Calculation of the $A = 6$ bound states within the Hyperspherical Harmonic basis

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Solving the Schrödinger equation

$$H = \sum_i \frac{p_i^2}{2M} + \sum_{i<j} V(i,j) + \sum_{i<j<k} W(i,j,k) + \ldots$$

Search for accurate solution of the Schrödinger equation

- Variational approach (focus on bound state only)

$$H\Phi = E\Phi \qquad\qquad \Phi = \sum_{i=1}^{N} c_i \psi_i \tag{1}$$

- The ψ_i form a complete basis

$$\hat{O}\psi_i = O_i \psi_i \tag{2}$$

- Transform the Schrödinger eq. in an eigenvalue problem

$$\sum_{j=1}^{N} c_j \langle \psi_i | H | \psi_j \rangle = E(N) \sum_{j=1}^{N} c_j \langle \psi_i | \psi_j \rangle \tag{3}$$

- $E(N) \xrightarrow{N\to\infty} E \Rightarrow$ Check convergence

Hyperspherical Harmonics (HH)

Some literature on HH

- Reviews on $A = 3$ and 4 systems
  - A. Kievsky, *et al.*, J. Phys. G, **35**, 063101 (2008)
  - L.E. Marcucci, *et al.*, Front. Phys. **8**, 69 (2020)

- Articles on $A = 6$
  - AG, M. Viviani and L.E. Marcucci, Phys. Rev. C **102**, 014001 (2020)
  - AG, L.E. Marcucci, R. Schiavilla, M. Viviani, arXiv:2106.07439 (2021)

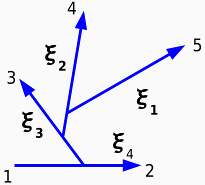
Outline

- The Hyperspherical Harmonics method
  - General introduction
  - Hands on
- The $A = 6$ bound state
  - Convergence and spectrum
  - ^{6}He beta decay
- Towards scattering states
  - $\alpha + d$ clusterization of ^{6}Li
- Conclusions

The HH basis

- Jacobi vectors $\vec{\xi}_1, \ldots, \vec{\xi}_N \Rightarrow$ CoM completely decoupled

$$T = -\frac{\hbar^2}{2m}\sum_{i=1}^{A}\nabla^2_{\vec{r}_i} = T_{CoM} - \frac{\hbar^2}{m}\sum_{i=1}^{A-1}\nabla^2_{\vec{\xi}_i}$$



- Hyperangular variables $\rho = \sum_{k=1}^{5}(\xi_k)^2$, $\Omega = \{\hat{\xi}_i, \phi_i\}$, $\cos\phi_k = \frac{\xi_k}{\sqrt{\xi_1^2+\ldots+\xi_k^2}}$

$$T_0 = -\frac{\hbar^2}{m}\left(\frac{\partial^2}{\partial\rho^2} + \frac{3A-4}{\rho}\frac{\partial}{\partial\rho} - \frac{L^2(\Omega)}{\rho^2}\right)$$

- Eigenbasis of $L^2(\Omega) \Rightarrow$ Hyperspherical Harmonics (HH)

$$L^2(\Omega)\mathcal{Y}_{[K]}(\Omega) = K(K+3A-5)\mathcal{Y}_{[K]}(\Omega)$$

The HH basis

- The variational wave function

$$\psi_A^{J^\pi} = \sum_p \sum_{l,[KST]} c_{l,[KST]} f_l(\rho) \underbrace{\left[\mathcal{Y}_{[K]}(\Omega_{A-1}^p)\left[\chi_{[S]}^p \otimes \chi_{[T]}^p\right]\right]_{J^\pi}}_{\phi_{[\alpha]}^{KLSTJ}(\Omega^p)},$$

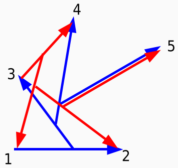
- Hyperradius $\Rightarrow$ Laguerre polynomials $f_l(\rho)$
- Spin and Isospin degrees of freedom $\chi_{[S]}$ and $\chi_{[T]}$
- $p \rightarrow$ even permutation of the particles
- $c_{l,[KST]}$ variational coefficients

Properties of HH

- Antisymmetrization selecting quantum numbers $\Rightarrow$ exploiting permutations

- Transformation Coefficients (TC)

$$\mathcal{Y}_{[K]}(\Omega^p) = \sum_{[K']}^{K=K'} a_{[K],[K']} \mathcal{Y}_{[K']}(\Omega)$$



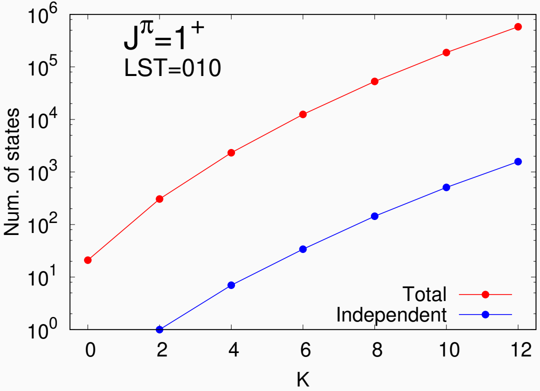
- Sum over the permutations rewritten in terms of the transformation coefficients

$$\sum_p \phi_{[\alpha]}^{KLSTJ}(\Omega^p) = \sum_{[\alpha']} a_{[\alpha],[\alpha']}^{KLSTJ} \phi_{[\alpha']}^{KLSTJ}(\Omega)$$

