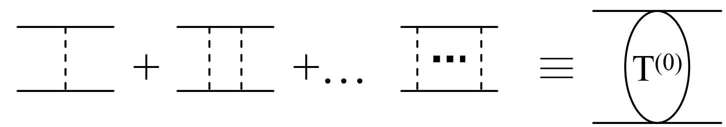
LO: Still iterate to all order (at least for *l*<2).

Reason: van Kolck, Bedaque,… etc.



$\sim \frac{g_A^2 M}{8\pi f_\pi^2} Q$

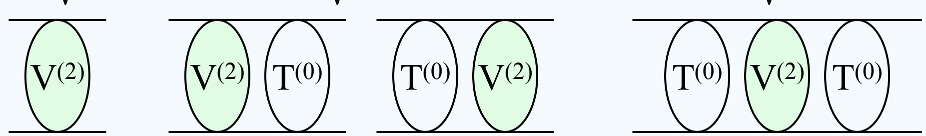
Thus, $O(Q^0)$:



Start at NLO, do perturbation. $(T = T^{(0)} + T^{(1)} + T^{(2)} + T^{(3)} + \ldots)$

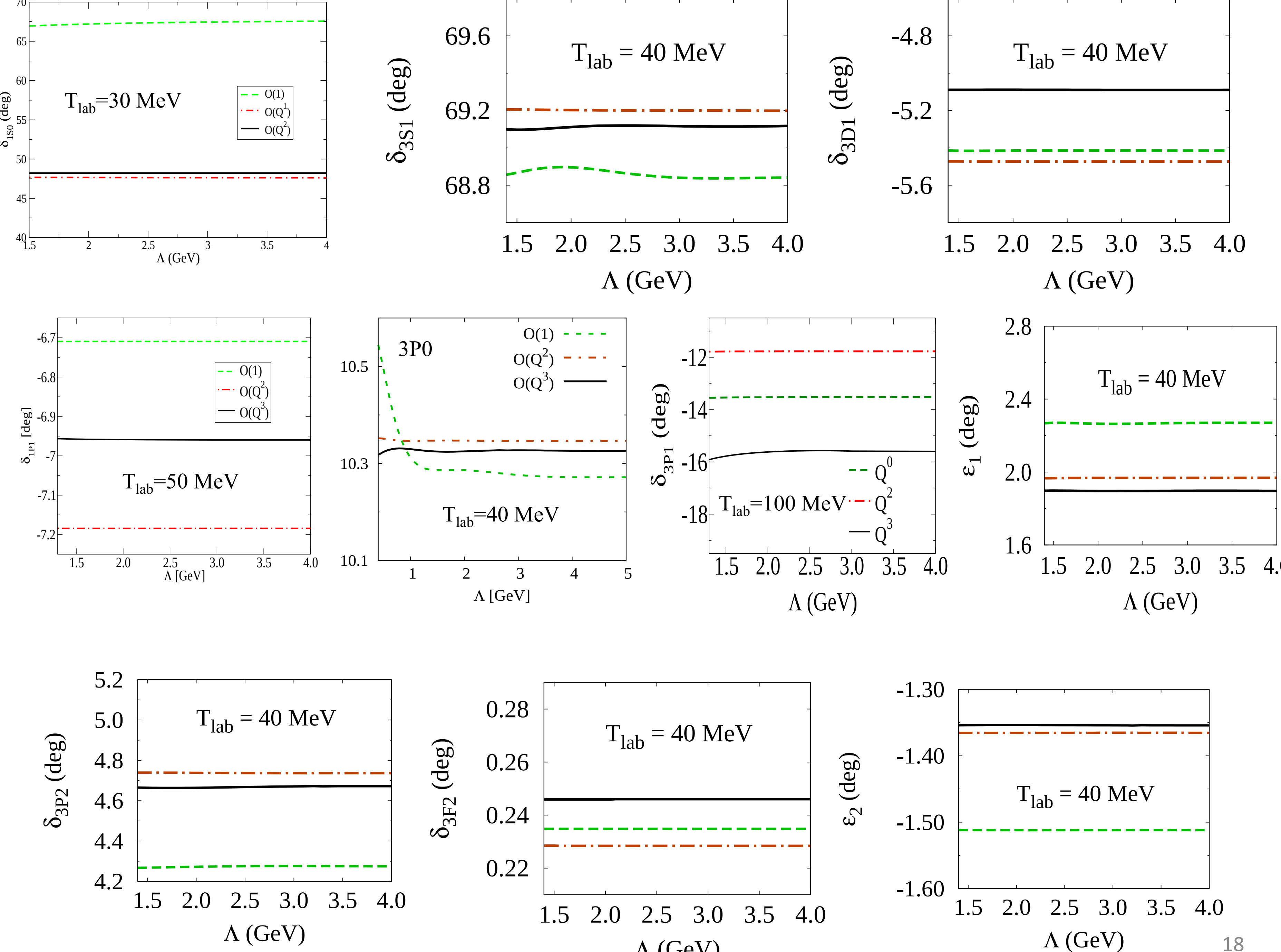
If $V^{(1)}$ is absent:

$$T^{(2)} = V^{(2)} + 2V^{(2)}GT^{(0)} + T^{(0)}GV^{(2)}GT^{(0)}.$$



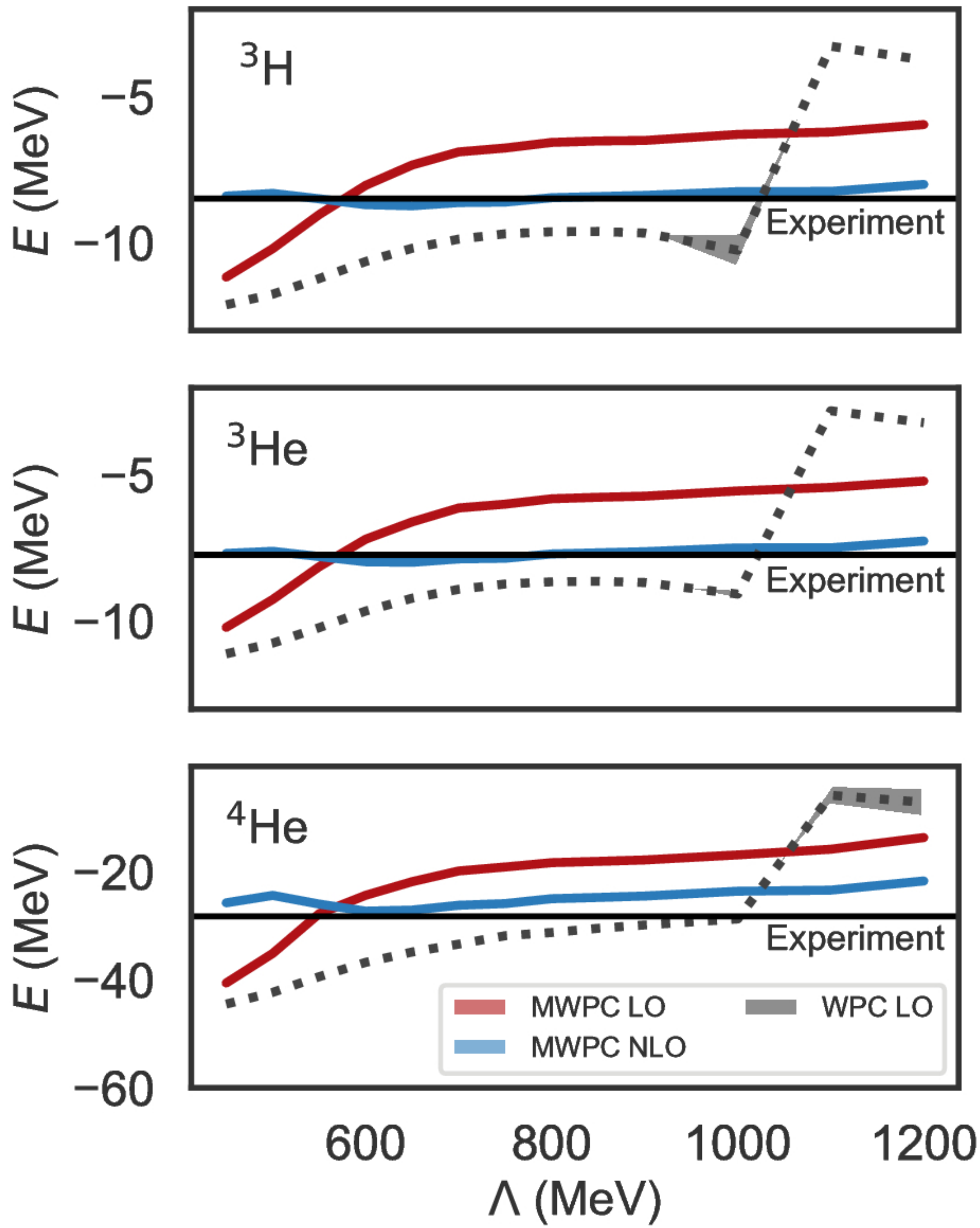
$$G \equiv \frac{2M_N}{\pi}\int_0^\Lambda \frac{p^2\,dp}{p_0^2 - p^2 + i\varepsilon}$$

