CDCC model space

$$[E-K-V(r)-U(r_p)-U(r_n)]\,\psi=0$$

The CDCC model space P:

selects low angular momenta l associated with $r=r_p-r_n$, up to a maximum l_m.

$$1-P=Q$$

Principal point:

· Asymptotic two-body channels are located uniquely either in component $P\psi$ or in $Q\psi$

· Asymptotic boundary conditions for each two-body channel are expressed in finite form in terms of the natural variables of its Faddeev component.

$P\psi$: no asymptotic amplitudes in the two-body rearrangement channels.

$n - A$ bound-state channel:

$$P\left[\phi_n\left(r_n\right)\chi_p\left(r_p\right)\right] \to O\left(1/r_p^3\right)$$

Natural variables: $r = r_p - r_n$, $R = (r_p + r_n)/2$.

$Q\psi$: no asymptotic amplitude in the deuteron channel.

Natural variables: r_n, r_p.

$$(E - K - V - PU)P\psi = PUQ\psi,$$

$$(E - K - V - QU)Q\psi = QUP\psi.$$

Additional approximations: use eigenstates of $K_r + V(r)$ to expand.

Coupling potentials in these equations arise fromPUP have long tails; depend somewhat on the method of discretization.

Term $PUQ\psi$ tends to be weak, especially for the smooth potentials U_p, U_n in current CDCC applications, because:

U has small matrix elements between significantly different l states

U only links $l \approx l'$ states in the P and Q spaces if $l \approx l' \approx l_m$, a fairly large angular momentum.

For such values of angular momenta centrifugal repulsion reduces the wave function at small radii; in general $PUQ\psi$ vanishes rapidly at large radii, giving an overall reduction.

$$(E - K - V - PUP)\psi^{\mathrm{CDCC}} = 0$$

Effects of the coupling term $PUQ\psi$

1. realistic nucleon optical potentials:

U_p and U_n have absorptive imaginary parts;

rearrangement channels are ”closed by absorption” and have no very-high-l components.

components of $Q\psi$ that are appreciable will be ones that can be reached directly from the deuteron channel or through a few steps of continuum-continuum coupling.

2. U_p and U_n are real:

can have open rearrangement channels.

recognize $PUQ\psi$ as a complicated long-ranged eff’ective potential in P space that takes account of the higher-angular-momentum states in Qspace.

despite the coupling to Q space, ψ may still be a good approximation in a limited region $R < R_m$.

standard Faddeev differential equations for a deuteron-nucleus example:

$$
\begin{aligned}
(E-K-V)\psi_d &= V\left(\psi_p+\psi_n\right),\\
\left(E-K-U_p\right)\psi_p &= U_p\left(\psi_d+\psi_n\right),\\
\left(E-K-U_n\right)\psi_n &= U_n\left(\psi_d+\psi_p\right),
\end{aligned}
$$

Faddeev equations with distorting potentials:

$$
\begin{aligned}
(E-K-V)\psi_d &= V\left(\psi_p+\psi_n\right),\\
\left(E-K-U_p\right)\psi_p &= U_p\left(\psi_d+\psi_n\right),\\
\left(E-K-U_n\right)\psi_n &= U_n\left(\psi_d+\psi_p\right),
\end{aligned}
$$

Addition:

$$
\left(E-K-U_p-U_n\right)\left(\hat{\psi}_p+\psi_n\right)=(U-PUP)\hat{\psi}_d
$$

$$(E - K - U_p - U_n)\left(\hat{\psi}_p + \psi_n\right) = (U - PUP)\hat{\psi}_d$$

$$(E - K - U_p - U_n)\left(\hat{\psi}_p + \hat{\psi}_n\right) \approx QUP\hat{\psi}_d$$

Iterative approach

1. solve the CDCC equation;

2.insert it as a zero-order approximation for $\hat{\psi}_d$ in belowing equation to produce $\hat{\psi}_p + \hat{\psi}_n$.

$$(E - K - U_p - U_n)\left(\hat{\psi}_p + \psi_n\right) = (U - PUP)\hat{\psi}_d$$

3.insert $\hat{\psi}_p + \hat{\psi}_n$ to produce $\hat{\psi}_d$.

$$(E - K - V)\psi_d = V\left(\psi_p + \psi_n\right)$$

4.repeat 2 and 3.

Iterative approach

separate $\hat{\psi}_p + \hat{\psi}_n$ into arrangement channels with bound n-A states and bound p-A states:

$$\begin{aligned}
&[E - K - U_p - P_p U_n P_p] \tilde{\psi}_p \\
&\quad = [U_p - P_n U_p P_n] \tilde{\psi}_n + [U_p - P U_p P] \tilde{\psi}_d, \\
&[E - K - U_n - P_n U_p P_n] \tilde{\psi}_n \\
&\quad = [U_n - P_p U_n P_p] \tilde{\psi}_p + [U_n - P U_n P] \tilde{\psi}_d,
\end{aligned}$$

1. solve the CDCC equation;

2.insert it as a zero-order approximation for $\hat{\psi}_d$ in (7') to produce $\hat{\psi}_p + \hat{\psi}_n$.

3.insert $\hat{\psi}_p + \hat{\psi}_n$ to produce $\hat{\psi}_d$.

4.repeat 2 and 3.

Iterative approach

Another way to solve

$$(E - K - U_p - U_n)\left(\hat{\psi}_p + \psi_n\right) = (U - PUP)\hat{\psi}_d$$

might be to expand in a truncated basis of homogeneous eigensolutions of the LHS of the equation.

choose the parameter l large enough so iteration is unnecessary.