- Basis states are linear dependent $\Rightarrow$ orthogonalization

Orthonormalization

- Ground state of ^{6}Li is $J^\pi = 1^+$ (mainly $T = 0$)



$$\frac{\#tot}{\#ind} \sim cost. = 370$$

Matrix elements calculation

- Variational wave function

$$\Psi_A = \sum_{l,[\alpha]} c_{l,\alpha} \Phi_{l,\alpha} \qquad \Phi_{l,\alpha} = f_l(\rho) \sum_{\alpha'} a_{[\alpha],[\alpha']} \phi_{[\alpha']}(\Omega)$$

- Evaluation of the matrix elements

$$H_{l,\alpha,l',\beta} = \langle \Phi_{l,\alpha} | H | \Phi_{l',\beta} \rangle$$

- Analytical for the kinetic energy
- Potential matrix elements

$$\langle \Phi_{l,\alpha} | \sum_{i<j} V(i,j) | \Phi_{l',\beta} \rangle = \frac{A(A-1)}{2} \langle \Phi_{l,\alpha} | V(1,2) | \Phi_{l',\beta} \rangle$$

$\Rightarrow$ By using the sum over the permutations and the TC

Hands on

$$\langle \Phi_{l,\alpha} | V(1,2) | \Phi_{l',\beta} \rangle = \sum_{[\alpha'],[\beta']} a_{[\alpha],[\alpha']} a_{[\beta],[\beta']} \underbrace{V_{[l,\alpha'],[l',\beta']}(1,2)}_{\text{depend on } \mu \text{ only}}$$

$\alpha, \beta =$ qn of 6-bodies, $\mu =$ qn of the couple (1,2)

$$V_{[l,\alpha'],[l',\beta']}(1,2) = \int_{\rho,\Omega} f_l(\rho) \phi_{[\alpha']} V_\mu(\boldsymbol{r}_2 - \boldsymbol{r}_1) f_{l'}(\rho) \phi_{[\beta']}$$

To be notice that

1. The sum over $[\alpha'], [\beta']$ can run over millions of states
2. The potential involves only the particles (1,2) and so it depends on a small set μ of qn
3. The sum over the qn which do not involve the couple (1,2) is independent of the particular potential model

qn= quantum numbers

Hands on

Therefore, we can rewrite

$$\langle \Phi_{l,\alpha} | V(1,2) | \Phi_{l',\beta} \rangle = \sum_{\mu} D_{\mu}^{[\alpha],[\beta]} V_{\mu,l,l'}(1,2)$$

- $D_{\mu}^{[\alpha],[\beta]}$ independent on the potential model
  $\Rightarrow$ evaluated and stored only once
- Small number of combinations μ
  - small disc space required for saving $\sim$ 100GB
  - fast construction of potential matrix elements
- Extensible to 3N forces (in progress...)

Hands on

- Completely antisymmetrized
- Orthogonalization $\Rightarrow$ relatively small basis
  Full calculation $N_{HH} \sim 7000$
  Hamiltonian dimension $\sim 110000 \times 110000$
- Easy computation of the matrix elements
- No need to save the matrix elements only the D coefficients
- $\sim$ 3 hours for constructing and diagonalize the Hamiltonian
  $\Rightarrow$ easy to test various potentials

Warning!

- We will use SRG evolved N^3LO500 *NN* interaction [1-2]
  - SRG evolution parameter $\Lambda = 1.2, 1.5, 1.8\ \text{fm}^{-1}$
  - The Coulomb interaction is included as "bare" (not SRG evolved)
- Explorative study with NNLO^*_{sat} [3]
- No 3-body forces (for now)
- We compute the mean values of "bare" operators

[1] S.K. Bogner, R.J. Furnstahl, and R.J. Perry, PRC **75**, 061001(R) (2007)
[2] D.R. Entem and R. Machleidt, PRC **68**, 041001(R) (2003)
[3] A. Ekström, *et al.*, PRC **91**,051301 (2015)

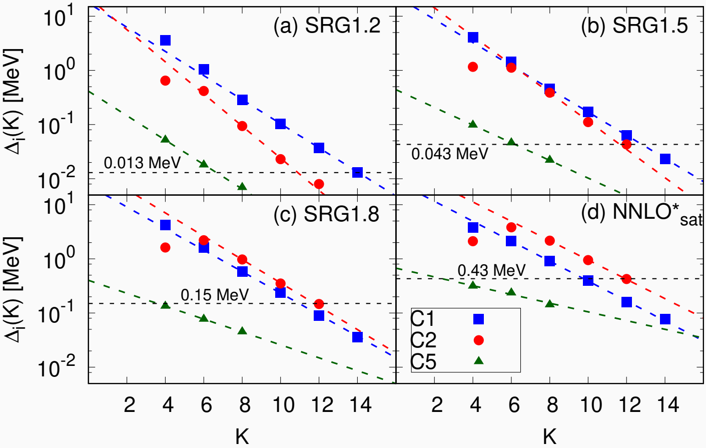
Selection of the states

- Include all the states up to a $K_{max} \Rightarrow$ FAIL!!
- NOT all the states gives the same contribution $\Rightarrow$ division by class [1]
  - Centrifugal barrier $\Rightarrow \ell_1 + \cdots + \ell_5 \leq 4$
  - two-body correlations are more important
- The ^{6}Li ground state is a $J^\pi = 1^+$ state

wave	class	corr.	K_{max}
S	C1	two-body	14
	C3	many-body	10
D	C2	two-body	12
	C4	many-body	10
P	C5	all	8
F-G	C6	all	8