$$T^{(3)} = V^{(3)} + 2V^{(3)}GT^{(0)} + T^{(0)}GV^{(3)}GT^{(0)}.$$



T_{lab}=30 MeV
O(1)
O(Q^1)
O(Q^2)
δ_{1S0} (deg)
Λ (GeV)
T_{lab} = 40 MeV
δ_{3S1} (deg)
Λ (GeV)
T_{lab} = 40 MeV
δ_{3D1} (deg)
Λ (GeV)
O(1)
O(Q^2)
O(Q^3)
T_{lab}=50 MeV
δ_{1P1} [deg]
Λ [GeV]
3P0
O(1)
O(Q^2)
O(Q^3)
T_{lab}=40 MeV
Λ [GeV]
Q^0
T_{lab}=100 MeV
Q^2
Q^3
δ_{3P1} (deg)
Λ (GeV)
T_{lab} = 40 MeV
ε_1 (deg)
Λ (GeV)
T_{lab} = 40 MeV
δ_{3P2} (deg)
Λ (GeV)
T_{lab} = 40 MeV
δ_{3F2} (deg)
Λ (GeV)
T_{lab} = 40 MeV
ε_2 (deg)
Λ (GeV)

Few-body results



Conclusion:
New power counting (MWPC) works, at least for A≤4 systems.

C.-J. Yang, A. Ekstrom, C. Forssen, G. Hagen,
https://arxiv.org/abs/2011.11584

For A up to 3 see also:
Nogga et al, PRC 72 (2005), 054006
Song et al, PRC 96 (2017), 024002.

Wrong ^{16}O pole

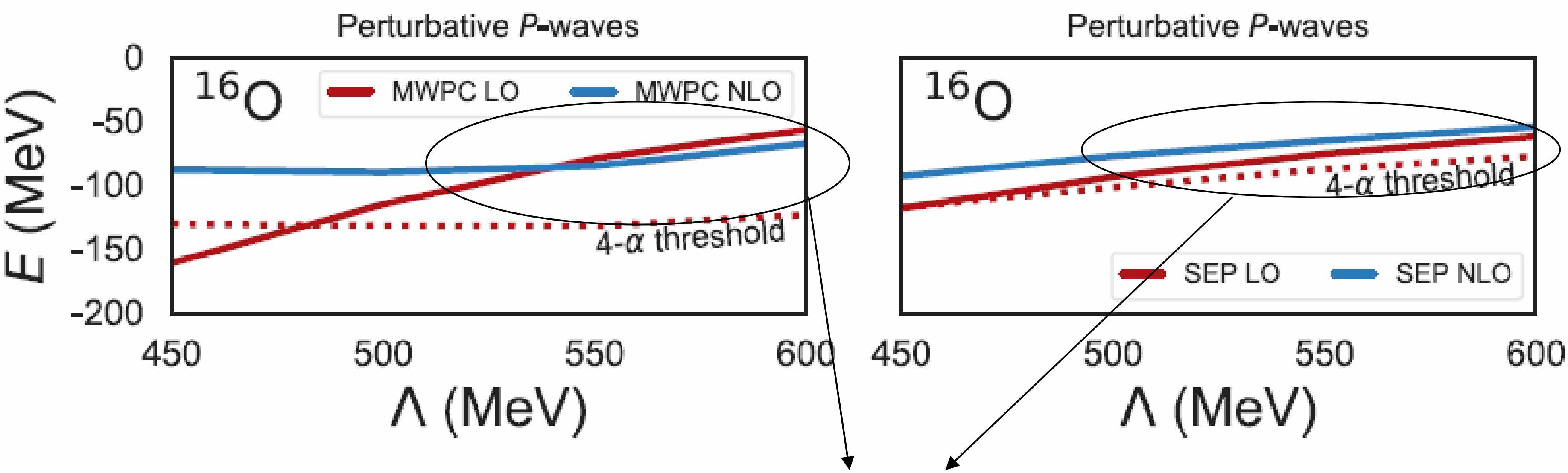
The same NN interaction generates ^{16}O with the *wrong* pole structure (not stable w.r.t. 4α decay) at LO. Also, deformed state becomes deeper than spherical state.

Same thing for PC improved with auxiliary dibaryon fields, Weinberg counting and pionless EFT.

M. S. Sánchez, C.-J. Yang, Bingwei Long, U. van Kolck, Phys.Rev. C97 (2018) no.2, 024001.

In fact, nobody got ^{16}O right at LO yet!

^{16}O results (LO, NN only)



^{16}O non-physical !

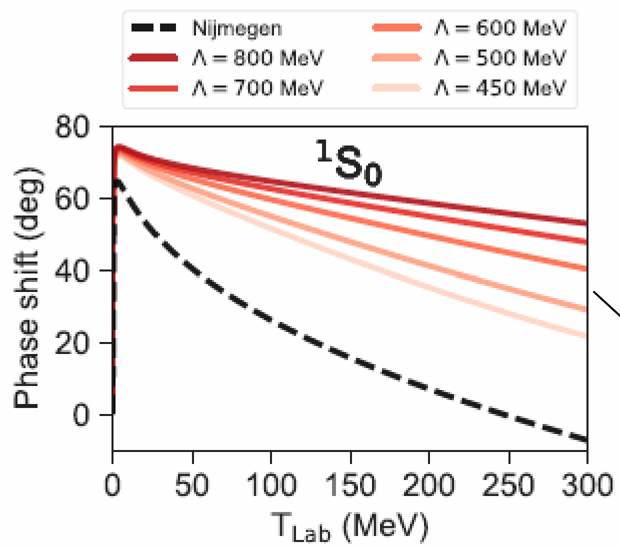
MWPC:
At LO, Nogga, Timmerman, van Kolck PC
(*Phys.Rev.C 72 (2005) 054006)*

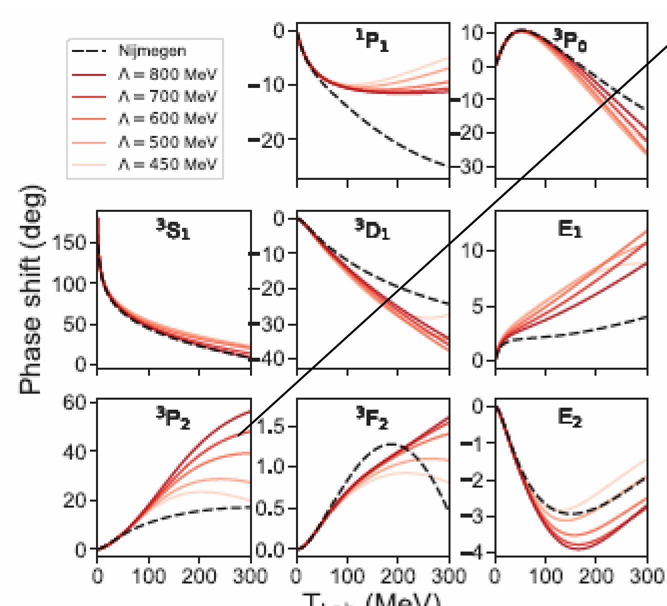
NLO, plus Long & Yang PC
(*Phys.Rev.C* 86 (2012) 024001)

SEP: NN 1s0 adopts dibaryon field
(Phys.Rev.C 97 (2018) 2, 024001)

Perturbative P-waves: PC by S. Wu & B. Long (*Phys.Rev.C* 99 (2019) 2, 024003)

