

Group Meeting 11.28

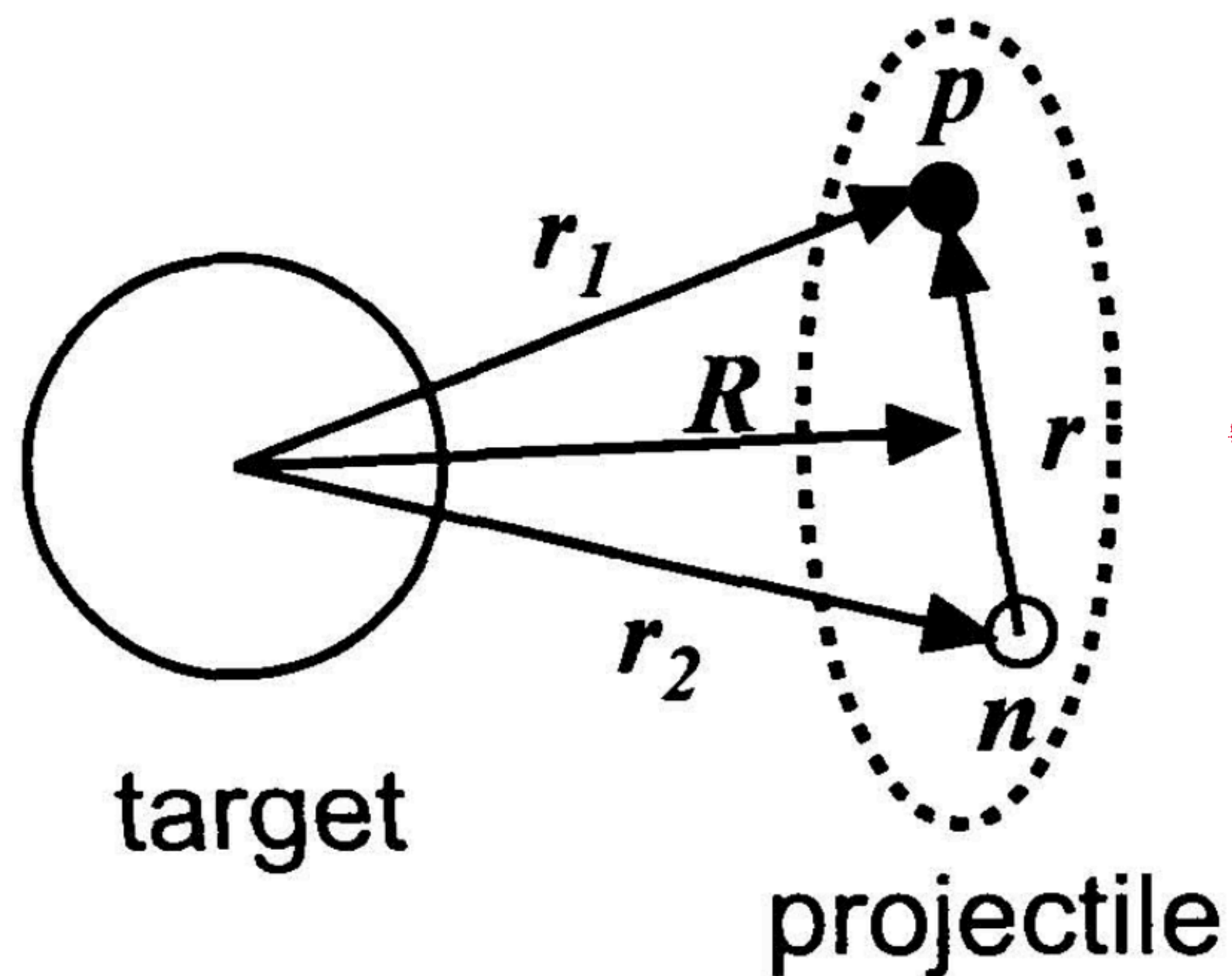
**COMPLEX-SCALED CDCC METHOD FOR NUCLEAR BREAKUP  
REACTIONS**

**Hao Liu  
Zetian Ma**