[1] M. Viviani, *et al.*, PRC **71**,024006 (2005)

Convergence of the classes



$$\Delta_i(K) = B_i(K) - B_i(K-2)$$

Extrapolation to $K = \infty$

- Fit the quantity Δ_i with [1]

$$\Delta_i(K) = A_i e^{-b_i K}(1 - e^{-2b_i})$$

- Missing energy for class

$$\Delta_i(\infty) = \sum_{K=\overline{K}}^{\infty} \Delta_i(K) = A_i e^{-b_i \overline{K}}$$

  where $\overline{K}$ maximum K used for the class i

- Extrapolated energy

$$B(\infty) = B_{full} + \sum_i \Delta_i(\infty)$$

  where B_{full} = binding energy with all the states

[1] S.K. Bogner *et al.*, NPA **801**, 21 (2008)

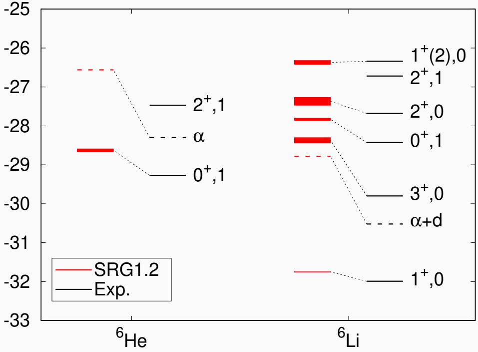
Final extrapolation

	"bare" Coul.		SRG Coul.		
	B_{full}	$B(\infty)$	B_{full}	$B(\infty)$	Ref. [1]
SRG1.2	31.75	31.78(1)	31.78	31.81(1)	31.85(5)
SRG1.5	32.75	32.87(2)	32.79	32.91(2)	33.00(5)
SRG1.8	32.21	32.64(9)	32.25	32.68(9)	32.8(1)
$NNLO^*_{sat}$	29.77	30.71(15)			–

- All the energies are in MeV
- The errors come form the fit
- Results of Ref. [1] extrapolated from $N_{max} = 10$
- Experimental value $B = 31.99$ MeV

[1] E.D Jungerson, P. Navrátil and R.J. Furnstahl, PRC **83**, 034301 (2011)

$A = 6$ spectra with SRG1.2 (preliminary)



- States $(2^+, 0)$ and $(3^+, 0)$ from $K = 8$.
- States $(0^+, 1)$ from $K = 12$.

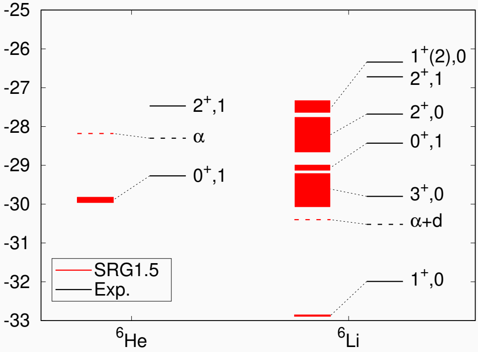
^{6}Li

J^π, T	Exp.	SRG1.2
$1^+, 0$	-31.99	-31.78(1)
$\alpha + d$	-30.52	-28.78
$3^+, 0$	-29.80	-28.37(7)
$0^+, 1$	-28.43	-26.37(5)
$2^+, 0$	-27.86	-27.4(1)
$2^+, 1$	-26.72	–
$1_2^+, 0$	-26.34	-27.83(3)

^{6}He

J^π, T	Exp.	SRG1.2
$0^+, 1$	-29.27	-28.63(4)
α	-28.30	-26.56
$2^+, 1$	-27.47	–

$A = 6$ spectra with SRG1.5 (preliminary)



- States $(2^+, 0)$ and $(3^+, 0)$ from $K = 8$.
- States $(0^+, 1)$ from $K = 12$.