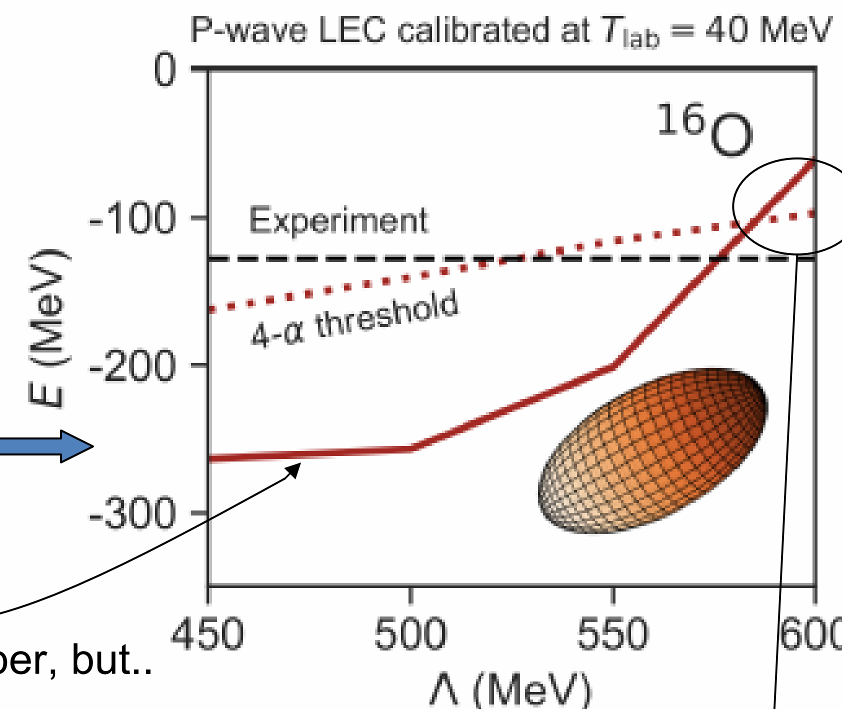
Further fine-tune (LO, NN only)





over-attractive

Make ^{16}O deeper, but..



^{16}O is still non-physical
(deformed and unbound) !

- We have exhausted all possibilities (dibaryon, perturbative P-waves, different fitting of LECs) we could think of in the NN sector.

- Maybe need NNN force at LO.

Estimations

- Combining NDA and "A choose n":

Combine both:

Approx. with nuclear saturation density

$$\frac{C_n^A F_n}{C_m^A F_m} = \frac{A-m}{n}\left(\frac{\rho_0}{f_\pi^2 M_{\rm hi}}\right)^{n-m} = \frac{A-m}{n}\left(\frac{142(\mathrm{MeV})}{M_{\rm hi}}\right)^{n-m}$$

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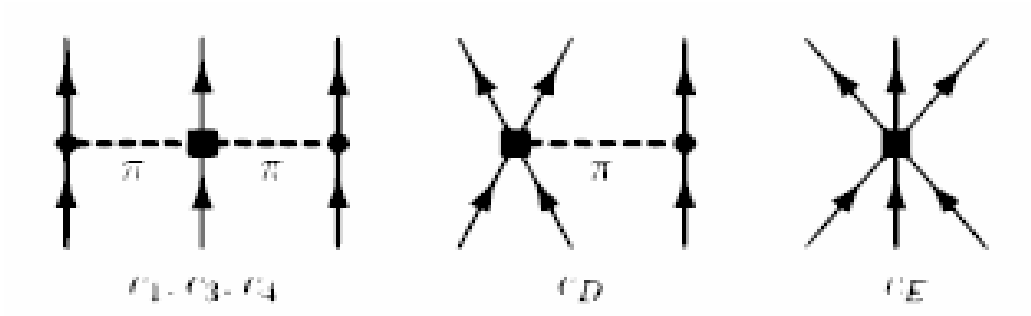
***Might be even weaker due to the previous page's argument.**

NN and NNN becomes the same important starting from A=13-26 !

^{16}O has A=16!

3-body force at LO for ^{16}O

What to include?



NNLO in WPC

All LECs are fixed by NN, π N

This diagram contributes repulsively In ^{16}O or larger nuclei. For A$\leq$4 it's also weakly repulsive.

(**Values $C_{1,3,4}$ from Siemens, et al, PLB 770, 27 (2017)**)

C_D, C_E are free parameters. Renormalization needed.

It turns out, adding only one of them will destroys RG.

Boundary condition is important

Including only 1 diagram won't work (Long and Bira, Annal. phys. 323 (2008) 1304)

Λ (MeV)	c_E	E(^{3}H) (MeV)	E(^{4}He) (MeV)
450	-0.40	-8.48	-24.37
500	-0.205	-8.48	-24.96
550	-0.066	-8.48	-25.69
600	0.0534	-8.48	-26.7
650	0.158	-8.48	-28.09

C_D =0, C_E fixed to ^{3}H binding

^{4}He **diverges w.r.t. cutoff!**

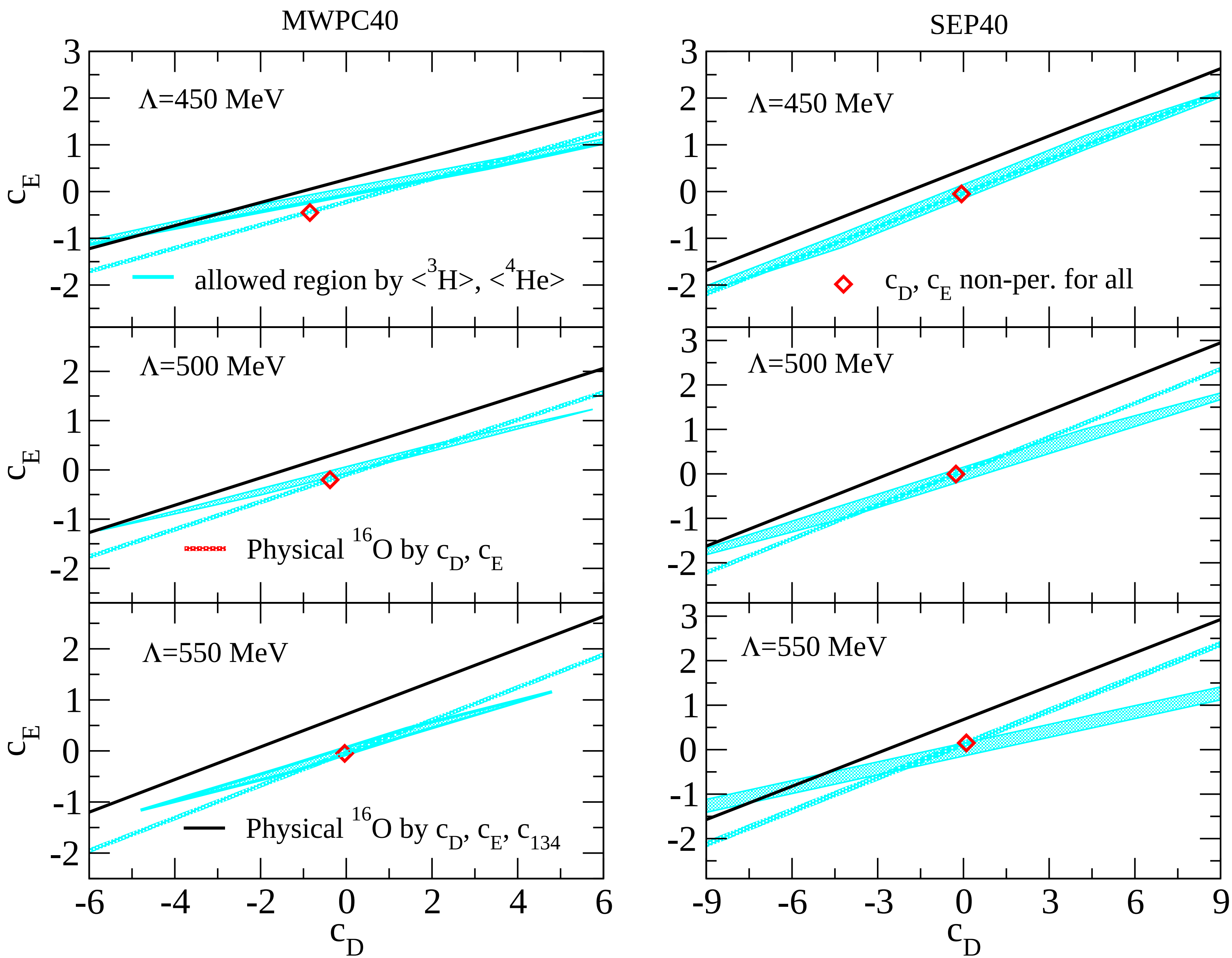
Need both C_D and C_E

how to fix them?

Linear correlated C_D, C_E

- C_D, C_E, C_{134} should enter at higher order (probably at NNLO or later) for $A \leq 4$ nuclei.
- But they will enter non-pertur. at LO for ^{16}O.

