Phys 6021: Homework III

due February 8, 2019

1. Scattering from a Spherical Square Well

Two particles of mass m scatter. The potential between them is approximated by an attractive square well:

$$V(r) = \begin{cases} -V_0 & r < b \\ 0 & r > b \end{cases}, \tag{1}$$

where V_0 is a positive number.

(a) To warm up work through problem 11.4 in Zettili

(b) *(6 p)* Determine the scattering length a_0 for this potential.
(Hint: Consider the logarithmic derivative when deriving your expressions, though Zettili does not use it.)

(c) *(6 p)* Sketch a_0 versus the potential strength V_0 with V_0 starting at zero and getting large enough to produce a bound state.
(Use your favorite plot program or Wolfram-alpha.)

(d) For making a connection to bound states, review Section 6.3.3 in Zettili, which calculates the bound state for a spherical square well potential.

(e) *(6 p)* Expand a_0 in powers of V_0, thereby creating a *Born Series* for the scattering length.

(f) *(6 p)* Show that this Born series diverges when the potential is strong enough to form a bound state.

(g) *(6 p)* The same analysis can be used for repulsive potentials by changing the sign of V_0. Sketch a_0 for a repulsive potential as a function of potential strength and deduce if the Born series may fail for a repulsive potential.

(h) *(6 p)* Obtain an expression for the total cross section for low energy scattering in terms of the scattering length. Compare your expression with the total cross section for scattering off a hard sphere. Use your expansion from **(e)** to determine which information about the potential can be extracted from the total cross section.

(i) *(6 p)* The effective range expansion for the s-wave ($\ell = 0$) is given by

$$k \cot \delta_0 = \frac{1}{a_0} + \frac{1}{2} r_0 k^2, \tag{2}$$

where r_0 is called the *effective range*. Proceed as in **(a)**, but keep the next order to extract r_0 for the spherical square well.