# CSCDCC

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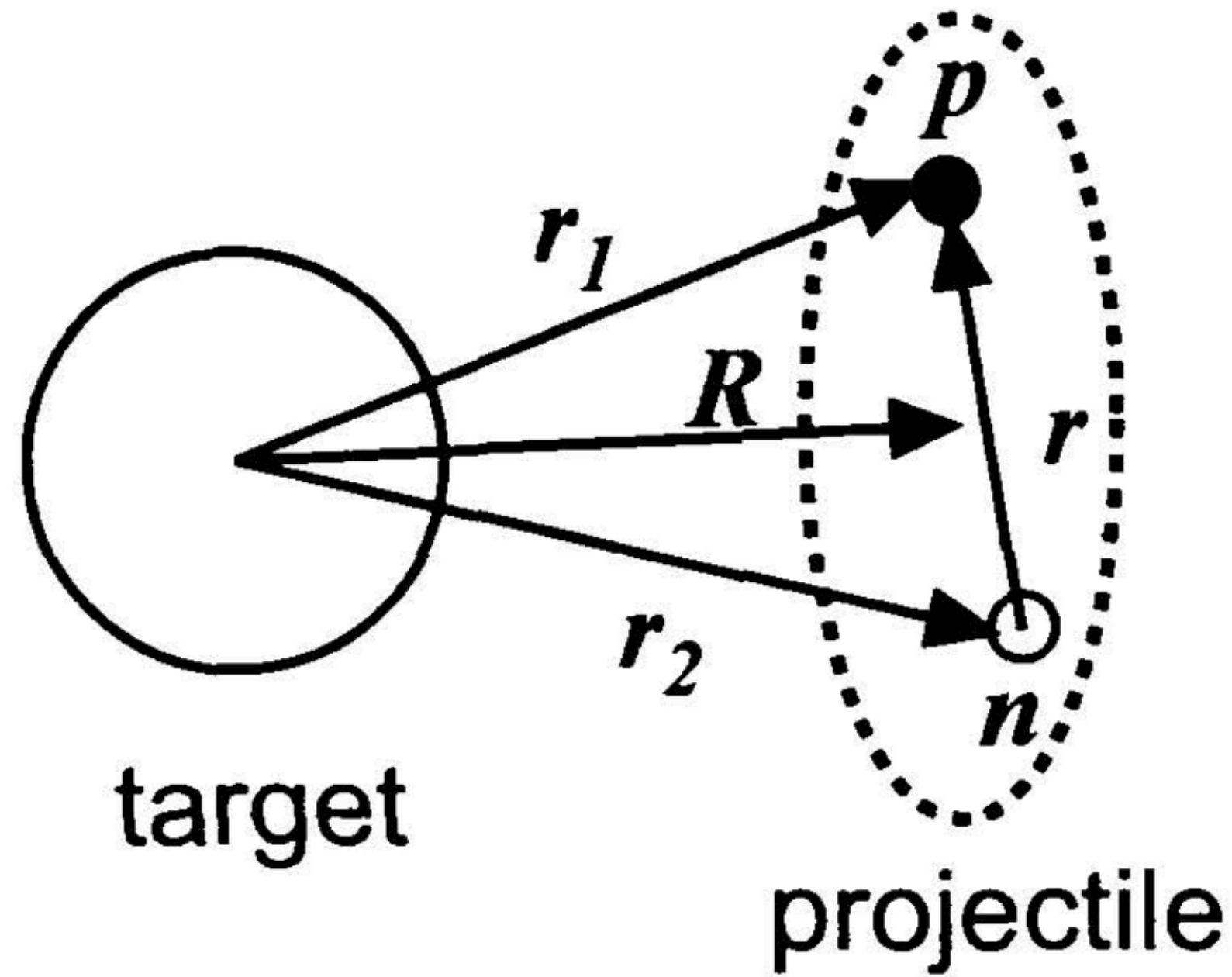
In this article, they only apply the Complex Scaling (CS) method in the projectile.



