Resonant states of deformed nuclei in the complex scaling method

Jizheng,Bo

November 20, 2023

Index

Introduction

why complex scaling method(CSM)

- the resonances of deformed nuclei have attracted additional attention.
- the interplay between the deformation and the low-lying resonance is interesting for the open shell nucleus close to the dripline.

experimental background(PRL 103, 262501 (2009))

- single-neutron removal from the very neutron-rich nucleus $^{31}Ne(Neon)$ on Pb(Plumbum) and C target (at Riken)

- the neutron halo of ^{31}Ne to be calculated

Formalism

purpose: to extend CSM to describe the resonances of deformed nuclei

CSM

- start with s.p Hamiltonian

$$H = T + V \tag{1}$$

- V: the axially symmetric quadruple-deformed potential

$$\begin{aligned} V_{cent}(r) &= V_0 f(r) \\ V_{cou}(r) &= -\beta_2 V_0 k(r) Y_{20}(\theta, \varphi) \\ V_{so}(r) &= -\frac{1}{2} v V_0 g(r)(\vec{s} \cdot \vec{l}) \end{aligned} \tag{2}$$

with $k(r) = r\frac{df(r)}{dr}, g(r) = \frac{\Lambda^2}{r}\frac{df(r)}{dr}$

Λ: the reduced Compton wavelength of nucleon $\frac{\hbar}{M_r c}$

WS factor: $f(r) = \frac{1}{1 + exp(\frac{r-R}{a})}$

Formalism

purpose: to extend CSM to describe the resonances of deformed nuclei

CSM

- a relative coordinate r in Hamiltonian & wf ψ is complex scaled

$$U(\theta) : r \rightarrow re^{i\theta} \tag{3}$$

- the transformed Hamiltonian and wf

$$H_\theta = U(\theta)HU(\theta)^{-1} \tag{4}$$

$$\psi_\theta = U(\theta)\psi \tag{5}$$

- the corresponding Schrodinger equation

$$H_\theta\psi_\theta = E_\theta\psi_\theta \tag{6}$$

Formalism

purpose: to extend CSM to describe the resonances of deformed nuclei

solve the complex scaled equation

- the axially symmetrically deformed system: the projection of total J along the symmetry axis $\Omega \rightarrow$ good quantum number
- basis expansion

$$\psi_\theta = \sum_i c_i(\theta)\phi_i \tag{7}$$

with

$$\phi_i = R_{nl}(r)Y_{lm_l}(\theta,\phi)\chi_{m_s}(\sigma_z) \tag{8}$$

- index $i = \{n, l, m_l\}$, where m_l satisfies $\Omega = m_l + m_s$

Formalism

purpose: to extend CSM to describe the resonances of deformed nuclei

basis & matrix elements

- spherical harmonic oscillator basis

$$R_{nl}(r) = \frac{1}{b^{3/2}}\sqrt{\frac{2(n-1)!}{\Gamma(n+l+1/2)}}x^l L_{n-1}^{l+1/2}(x^2)e^{-x^2/2} \quad (9)$$

$$n = 1, 2, 3\ldots$$

- the matrix diagonalizaion problem

$$\sum_i [T_{i',i} + V_{i',i}]c_i = E_\theta c_{i'} \quad (10)$$

Formalism

purpose: to extend CSM to describe the resonances of deformed nuclei

matrix elements

- specifically

$$T_{i',i} = e^{-i2\theta} \int \phi_{i'} \left[-\frac{\hbar^2}{2M} \left(\frac{d^2}{dr^2} + \frac{2}{r} \frac{d}{dr} \right) + \frac{\vec{l}^2}{2Mr^2} \right] \phi_i d\vec{r} \quad (11)$$

$$V_{i',i} = \int \phi_{i'} V(\vec{r} e^{i\theta}) \phi_i d\vec{r} \quad (12)$$

Formalism

purpose: to extend CSM to describe the resonances of deformed nuclei

concrete matrix elements

- T matrix elements

$$
\begin{aligned}
T_{i,i'} =& e^{-i2\theta}\frac{\hbar^2}{2Mb_0^2}[\sqrt{n(n+l+1/2)}\delta_{n',n+1} \\
&+ (2n+l-1/2)\delta_{n',n} \\
&+ \sqrt{(n-1)(n+1-1/2)}\delta_{n',n+1}]\delta_{l'l}\delta_{m_l'm_l}\delta_{m_s'm_s}
\end{aligned} \tag{13}
$$

- V matrix elements (3 components)

Formalism

purpose: to extend CSM to describe the resonances of deformed nuclei

concrete matrix elements (V terms)

- component 1

$$\begin{aligned} V_{i',i}^{cent} &= \langle \phi_{i'} | V_{cent}(re^{i\theta}) | \phi_i \rangle \\ &= V_0 \langle n'l' | f(re^{i\theta}) | nl \rangle \delta_{l'l} \delta_{m_l' m_l} \delta_{m_s' m_s} \end{aligned} \tag{14}$$