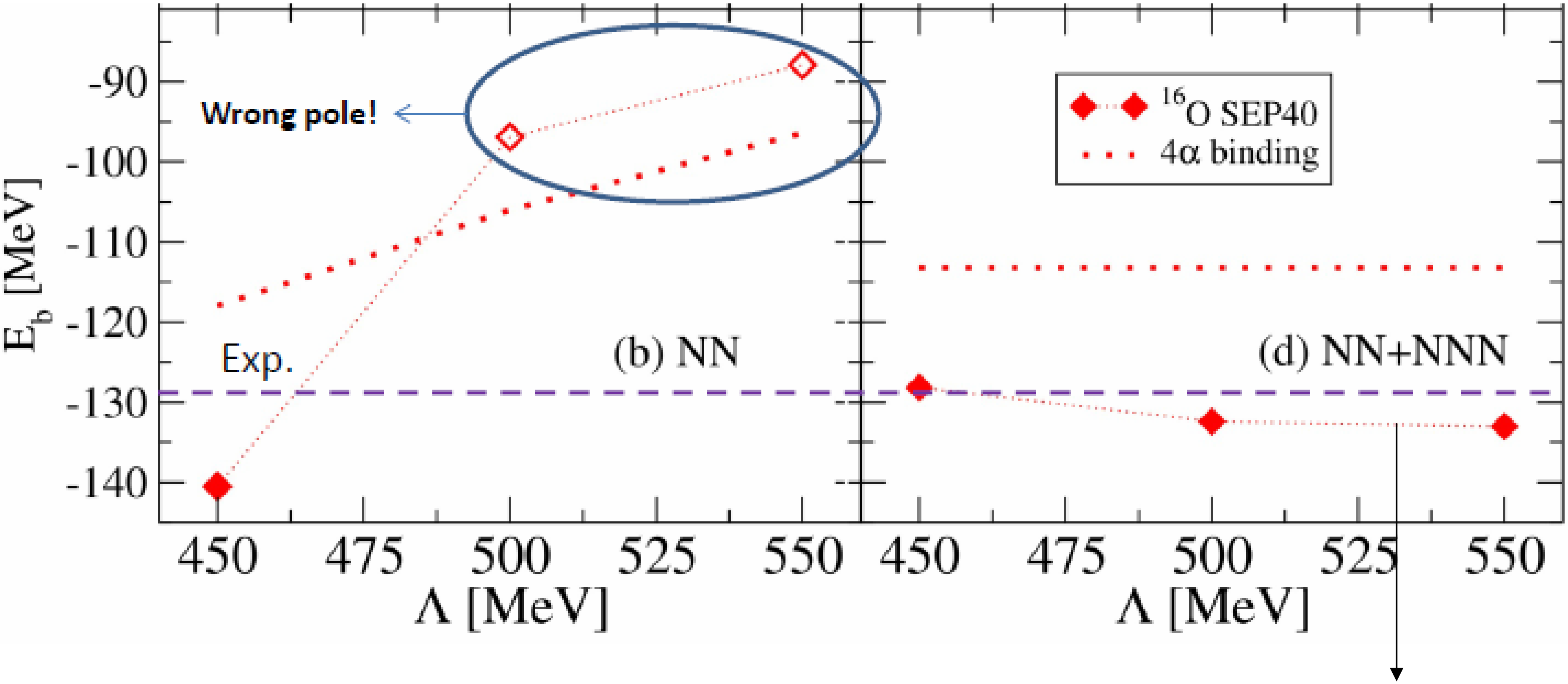
=>Require $|\langle \phi_{LO,3or4} | C_D + C_E | \phi_{LO,3or4} \rangle|$ to be smaller than 1/6 of the ^3H, ^4He binding (need to satisfy in both A=3 &4 cases). ➔ **Denoted as the blue region in next page.**



Red and **black** lines are the (C_D,C_E) and (C_D,C_E,C_{134}) reproduce $^{16}O=-128\pm10\%$.

Using the overlap of the red and blue lines, i.e., (C_D, C_E) fixed by <^{3}H> and <^{4}He> , we got:

before After

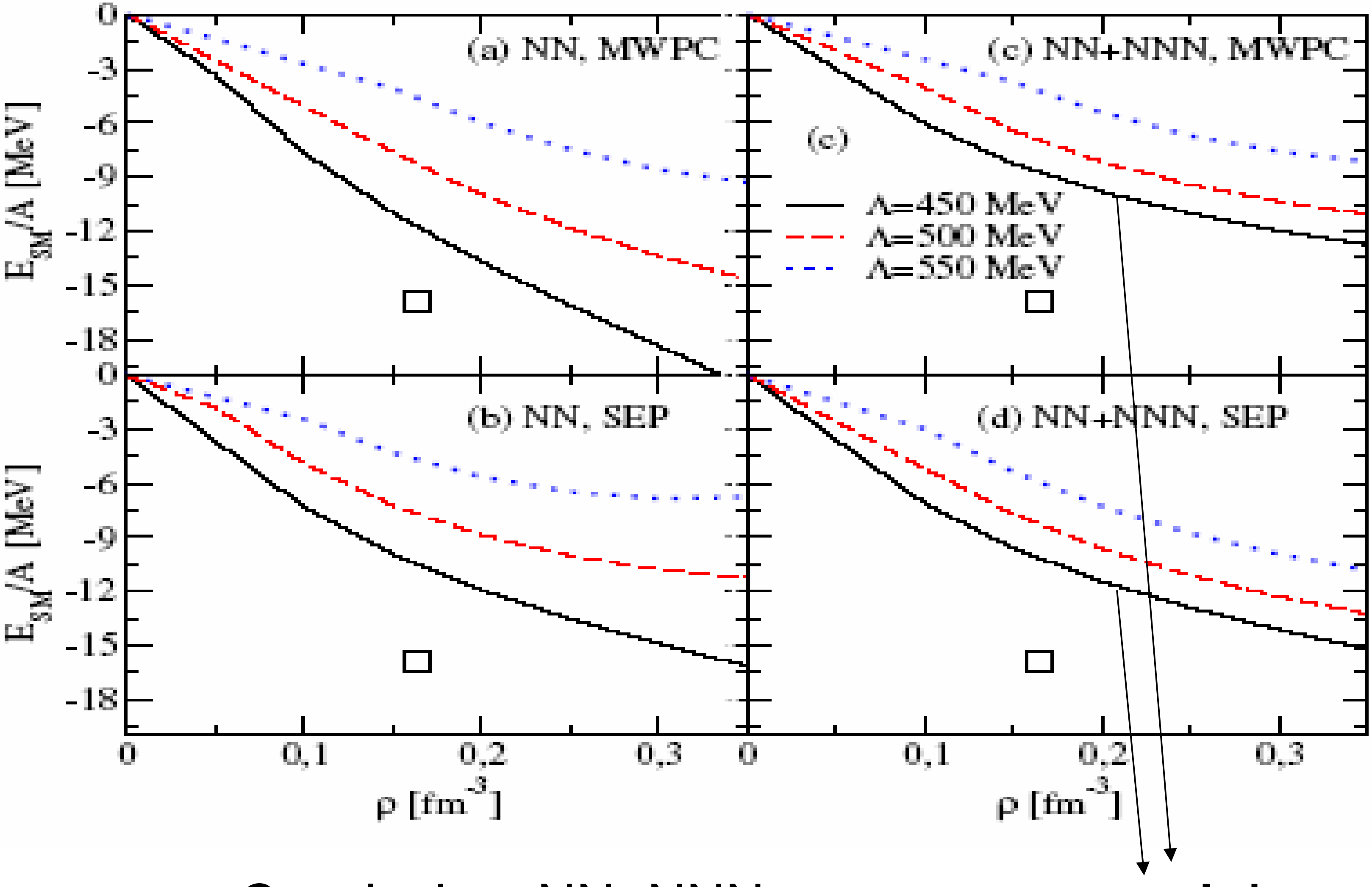


Problem solved! ^{16}O great already at LO!

This suggests:
3NFs are LO at least for A$\geq$16

NNNN at LO?

No-go test by nuclear matter (EoS)



Conclusion: NN+NNN seems **no enough** !

Comments on current status of ab-initio approaches

- With the advance of ab-initio techniques, today the biggest uncertainty of nuclear structure calculations comes from the input potential. => **One needs to improve this first.**

Without A-dependent PC

- State-of-the-art calculations includes at least V_{NNLO} in WPC, i.e., where 3NFs first appears.
- Fitting LECs at few-body sector ➔ Bad saturation properties.
- Fitting LECs overall ➔ Sacrifices NN and few-body sector.

Our study suggests

- **Maybe we really need to include the A-dependent power counting! Where the orders 3NFs and 4NFs enter dep. on A.** (more flexible in some sense, as their LECs don't need to accommodate every nuclei at the same time.)

Summary

- Starting from A=16 (or lower), we need to promote 3-body force (c_D,c_E) at LO in chiral EFT.
- For saturation properties to be reasonably described, a further promotion of 4-body forces to LO is likely.