$$\left\{ \begin{array}{l} h = T_{\mathbf{r}} + V(\mathbf{r}) \\ h\varphi_{\alpha} = \varepsilon_{\alpha}\varphi_{\alpha} \\ h^{\theta} = U(\theta)hU^{-1}(\theta) \\ h^{\theta}\varphi_{\alpha}^{\theta} = \varepsilon_{\alpha}^{\theta}\varphi_{\alpha}^{\theta} \end{array} \right.$$

# CSCDCC

The rotated operator  $U(\theta)$  only affects on  $\mathbf{r}$ .



$$\sum_{\alpha,\beta} \left\langle \left[ \varphi_{\alpha}(\mathbf{r}) \otimes y_{L_{\alpha}}(\hat{\mathbf{R}}) \right]_{JM} \left| \left[ \left( T - (E - \varepsilon_{\alpha}) + V \right) \right] \right. \right.$$

$$\times \left. \left| \left[ \varphi_{\beta}(\mathbf{r}) \otimes y_{L_{\beta}}(\hat{\mathbf{R}}) \right]_{JM} \right\rangle \left\langle \left[ \varphi_{\beta}(\mathbf{r}) \otimes y_{L_{\beta}}(\hat{\mathbf{R}}) \right]_{JM} \right| \Psi \right\rangle = 0$$

$$\rightarrow \sum_{\alpha,\beta} \left\langle \left[ \varphi_{\alpha}(\mathbf{r}) \otimes y_{L_{\alpha}}(\hat{\mathbf{R}}) \right]_{JM} \left| U^{-1}(\theta) U(\theta) \left[ \left( T - (E - \varepsilon_{\alpha}) + V \right) \right] \right. \right.$$

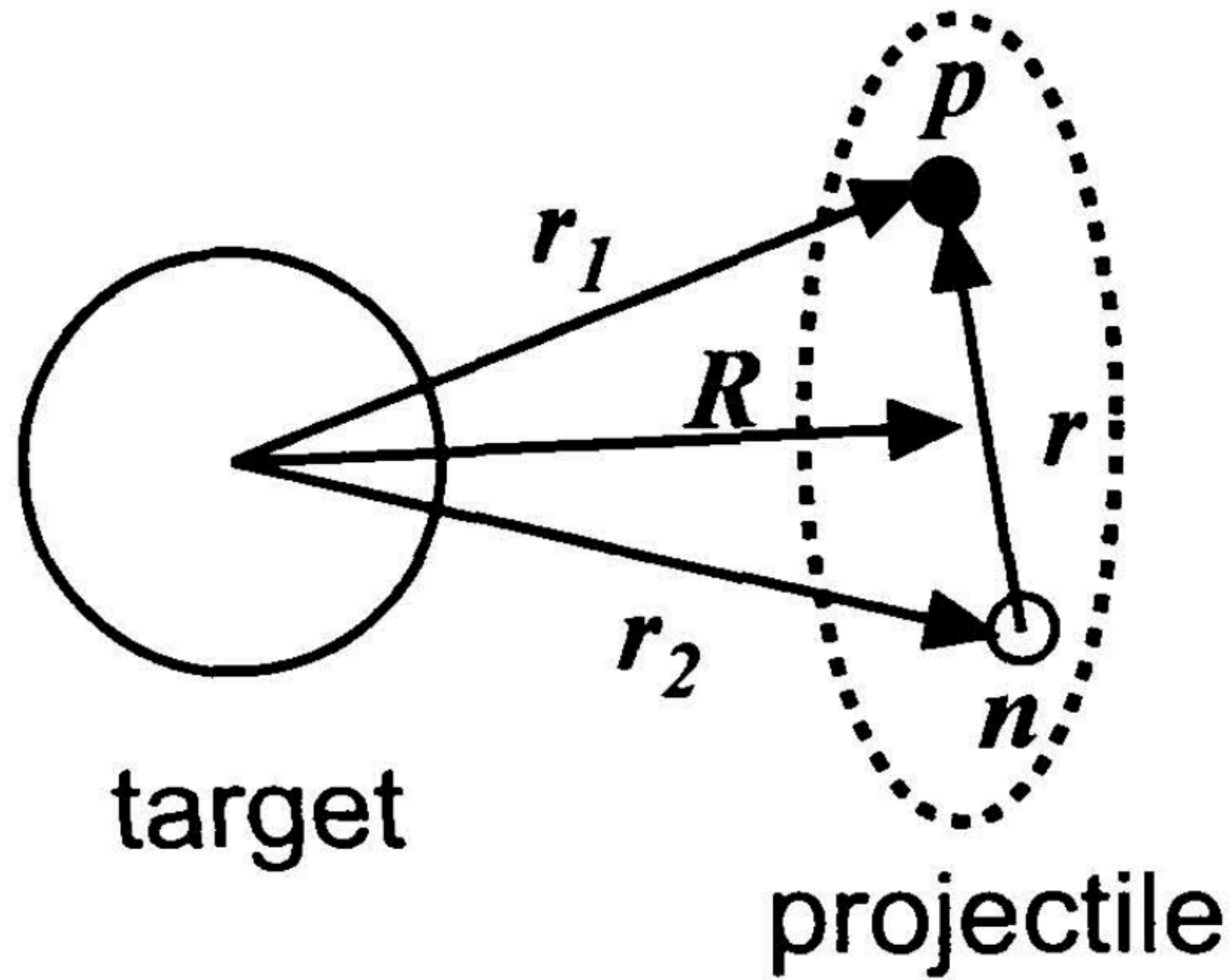
$$\times \left. U^{-1}(\theta) \left| \left[ \varphi_{\beta}(\mathbf{r}) \otimes y_{L_{\beta}}(\hat{\mathbf{R}}) \right]_{JM} \right\rangle \left\langle \left[ \varphi_{\beta}(\mathbf{r}) \otimes y_{L_{\beta}}(\hat{\mathbf{R}}) \right]_{JM} \right| \Psi \right\rangle = 0$$