^{6}Li

J^π, T	Exp.	SRG1.5
$1^+, 0$	-31.99	-32.87(2)
$\alpha + d$	-30.52	-30.40
$3^+, 0$	-29.80	-29.6(5)
$0^+, 1$	-28.43	-29.06(8)
$2^+, 0$	-27.86	-28.2(5)
$2^+, 1$	-26.72	–
$1^+_2, 0$	-26.34	-27.5(3)

^{6}He

J^π, T	Exp.	SRG1.5
$0^+, 1$	-29.27	-29.89(7)
α	-28.30	-28.18
$2^+, 1$	-27.47	–

The ^{6}He beta decay

- Pure Gamow-Teller transition ($0^+ \rightarrow 1^+$)

$$M_{fi} = \langle {}^6\text{Li} | J_5^+ | {}^6\text{He} \rangle$$

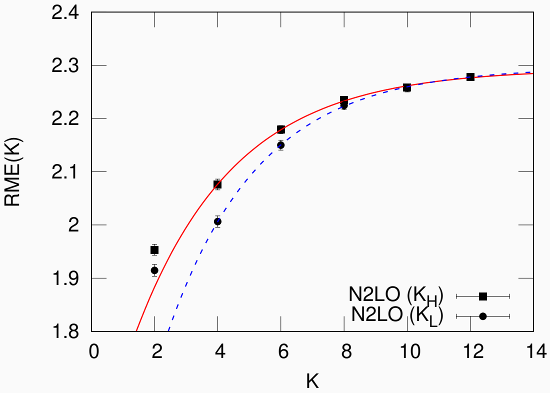
$$J_5^+ = -\frac{g_a}{2} \sum_{i=1,A} \sigma_i \tau_i^+ + J^{RC}(1) + J^{OPE}(2) + J^{CT}(2) + \cdots$$

- First accessible beta decay after ^{3}He β-decay
  $\Rightarrow$ first non trivial test of 2-body currents
- QMC calculations show a different sign for the two-body currents compare to NCSM [1,2]
- We used the N2LO450 EM potential [3] evolved with $\Lambda = 1.2, 1.5, 1.8, 2.0, \infty$

[1] P. Gysbers, *et al.* Nature Phys. **15**, 428 (2019)
[2] G.B. King, *et al.* Phys. Rev. C **102**, 025501 (2020)
[3] D.R. Entem, *et al.* Phys. Rev. C **91**, 024003 (2017)

Convergence of the observable

$$\text{RME} = \frac{\sqrt{2J_f+1}}{g_A}\frac{< J_f M(^6\text{Li})|J_5^+|J_i M(^6\text{He}) >}{< J_i M, 10|J_f M >}$$



Results for the ^{6}He β-decay

	LO(GT)	NLO(RC)	N2LO(OPE)	N2LO(CT)	Tot.
SRG1.2	2.345(2)	−0.019	−0.038(1)	−0.018	2.272(3)
SRG1.5	2.342(3)	−0.021	−0.029(1)	−0.012	2.281(2)
SRG1.8	2.327(3)	−0.022	−0.019(1)	−0.009	2.280(4)
SRG2.0	2.338(3)	−0.022	−0.013(1)	−0.008	2.297(2)
bare	2.321(9)	−0.023	0.002(1)	−0.004	2.303(11)
Exp.					2.1609(40)

- Errors come from the extrapolation
- Results pretty consistent with [1]
- Large dependence on SRG parameter of OPE

[1] P. Gysbers, *et al.* Nature Phys. **15**, 428 (2019)

$\alpha + d$ cluster form factor of ^{6}Li

- ^{6}Li can be considered mainly as an $\alpha + d$ state

$$\Psi_{^6\mathrm{Li}} \simeq \sum_{L=0,2} \left[\left(\Psi_\alpha \otimes \Psi_d \right)_1 Y_L(\hat{r}) \right]_1 \frac{f_L(r)}{r}$$

- The cluster form factor is defined as

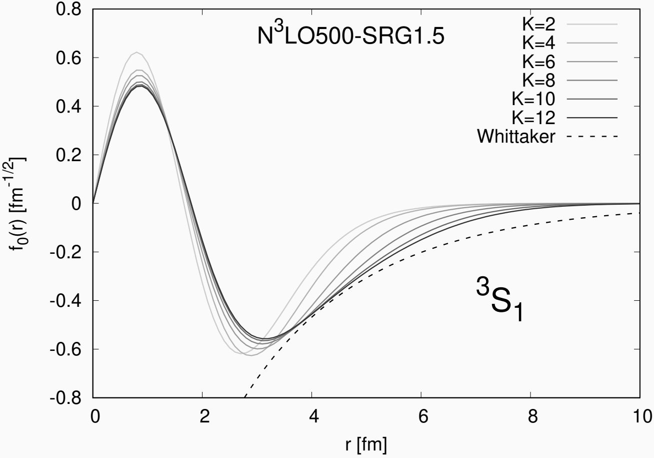
$$\frac{f_L(r)}{r} = \langle \left[\left(\Psi_\alpha \otimes \Psi_d \right)_1 Y_L(\hat{r}) \right]_1 | \Psi_{^6\mathrm{Li}} \rangle$$

- Extract the Asymptotic Normalization Coefficients (ANCs)

$$C_L(r) = \frac{f_L(r)}{W_{-\eta, L+1/2}(2kr)} \xrightarrow{r \to \infty} C_L$$

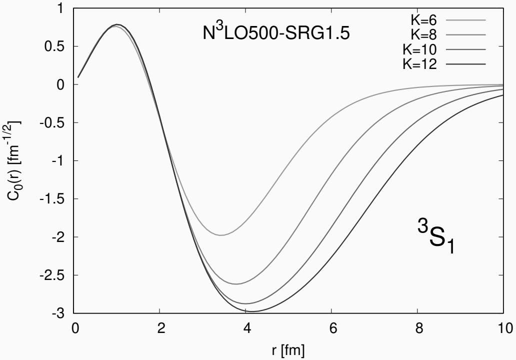
$\alpha + d$ cluster form factor of ^{6}Li

$$\frac{f_L(r)}{r} = \langle \left[(\Psi_\alpha \otimes \Psi_d)_S \, Y_L(\hat{r}) \right]_J | \Psi_{^6\mathrm{Li}} \rangle$$