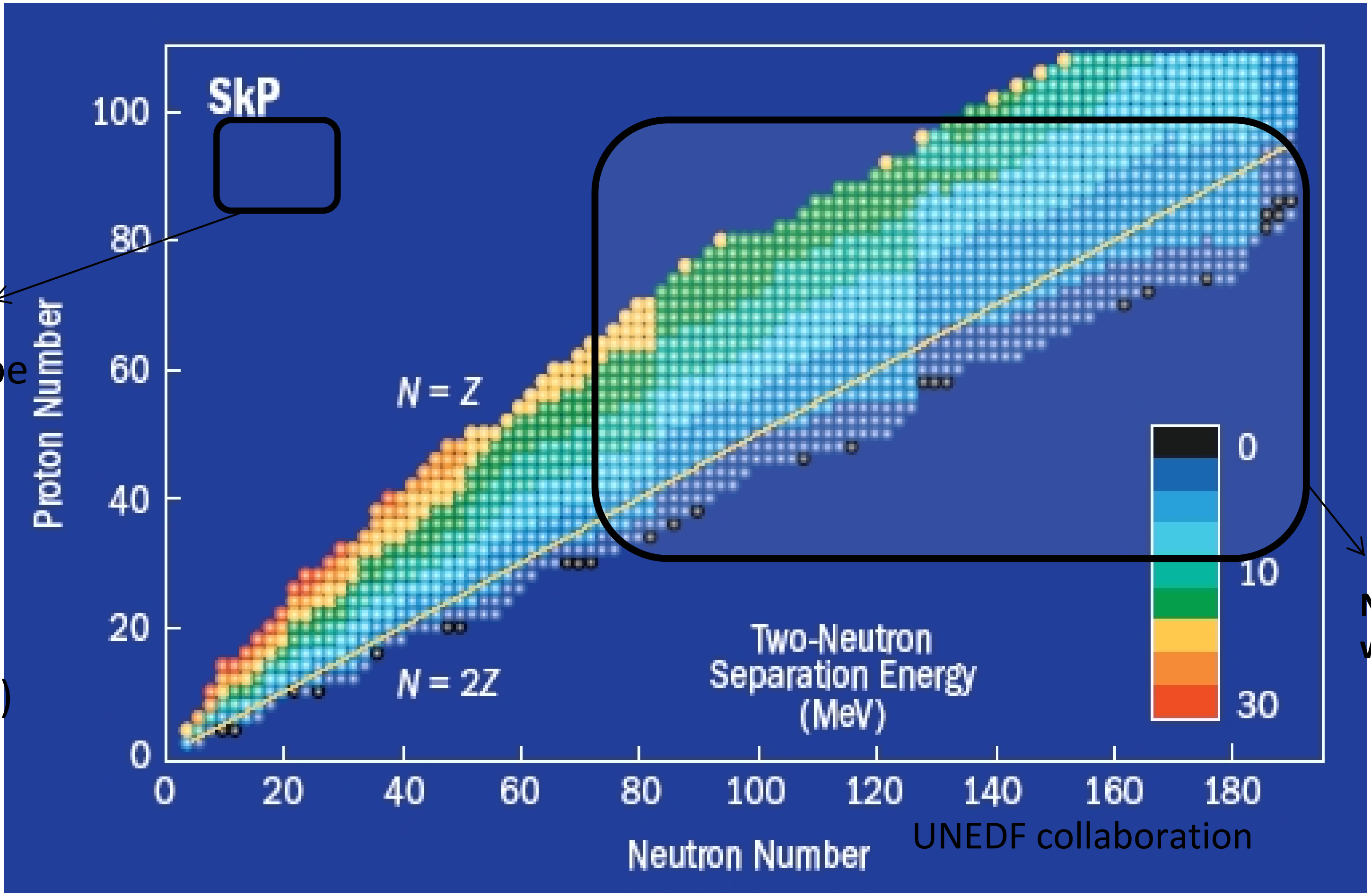
Solutions:

- In short term, might play with Λ and regulator form to avoid 4-body forces ($c_D(\Lambda)$,$c_E(\Lambda)$… go through zero).
- In the long term, need to further develop ab-initio codes/technique to include up to 4-body forces.

Any other way (to get heavier nuclei)?

Mean field with Skyrme-type

Skyrme-type interaction works o.k. (able to do the fitting in EDF framework)

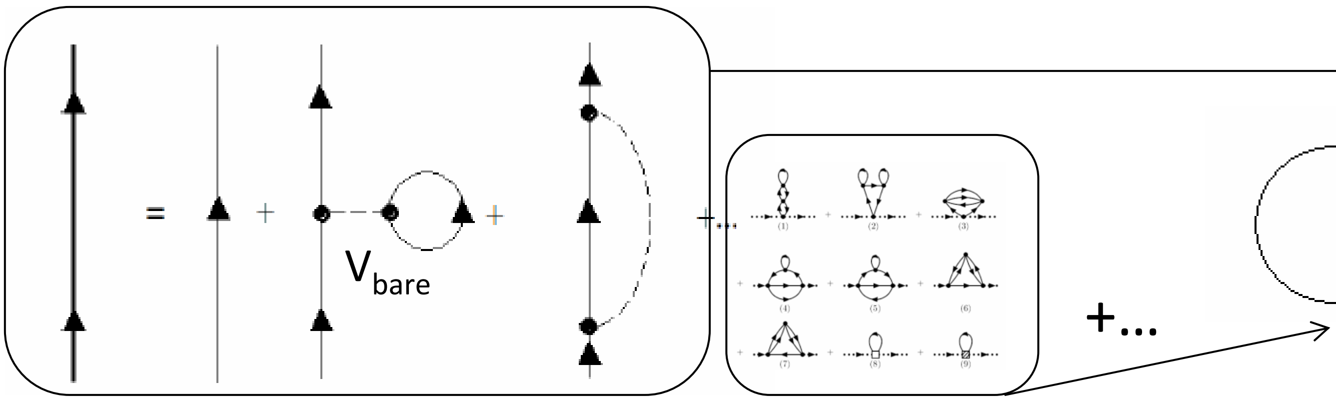


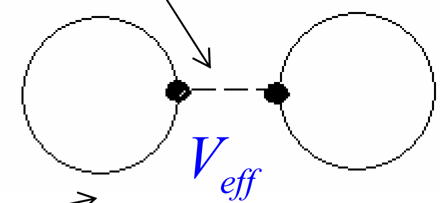
No way to get with ab-initio!

Need to think about other expansion (than on NN d.o.f.).

From bare interaction → V_{eff}

Analogue: from QCD to V_{NN}





Mean field=

$$\int_0^{k_F} dk' \int_0^{k_F} dk \left\langle k \,|\, V_{eff} \,|\, k' \right\rangle$$

$$V_{eff} = \underbrace{t_0(1+x_0P_\sigma)}_{S-wave\ \mathrm{O}(0)} + \frac{1}{2}\underbrace{t_1(1+x_1P_\sigma)(k'^2+k^2)}_{S-wave\ \mathrm{O}(q^2)} + \underbrace{t_2(1+x_2P_\sigma)\mathbf{k}'\cdot\mathbf{k}}_{p-wave\ \mathrm{O}(q^2)}$$

$$+\frac{1}{6}\underbrace{t_3(1+x_3P_\sigma)\rho^\alpha}_{s-wave,\ \text{higher body force}} .$$

$$\mathrm{P}_\sigma = \frac{1}{2}(1+\sigma_1\cdot\sigma_2)$$

I.e., CTs in pionless EFT

Has many-body force to begin with!

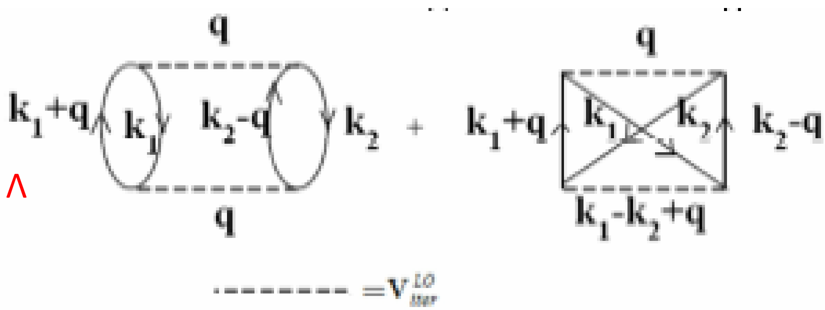
By the way, this (MF EDF) is a pure perturbative approach (i.e., you *do not* iterate V_{eff} at all).