$$\sum_{\alpha,\beta} \left\langle \left[ \varphi_{\alpha}^{\theta}(\mathbf{r}) \otimes y_{L_{\alpha}}(\hat{\mathbf{R}}) \right]_{JM} \left| U(\theta) \left[ \left( T - (E - \varepsilon_{\alpha}) + V \right) \right] \right. \right.$$

$$\times \left. U^{-1}(\theta) \left| \left[ \varphi_{\beta}^{\theta}(\mathbf{r}) \otimes y_{L_{\beta}}(\hat{\mathbf{R}}) \right]_{JM} \right\rangle \left\langle \left[ \varphi_{\beta}(\mathbf{r}) \otimes y_{L_{\beta}}(\hat{\mathbf{R}}) \right]_{JM} \right| \Psi \right\rangle = 0$$

# CSCDCC

The rotated operator  $U(\theta)$  only affects on  $\mathbf{r}$ .



$$\sum_{\alpha, \beta} \left\langle \left[ \varphi_{\alpha}^{\theta}(\mathbf{r}) \otimes y_{L_{\alpha}}(\hat{\mathbf{R}}) \right]_{JM} \left| U(\theta) \left[ \left( T - (E - h_{\alpha}) + V \right) \right] \right. \right. \\ \left. \left. \times U^{-1}(\theta) \left[ \varphi_{\beta}^{\theta}(\mathbf{r}) \otimes y_{L_{\beta}}(\hat{\mathbf{R}}) \right]_{JM} \right\rangle \left\langle \left[ \varphi_{\beta}(\mathbf{r}) \otimes y_{L_{\beta}}(\hat{\mathbf{R}}) \right]_{JM} \left| \Psi \right\rangle = 0 \right.$$