Asymptotic Normalization Coefficient

$$C_L(r) = \frac{f_L(r)}{W_{-\eta,L+1/2}(2kr)} \xrightarrow{r\to\infty} C_L$$



- B_c and so the Whittaker function depends K
- ANC obtained from the "plateau" around the minimum [1]

$B_c = B_{^6\mathrm{Li}} - B_\alpha - B_d$, $k = \sqrt{2\mu B_c/\hbar^2}$ and $\eta = 2.88\mu/k$

[1] M. Viviani, *et al.*, PRC **71**,024006 (2005)

A more precise approach for the ANC

- From the Schrödinger Equation $\Rightarrow$

$$\langle \Psi^{(L)}_{\alpha+d} | H_6 | \Psi_{^6\mathrm{Li}} \rangle = \langle \Psi^{(L)}_{\alpha+d} | B_{^6\mathrm{Li}} | \Psi_{^6\mathrm{Li}} \rangle$$

- $\Rightarrow$ An equation for the cluster form factor [1-2]

$$\left[-\frac{\hbar^2}{2\mu} \left(\frac{d^2}{dr^2} - \frac{L(L+1)}{r^2} \right) + \frac{2e^2}{r} + B_c \right] f_L(r) + g_L(r) = 0$$

- Source term

$$g_L(r) = \langle \Psi^{(L)}_{\alpha+d} | \left(\sum_{i\in\alpha} \sum_{j\in d} V_{ij} - \frac{2e^2}{r} \right) | \Psi_{^6\mathrm{Li}} \rangle |_{r \text{ fixed}}$$

$\Rightarrow$ Hard to compute in this form!

$\Rightarrow$ Same matrix elements needed for scattering!

- Correct asymptotic behavior

$$g_L(r) \xrightarrow{r\to\infty} 0 \Rightarrow f_L(r) \xrightarrow{r\to\infty} W_{-\eta, L+1/2}(2kr)$$

[1] N.K. Timofeyuk, NPA **632**,19 (1998)

[2] M. Viviani, *et al.*, PRC **71**,024006 (2005)

The "projection" method

- A cluster wave function

$$\Psi^{LSJ}_{\alpha+d} = \left\{ [\Psi_\alpha \otimes \Psi_d]_S \, Y_L(\hat{r}) \right\}_J f(r)$$

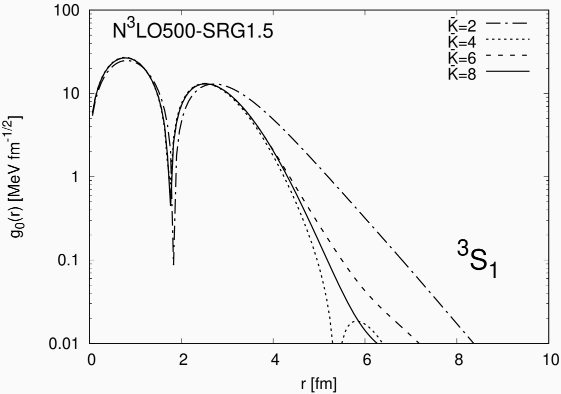
- where $f(r)$ intercluster wave function
- Expansion in term of the HH states $\phi_{[K]}$ =HH+spin+isospin

$$\Psi^{LSJ}_{\alpha+d} = \sum_{\bar{K}=0}^{\bar{K}_{max}} d^{LSJ}_{[\bar{K}]} \phi_{[\bar{K}]} \qquad d^{LSJ}_{[\bar{K}]} = \langle \phi_{[\bar{K}]} | \Psi_{\alpha+d} \rangle$$

- The equality holds only when $\bar{K}_{max} \to \infty$
- Advantages:
  - Easy computation of the matrix elements
  - Control of the convergence $\bar{K}$

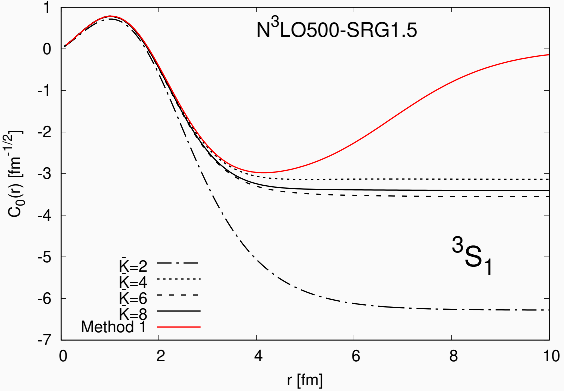
The source term

$$g_L(r) = \langle \Psi^{(L)}_{\alpha+d}(\bar{K})| \left(\sum_{i\in\alpha} \sum_{j\in d} V_{ij} - \frac{2e^2}{r} \right) |\Psi_{^6\mathrm{Li}}\rangle$$