One would like to add some EFT features to it.

First step, need an NLO correction to MF.

Between ab-initio and EDF

Diagrams of vgv:



diverge, need Λ

MBPT(ab-initio)

V defined in **vacuum**
Improvable: v+vgv+…
Exact when include all diagrams (iterated to all order).

Pros:
First principle calculations with clear foundation.
Issues:
Calculations too costly (model-space is large).
Could be very sensitive to detail of V.

Our approach

V **re-defined** at **each order**
Approach by: v+vgv+…, but we want to stop at certain order.

Advantages: Hopefully have advantages from both EDF & MBPT.

EDF

V defined in **medium** (mean field)
Improvable: <nlm|V|nlm>, i.e., V itself, or need multi-reference.

Pros: Simple and effective
Issues:
Missing correlations beyond MF,
V is a model !

Interpretation
LO=MF in EDF
Subleading: included via Perturbation theory

A novel approach toward finite nuclei: between *ab initio* and energy–density–functional theories

C.J. Yang,[1,2] W.G. Jiang,[1] S. Burrello,[3] and M. Grasso[4]

[1] *Department of Physics, Chalmers University of Technology, SE-412 96, Göteborg, Sweden*
[2] *Nuclear Physics Institute of the Czech Academy of Sciences, 25069 Řež, Czech Republic*
[3] *Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany*
[4] *Université Paris-Saclay, CNRS/IN2P3, IJCLab, 91405 Orsay, France*

We propose a novel idea to construct an effective interaction under energy–density–functional (EDF) theories which is adaptive to the enlargement of the model space. Guided by effective field theory principles, iterations of interactions as well as enlargements of the model space through particle-hole excitations are carried out for infinite nuclear matter and selected closed-shell nuclei (^{4}He, ^{16}O and ^{40}Ca) up to next-to-leading order. Our approach provides a new way for handling nuclear matter and finite nuclei within the same scheme, with advantages from both EDF and *ab initio* approaches.

arXiv:2110.01959

MBI

V defi

Improvable: v+vgv+…

Exact when include all diagrams (iterated to all order).

Pros:
First principle calculations with clear foundation.

Issues:
Calculations too costly (model-space is large).
Could be very sensitive to detail of V.

field)

Improvable: <nlm|V|nlm>, i.e., V itself, or need multi-reference.

Pros: Simple and effective

Issues:
Missing correlations beyond MF,
V is a model !

Our approach

V **re-defined** at **each order**

Approach by: v+vgv+…, but we want to stop at certain order.

Advantages: Hopefully have advantages from both EDF & MBPT.

Interpretation
LO=MF in EDF
Subleading: included via Perturbation theory

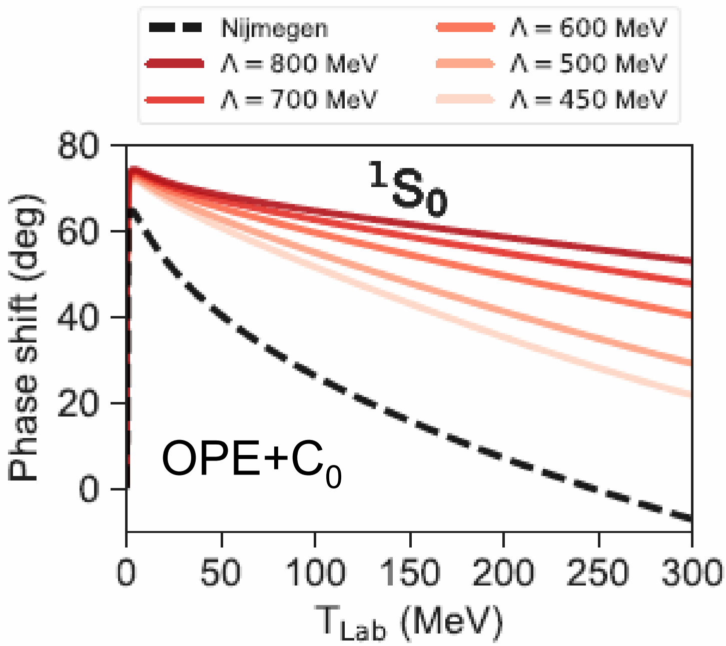
Outlook

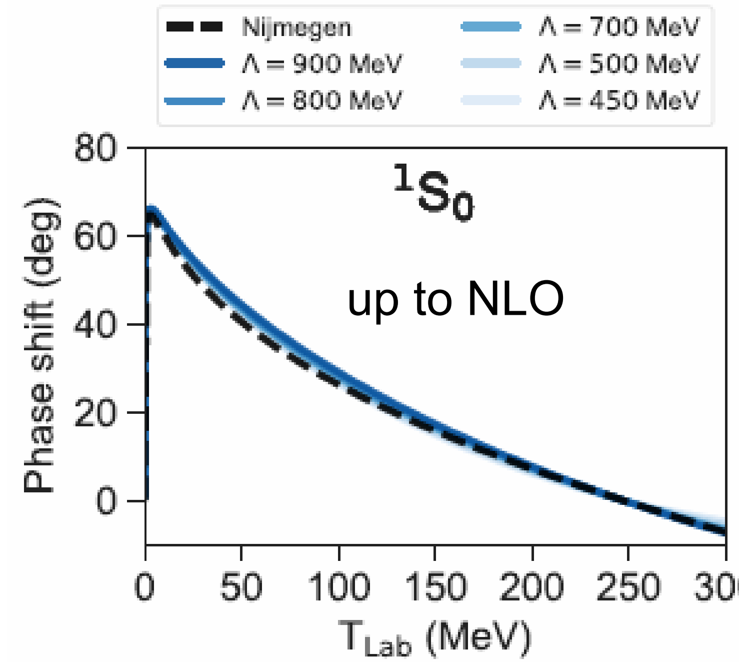
- We are at the **beginning** of a whole new, exciting paradigm!
- Lots of following works (NLO, NNLO, Δ(1232), PC for nuclei away from closed shell, core of the neutron star, etc.) to be done.

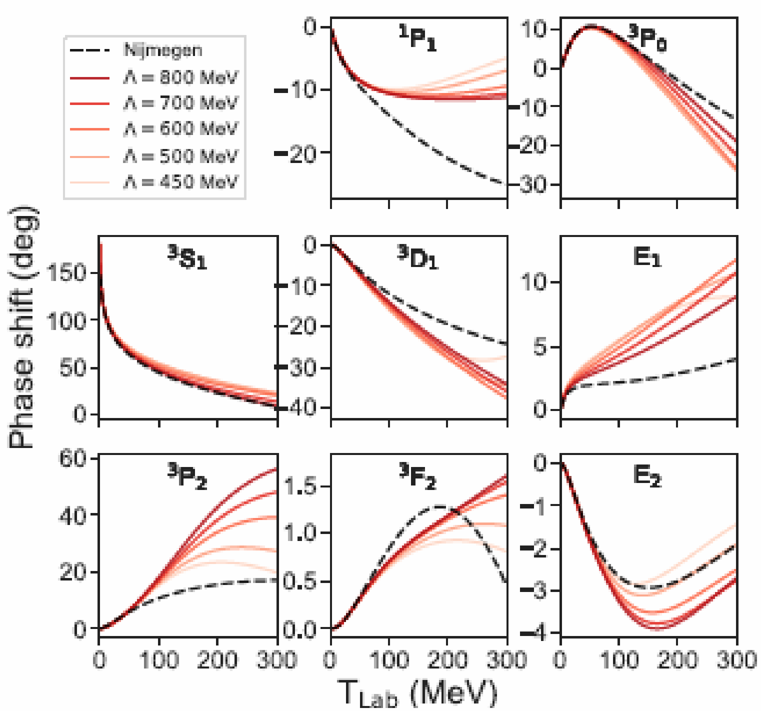
Thank you!