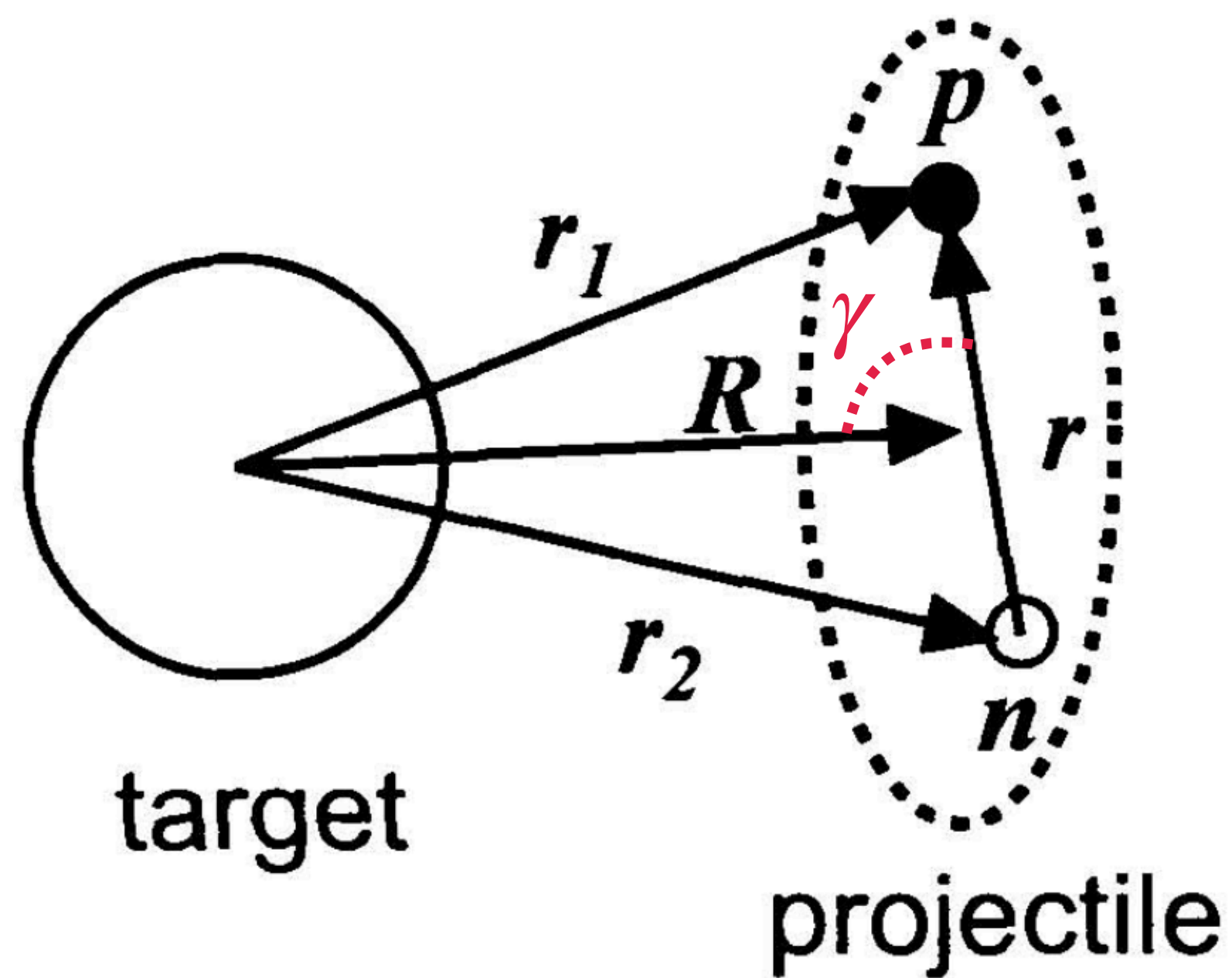
$$\left[ \left( T - (E - \varepsilon_{\alpha}^{\theta}) \right) \right] \chi_{\alpha}^{(J)\theta}(R) = - \sum_{\beta \neq \alpha} F_{\alpha, \beta}^{(J)\theta}(R) \chi_{\beta}^{(J)\theta}(R).$$

$$F_{\alpha, \beta}^{(J)\theta}(R) = \left\langle \left[ \varphi_{\alpha}^{\theta}(\mathbf{r}) \otimes y_{L_{\alpha}}(\hat{\mathbf{R}}) \right]_{JM} \left| U(\theta) \left[ V_1(r_1) + V_2(r_2) \right] \right. \right. \\ \left. \left. \times U^{-1}(\theta) \left[ \varphi_{\beta}^{\theta}(\mathbf{r}) \otimes y_{L_{\beta}}(\hat{\mathbf{R}}) \right]_{JM} \right\rangle.$$

