Phys 6021: Homework II

due February 1, 2019

Cross Sections and Phase Shift Analysis

An experiment measures the differential cross section for the elastic scattering of two particles with wave vector k in the center of momentum to have the form

$$\frac{d\sigma}{d\Omega}(\theta) = \frac{1}{k^2} e^{-2(1-\cos\theta)}. \tag{1}$$

1. *[2 pt]* Plot the differential cross section as function of the scattering angle θ for all allowed values of θ. For the plot decide which units you want to use (i.e. atomic, nuclear or particle) for k and make sure the cross section is given in the correct units. In addition, argue why you choose your specific value of k when you think of your specific physicical system and experimental situation. The axes of your plot are supposed to contain your units.

2. *[3 pts]* Without any detailed calculation, deduce the number of partial waves which may contribute to the scattering and indicate if this is compatible with scattering from a finite range potential.

3. *[3 pts]* What must be the modulus of the angle-dependent scattering amplitude, $|f_E(\theta)|$?
   Remark: A complex number $z = x + iy = Re^{i\alpha}$ has modulus $R = \sqrt{x^2 + y^2}$ and phase α.

   Next, the experimentalist measures the total cross section for the same particles and finds it to have the form

   $$\sigma_{tot} = \frac{4\pi}{k^2}. \tag{2}$$

4. *[3 pts]* What is the value of the scattering amplitude in forward direction, $f_E(0^o)$?

5. *[3 pts]* Assuming that the scattering amplitude has a constant phase, what is $f_E(\theta)$?

6. *[3 pts]* What is the total elastic (integrated elastic) cross section for this reaction? Comment on why this is the same or different from the total cross section.

7. *[3 pts]* Why must the phase shift $\delta_l(k)$ be complex for this reaction?

8. *[3 pts]* Find the $l = 0$ phase shift for this interaction.

Phase Shifts for Hard Sphere Scattering *[6 pts]*

(a) Find the phase shifts for scattering by a hard sphere

$$V(r) = \begin{cases} \infty & r < a \\ 0 & r > a \end{cases} \tag{3}$$

(b) Find the total cross section for an incoming energy

$$E = \frac{\hbar^2 k^2}{2m} \tag{4}$$

in the two limits

$$\begin{aligned} k &\to 0 \\ k &\to \infty. \end{aligned} \tag{5}$$

Give a physical interpretation of the factors 4 and 2 in your answers.

Hint 1: For $k \to \infty$ use the asymptotic form of j_l and n_l to obtain a simple form for $\sin^2 \delta_l$. Furthermore, replace the sum over l by an integral so that

$$\sigma = \sum_{l=0}^{l=ka} \sigma_l \approx \frac{4\pi}{k} \int_0^{ka} dl \ (2l+1) \sin^2 \delta_l. \tag{6}$$

Hint 2: look at Zettili, Problem 11.3.