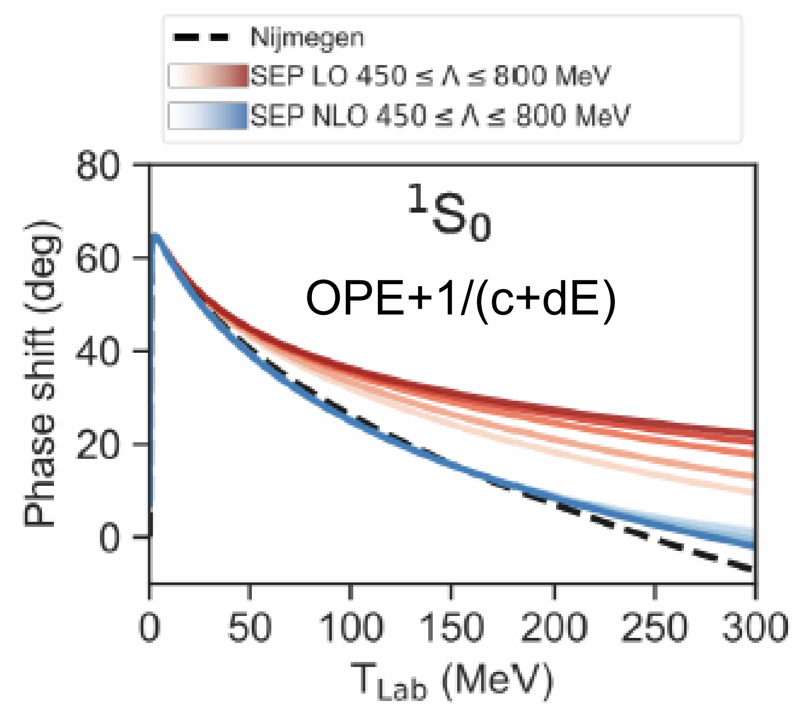
Potential	Lambda	B3LO	B4LO	<cD>_3	<cE>_3	<cD>_4	<cE>_4	<c1>_3	<c3>_3	<c4>_3	<c1>_4	<c3>_4	<c4>_4
MWPC40	450	-11.17156	-40.61872	-1.4	-3.66	-8.5	25.7	-0.0102	1.816	-2.25	1.065	25.175	-21.375
MWPC40	500	-10.19628	-35.12708	-0.71	1.97	-5.98	13.9	-0.0884	1.26	-2.49	0.768	25.24	-24.85
MWPC40	550	-9.07575	-29.02803	-0.0825	0.45	-0.61	2.7	-0.166	-0.192	-2.01	0.0387	11.24	-17.84

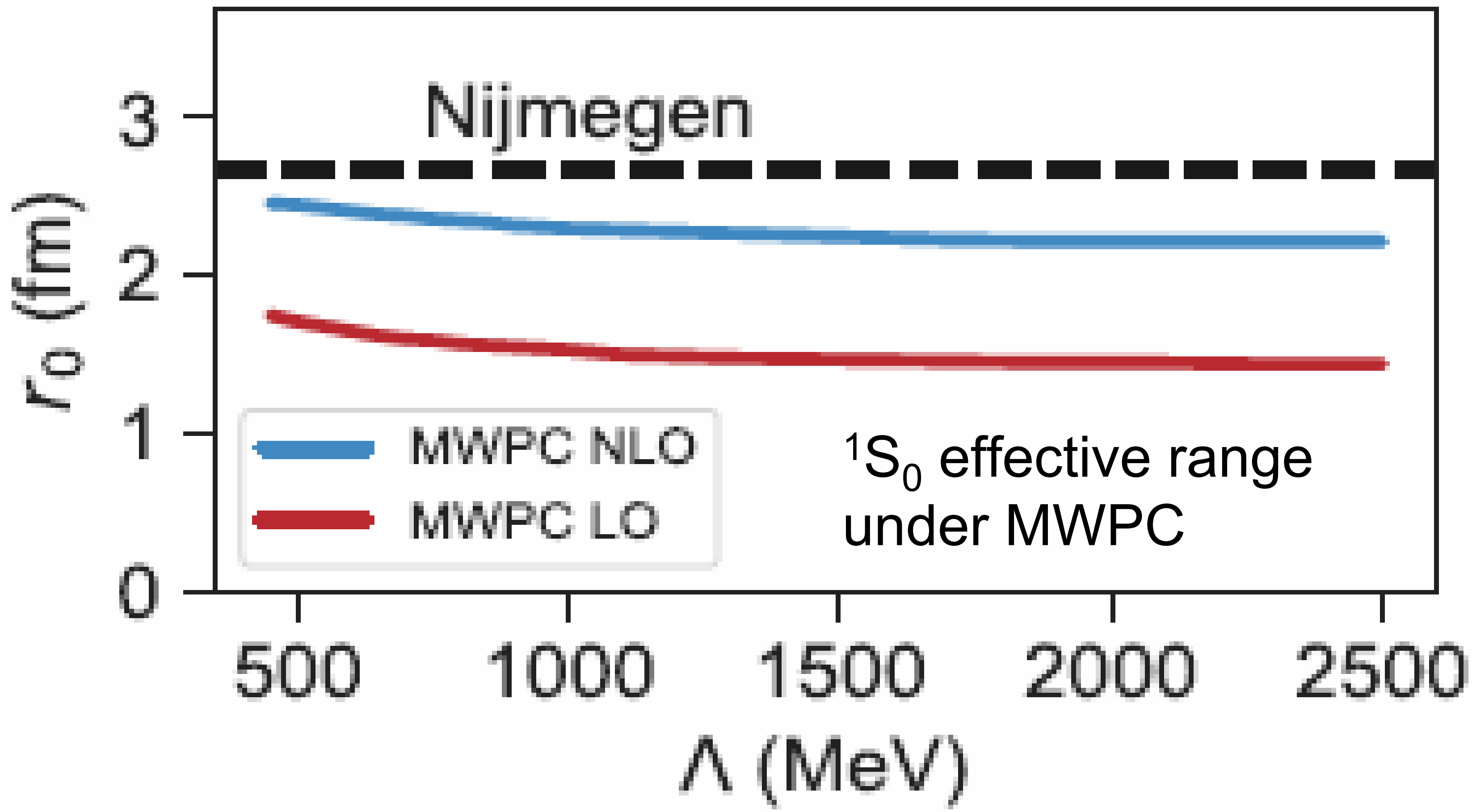
Potential	Lambda	B3LO	B4LO										
SEP40	450	-8.705024	-29.522573	-0.064	0.23	-0.518	1.59	-0.0484	0.439	-1.067	0.313	9.85	-10.21
SEP40	500	-8.197143	-26.527799	-0.235	0.026	-1.74	0.16	-0.106	-0.289	-0.989	-0.0736	4.85	-8.83
SEP40	550	-7.727850	-24.116944	0.067	-0.77	0.425	-4.53	-0.155	-1.36	-0.667	-0.46	-3.25	-5.1


Nijmegen
Λ = 800 MeV
Λ = 700 MeV
Λ = 600 MeV
Λ = 500 MeV
Λ = 450 MeV
$^1S_0$
OPE+C_0
Phase shift (deg)
T_{Lab} (MeV)