# CSCDCC

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In the coupling potential, there may be some problems when  $r$  becomes complex.



$$F_{\alpha,\beta}^{(J)\theta}(R) = \left\langle \left[ \varphi_{\alpha}^{\theta}(\mathbf{r}) \otimes y_{L_{\alpha}}(\hat{\mathbf{R}}) \right]_{JM} \left| U(\theta) \left[ V_1(r_1) + V_2(r_2) \right] \right. \right. \\ \left. \left. \times U^{-1}(\theta) \left[ \varphi_{\beta}^{\theta}(\mathbf{r}) \otimes y_{L_{\beta}}(\hat{\mathbf{R}}) \right]_{JM} \right\rangle.$$

In the integral, the angle  $\gamma$  couldn't be gotten from the law of cosine. They may use the multi-expansion here.

# CSCDCC

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With the boundary condition, the relative wave function is written by,

$$\chi_{\alpha}^{(J)\theta}(R) \rightarrow \left[ \delta_{\alpha 0} u_{L_0}^{(-)}(K_0 R) - \sqrt{\frac{v_0}{v_{\alpha}^{\theta}}} S_{\alpha 0}^{(J)\theta} u_{L_{\alpha}}^{(+)}(K_{\alpha}^{\theta} R) \right]$$

Project onto the real energy  $\varepsilon$

$$\left| S^{(J)}(\varepsilon) \right|^2 = \frac{1}{\pi} \mathcal{I}m \left[ \sum_{\alpha} \frac{\tilde{S}_{\alpha 0}^{(J)\theta*} S_{\alpha 0}^{(J)\theta}}{\varepsilon_{\alpha}^{\theta} - \varepsilon} \right]$$

# CSCDCC

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The result is very close to the PS-CDCC.