^{6}Li wave function computed with $K = 12$

Overlap vs. Equation



$$C_L(r) = \frac{f_L(r)}{W_{-\eta,L+1/2}(2kr)}$$

- Reproduced exactly the short range part
- Exact asymptotic behavior

^{6}Li wave function computed with $K = 12$

Results for the ANCs

	B_c [MeV]	C_0 [fm$^{-1/2}$]	C_2 [fm$^{-1/2}$]
SRG1.2	3.00(1)	−4.19(12)	0.116(18)
SRG1.5	2.46(2)	−3.44(7)	0.072(15)
SRG1.8	2.02(9)	−3.01(7)	0.047(10)
Exp.	1.4743	−2.91(9)	0.077(18)

- Extrapolated values of the ANC
- Errors from the convergence in K and $\overline{K}$
- Strong dependence on $B_c = E_{^6\text{Li}} - E_{\alpha+d}$
- Same order of magnitude of the experiment!

Conclusions

- HH formalism
  - Extensible up to $A = 8$
  - Inclusion of three-body forces in progress
- Bound states of $A = 6$ nuclei
  - Convergence for SRG potentials (similar to the NCSM)
  - ^{6}He β-decay
- $\alpha + d$ clusterization
  - Calculation of the Asymptotic Normalization Coefficients
  - Test of “projection” method to $A = 6 \Rightarrow$ Scattering

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M. Viviani (INFN Pisa, Italy)

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U.S. DEPARTMENT OF ENERGY

JSA

Computational resources:

NERSC National Energy Research Scientific Computing Center

CINECA

Sparse

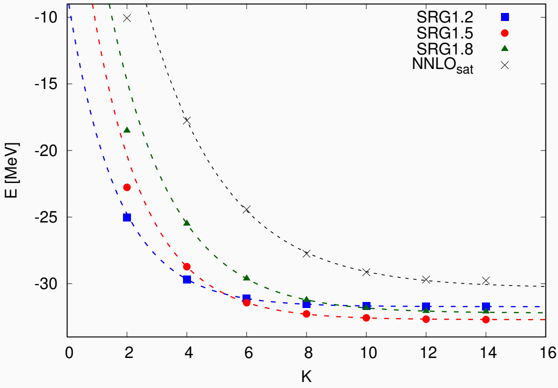
Validation

- $J^{\pi} = 1^{+}$ only $LST = 010$

K	This work	Ref. [1]
2	-61.142	-61.142
4	-62.015	-62.015
6	-63.377	-63.377
8	-64.437	-64.437
10	-65.354	-65.354
12	-65.884	-65.886

[1] M. Gattobigio *et al.*, PRC **83**, 024001 (2011)

Convergence (WRONG)



- Exponential behavior [1]

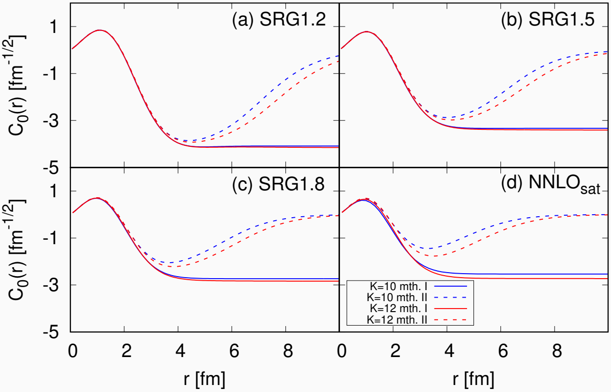
$$E(K) = E(\infty) + Ae^{-bK}$$

[1] S.K. Bogner *et al.*, NPA **801**, 21 (2008)

Extrapolation

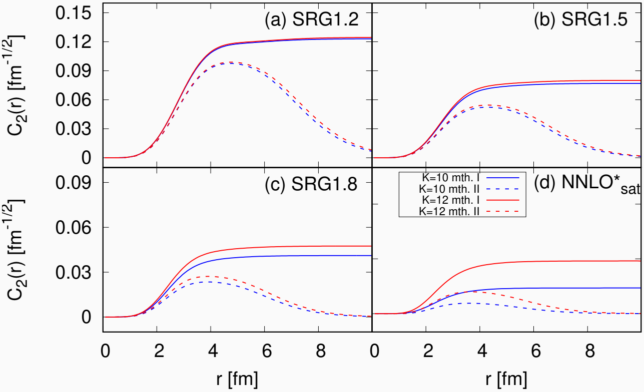
		SRG1.2			SRG1.5		
i	K_{iM}	$\Delta_i(K_{iM})$	b_i	$(\Delta B)_i$	$\Delta_i(K_{iM})$	b_i	$(\Delta B)_i$
1	14	0.013	0.51	0.007(0)	0.023	0.49	0.014(0)
2	12	0.008	0.68	0.003(1)	0.042	0.58	0.019(0)
3	10	0.015	0.37	0.014(7)	0.022	0.32	0.024(12)
4	10	0.008	0.60	0.004(2)	0.022	0.49	0.013(6)
5	8	0.007	0.52	0.004(0)	0.023	0.37	0.021(0)
6	8	0.004	0.44	0.003(1)	0.018	0.26	0.026(13)
$(\Delta B)_T$				0.034(7)			0.117(19)
		SRG1.8			$NNLO_{sat}$		
i	K_{iM}	$\Delta_i(K_{iM})$	b_i	$(\Delta B)_i$	$\Delta_i(K_{iM})$	b_i	$(\Delta B)_i$
1	14	0.035	0.46	0.023(0)	0.074	0.43	0.05(0)
2	12	0.144	0.50	0.084(11)	0.411	0.42	0.32(1)
3	10	0.024	0.30	0.029(15)	0.031	0.17	0.07(4)
4	10	0.045	0.38	0.039(20)	0.093	0.25	0.14(7)
5	8	0.049	0.26	0.070(1)	0.153	0.18	0.35(14)
6	8	0.048	0.11	0.19(9)	0.112	–	–
$(\Delta B)_T$				0.43(9)			0.93(20)