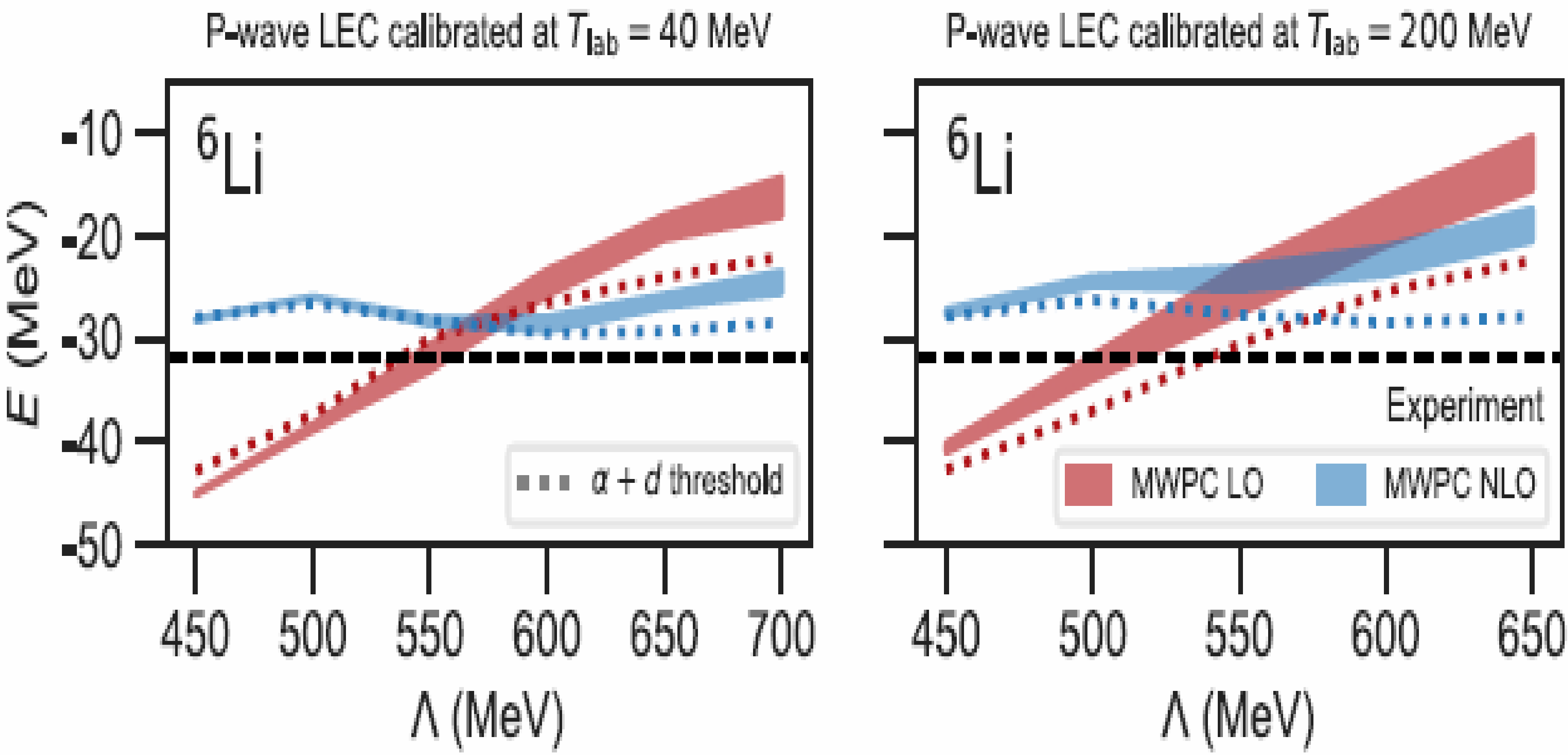

Nijmegen
Λ = 900 MeV
Λ = 800 MeV
Λ = 700 MeV
Λ = 500 MeV
Λ = 450 MeV
$^1S_0$
up to NLO
Phase shift (deg)
T_{Lab} (MeV)


Nijmegen
Λ = 800 MeV
Λ = 700 MeV
Λ = 600 MeV
Λ = 500 MeV
Λ = 450 MeV
$^1P_1$
$^3P_0$
$^3S_1$
$^3D_1$
E_1
$^3P_2$
$^3F_2$
E_2
Phase shift (deg)
T_{Lab} (MeV)


Nijmegen
SEP LO 450 ≤ Λ ≤ 800 MeV
SEP NLO 450 ≤ Λ ≤ 800 MeV
$^1S_0$
OPE+1/(c+dE)
Phase shift (deg)
T_{Lab} (MeV)



*SEP (dibaryon field) has r_0 fitted to data already at LO.

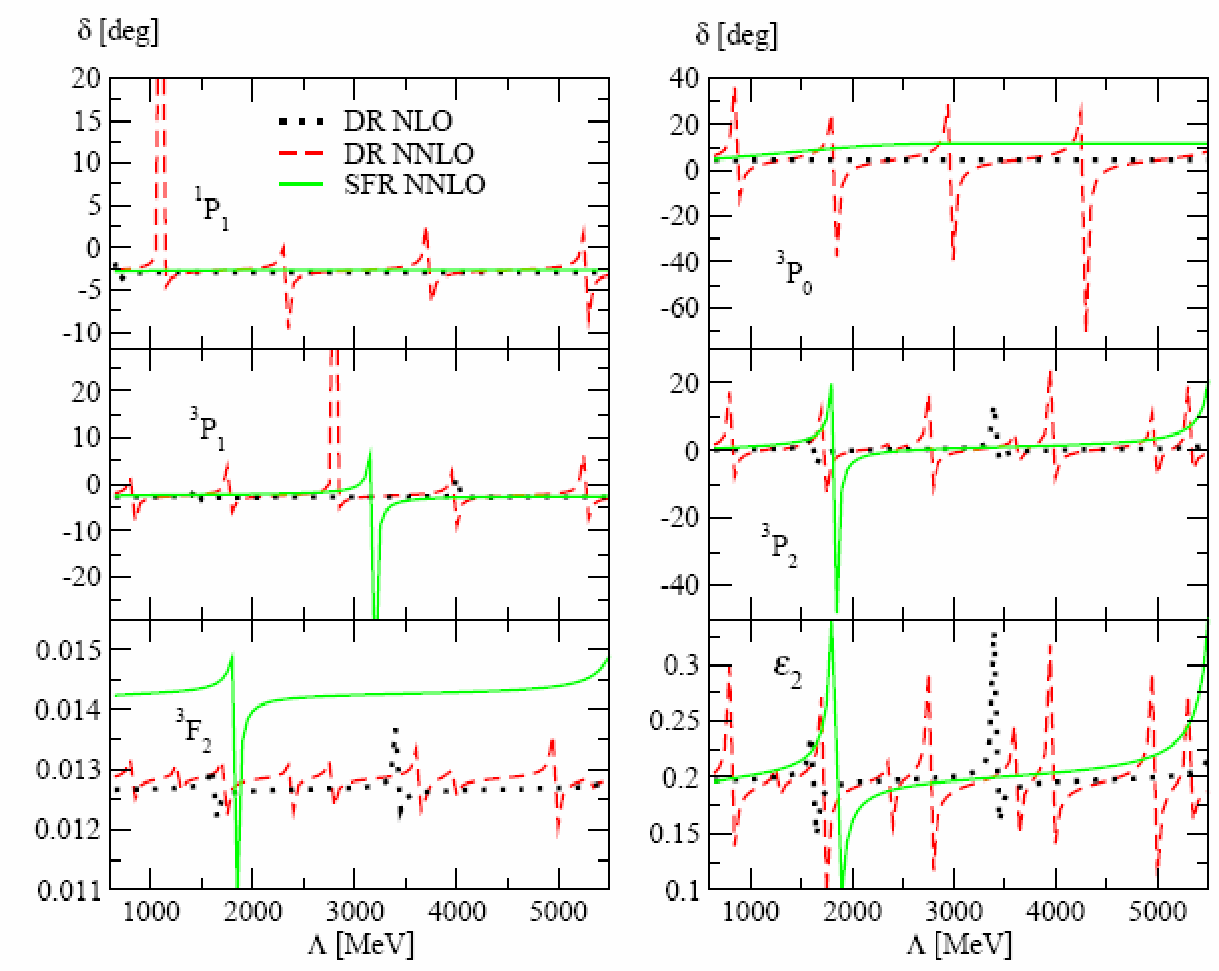


P-wave LEC calibrated at T_{lab} = 40 MeV
P-wave LEC calibrated at T_{lab} = 200 MeV
^{6}Li
^{6}Li
-10
-20
-30
-40
-50
E (MeV)
450
500
550
600
650
700
Λ (MeV)
α + d threshold
Experiment
MWPC LO
MWPC NLO
450
500
550
600
650
Λ (MeV)

Problems of WPC

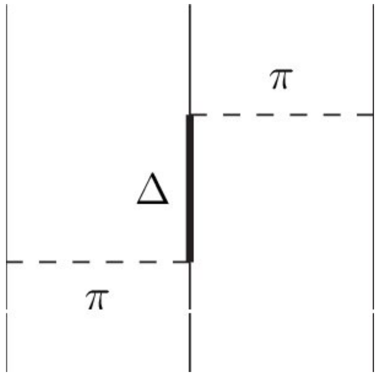
WPC is wrong at LO ! (Nogga, Timmermans, van Kolck, *PRC 72 (2005) 054006*)

•Beyond LO: (Yang, Elster, Phillips (2008-2010))



Δ(1232) NNN?

Fujita-Miyazawa



Appear at NLO, i.e., one order before c_D,c_E,c_{134}.

So far we include it by the "*resonant saturated*" way[1,2], but then it appears to be repulsive for ^{16}O, at least up to the highest cutoff (550 MeV) we can test.

[1] Epelbaum, et al, Nucl. Phys. A 806, 65 (2008).
[2] Ekstrom, et al, PRC 97, 024332 (2018).

EFT-like approaches

Select a model space
(since we cannot do calculations in (or even actually know the structure of) the entire Hilbert space)

Build/Derive the interactions in that model space
(normally utilize properties of symmetry)

Solve for any observable desired

Compare to data (check your assumptions)

Renormalization group (RG)

Select a model space
(since we cannot do calculations in (or even actually know the structure of) the entire Hilbert space)

Build/Derive the interactions in that model space

Solve for specified observable desired

Vary your model space (mostly enlarge→ more shouldn't hurt)

Compare to data (check your assumptions)

Minimum requirement of **EFT©**

For more details: see, e.g., publication list: 1

Select a model space
(since we cannot do calculations in (or even actually know the structure of) the entire Hilbert space)

Renormalize **the interactions in that model space**

Describe observable within the validity of EFT

Vary your model space (mostly enlarge→ more shouldn't hurt)

↓

Compare to data (check your assumptions)