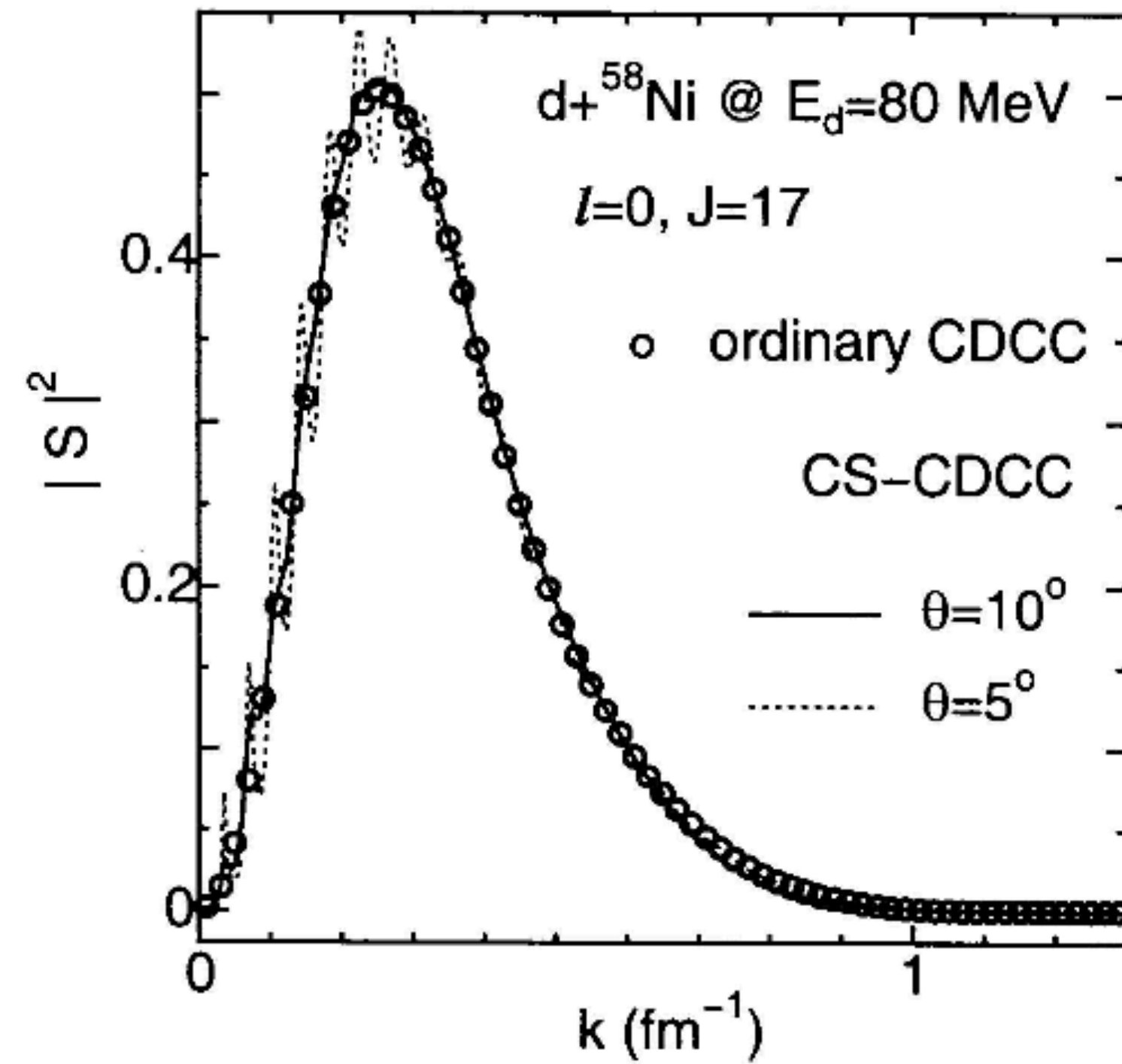


Fig. 2. Squared modulus of S matrix element as a function of  $k$ .

# Method

In another paper, they introduced the CS to solve the coupled channel problem.

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## Three-body model analysis of $\alpha + d$ elastic scattering and the ${}^2\text{H}(\alpha, \gamma){}^6\text{Li}$ reaction in complex-scaled solutions of the Lippmann-Schwinger equation

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We investigate the  $\alpha + d$  elastic scattering and the radiative capture reaction of  ${}^2\text{H}(\alpha, \gamma){}^6\text{Li}$  based on the  $\alpha + p + n$  three-body model. The  $\alpha + d$  scattering states are described by using the complex-scaled solutions of the Lippmann-Schwinger equation. We calculate the elastic phase shifts for the  $\alpha + d$  scattering and the radiative capture cross section of  ${}^6\text{Li}$ . We evaluate the contributions of the  $\alpha + p + n$  structures in those observables. It is found that in the  $\alpha + d$  scattering process, the deuteron breakup and the rearrangement to the  ${}^5\text{He} + p$  and  ${}^5\text{Li} + n$  channels play prominent roles in reproducing the observed phase shifts and radiative capture cross section.

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## Continuum Level Density of a Coupled-Channel System in the Complex Scaling Method

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We study the continuum level density (CLD) in the formalism of the complex scaling method (CSM) for coupled-channel systems. We apply the formalism to the  ${}^4\text{He} = [{}^3\text{H} + p] + [{}^3\text{He} + n]$  coupled-channel cluster model where there are resonances at low energy. Numerical calculations of the CLD in the CSM with a finite number of  $L^2$  basis functions are consistent with the exact result calculated from the  $S$ -matrix by solving coupled-channel equations. We also study channel densities. In this framework, the extended completeness relation (ECR) plays an important role.



# Method

They treated different channels like this.