Function $C_L(r)$ (S wave)



- $\bar{K} = 8$ in the “projection” of the cluster function
- Better convergence of ^{6}Li wf $\Rightarrow$ better agreement with Method I and II

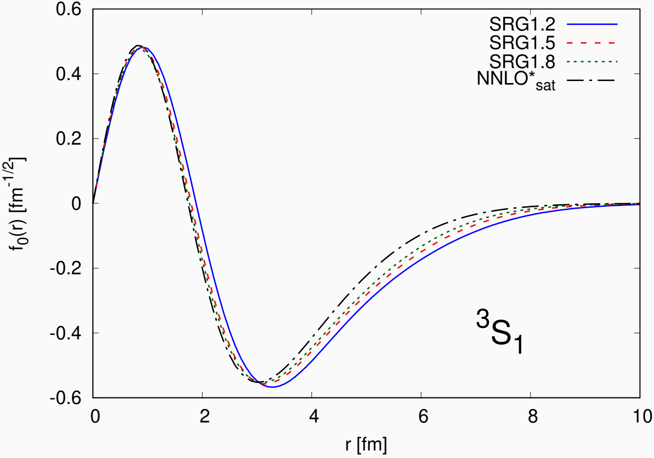
Function $C_L(r)$ (D wave)



- $\bar{K} = 8$ in the “projection” of the cluster function
- Better convergence of ^{6}Li wf $\Rightarrow$ better agreement with Method I and II

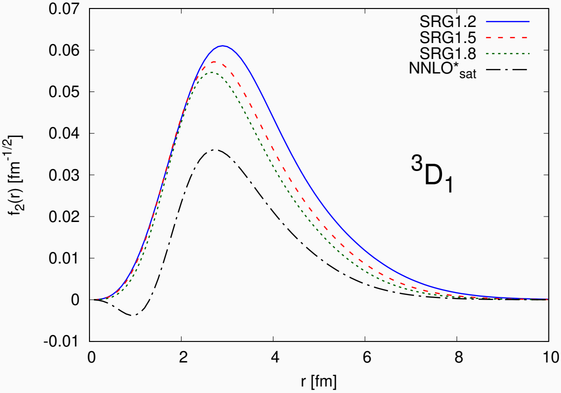
$\alpha + d$ cluster form factor of ^{6}Li

$$\frac{f_L(r)}{r} = \langle \left[(\Psi_\alpha \otimes \Psi_d)_S \, Y_L(\hat{r}) \right]_J | \Psi_{^6\mathrm{Li}} \rangle$$



Cluster form factor $\alpha + d$

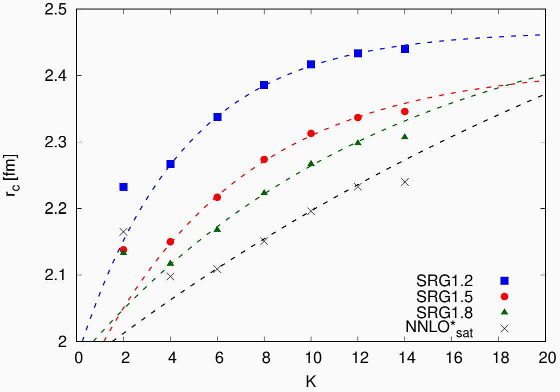
$$\frac{f_L(r)}{r} = \langle \left[(\Psi_\alpha \otimes \Psi_d)_S \, Y_L(\hat{r}) \right]_J | \Psi_{^6\mathrm{Li}} \rangle$$



- For the NNLO^*_{sat} a node appears ⇒ strength of the tensor forces[1]

[1] V.I. Kukulin, *et al.* NPA **586**,151 (1995)

Charge radius



	$r_c(\infty)$ [fm]
SRG1.2	2.47(1)
SRG1.5	2.42(2)
SRG1.8	2.52(10)
NNLO$^*_{sat}$	2.8(7)
Ref. [1]	2.40(6)
Exp.	2.540(28)

- Extrapolation $r_c(K) = r_c(\infty) + Ae^{-bK}$

[1] CDB2k-SRG1.5 C. Forssén, E. Caurier, P. Navrátil, PRC **71**, 021303 (2009)