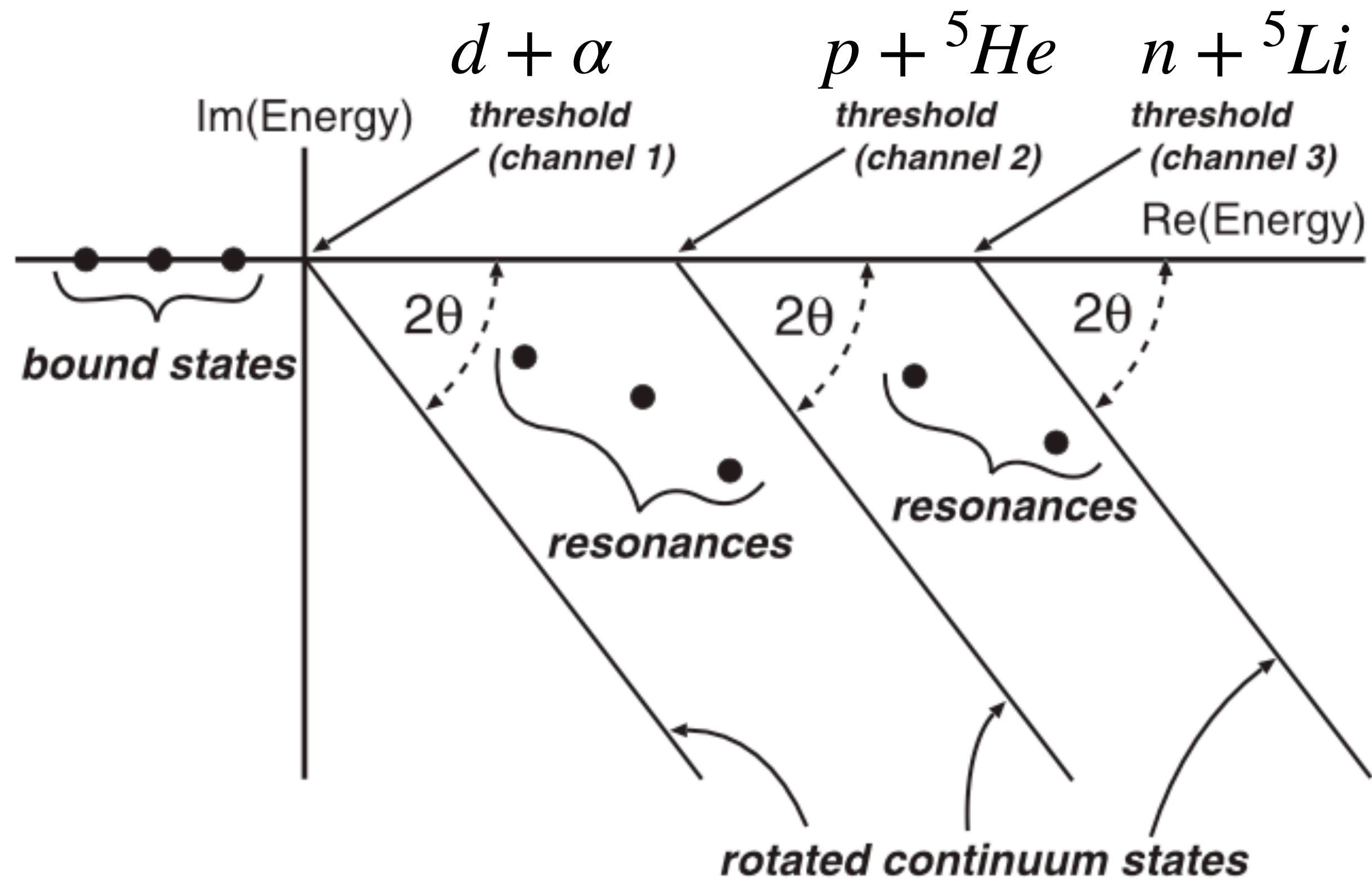


Fig. 1. Schematic energy eigenvalue distribution of a complex scaled Hamiltonian.