Magnetic dipole moment

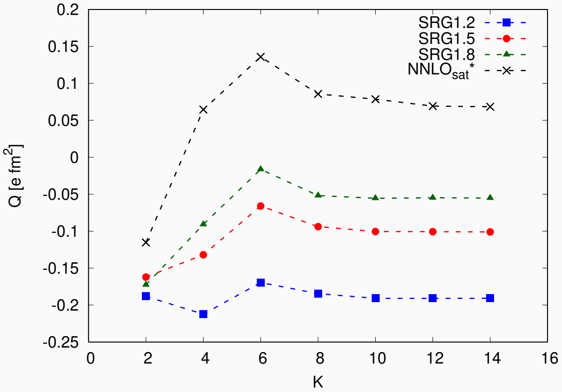
$$^6\text{Li} \simeq \alpha + d \Rightarrow \mu_z(^6\text{Li}) \simeq \mu_z(d)$$

Experiment tells us $\mu_z(^6\text{Li}) < \mu_z(d)$

	$\mu_z(d)$	$\mu_z(^6\text{Li})$
SRG1.2	0.872	0.865(1)
SRG1.5	0.868	0.858(2)
SRG1.8	0.865	0.852(2)
NNLO^*_{sat}	0.860	0.845(5)
Exp.	0.857	0.822

- Negative contribution only from the $L = 2$ $S = 1$ component $\Rightarrow$ NOT SUFFICIENT
- We need two body currents contribution!! [1]

[1] R. Schiavilla, *et al.*, PRC **99**, 034005 (2019)



	Q [e fm2]
SRG1.2	$-0.191(7)$
SRG1.5	$-0.101(7)$
SRG1.8	$-0.055(3)$
NNLO$^*_{sat}$	$+0.068(18)$
Ref. [1]	$-0.066(40)$
Exp.	$-0.0806(8)$

- Large cancellations between different K

[1] CDB2k-SRG1.5 C. Forssén, E. Caurier, P. Navrátil, PRC **71**, 021303 (2009)

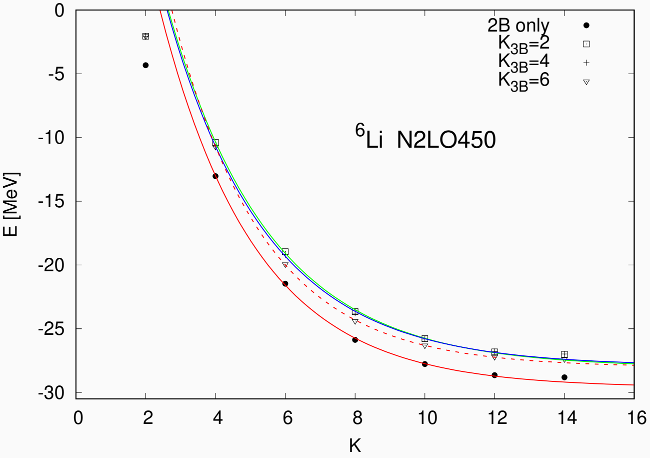
Electric quadrupole moment

Matrix elements between different waves

	$S-D$	$D-D$	$P-P$	$P-D$	remaining
SRG1.2	-0.187	-0.023	0.009	0.009	<0.001
SRG1.5	-0.102	-0.023	0.014	0.010	<0.001
SRG1.8	-0.058	-0.024	0.016	0.010	0.001
NNLO$^{*}_{sat}$	0.049	-0.018	0.023	0.011	0.003

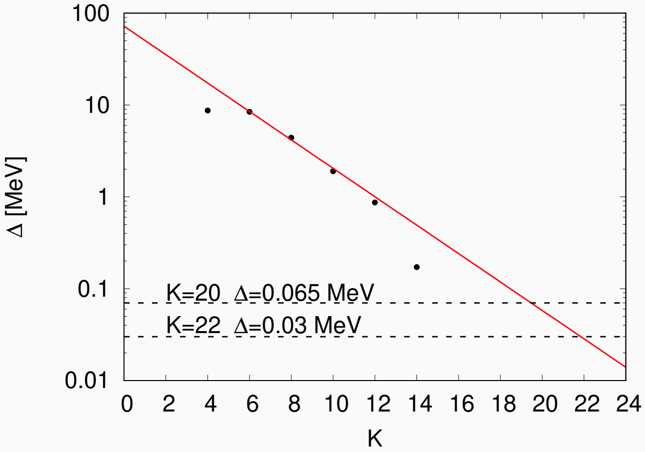
- Direct connection with the strength of the tensor term in the potential
- Two-body currents contribution could be necessary!!

Towards “bare” chiral potential



D.R. Entem *et al.*, PRC **91**, 014002 (2015)

Towards “bare” chiral potential



D.R. Entem *et al.*, PRC **91**, 014002 (2015)

How to reach $K = 20$?

- Increase the basis size up to $K = 20$
  From $\sim 30k$ h to $\sim 500 - 1000k$ h to compute D coefficients
- Better selection of the classes
  Only states with $n_1, n_2, n_3, n_4, n_5 = 0, 0, 0, 0, n$
- OpenMP $\Rightarrow$ OpenMPI
- Use of accelerators (OpenACC, CUDA,...)