- component 2

$$\begin{aligned} V_{i',i}^{cou} &= \langle \phi_{i'} | V_{cou}(\vec{r}e^{i\theta}) | \phi_i \rangle \\ &= -\beta_2 V_0 \langle n'l' | k(re^{i\theta}) | nl \rangle \langle l' m_{l'} | Y_{20}(\theta, \phi) | l m_l \rangle \delta_{m_s' m_s} \end{aligned} \tag{15}$$

- component 3

$$\begin{aligned} V_{i',i}^{so} &= \langle \phi_{i'} | V_{so} | \phi_i \rangle \\ &= -\frac{1}{2} v V_0 \langle n'l' | g(re^{i\theta}) | nl \rangle \langle l' m_l' m_s' | (\vec{s} \cdot \vec{l}) | l m_l m_s \rangle \end{aligned} \tag{16}$$

- To obtain the complex energy eigenvalue $E - i\Gamma/2$ by diagonalizing the matrix H_θ

Calculations: ^{31}Ne

complex energy eigenvalues

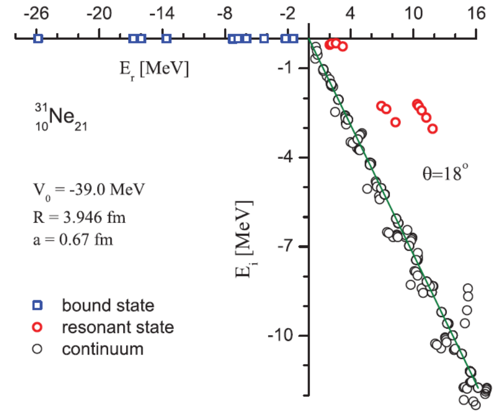

Figure 1: The complex energy eigenvalues with the complex scaling factor $\theta = 18^\circ$, quadruple deformation $\beta_2 = 0.1$, and 60 HO shells

Calculations: ^{31}Ne

distribution of complex energies with θ evolution

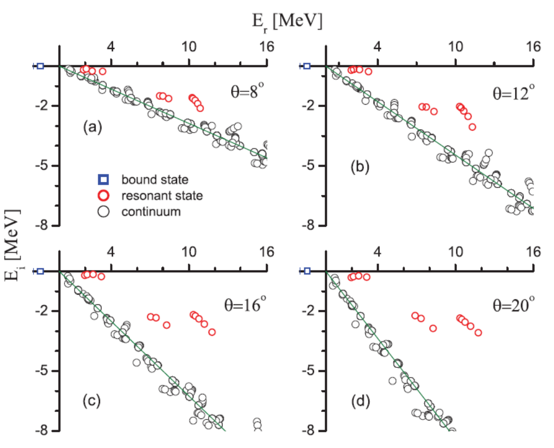

Figure 2: The resonant and continuous spectra varying with the complex rotation angle. (the same parameter as that in fig.1)

Calculations: ^{31}Ne

Deformation

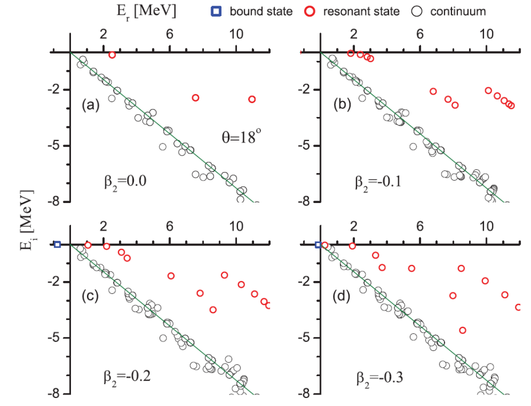

Figure 3: The movement of the resonant states in the position with deformation in the complex energy plane

Calculations: ^{31}Ne

- When the spherical symmetry of the system is broken ($\beta_2 \neq 0$), the 3 resonances are broken into 12 resonances.

Calculations: ^{31}Ne

θ trajectories

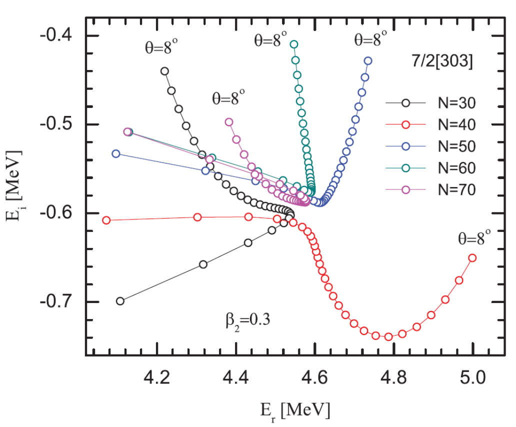

Figure 4: The θ trajectories corresponding to the several different numbers of oscillator shells of the basis for the resonant state 7/2[303] (quantum numbers $\Omega[Nn_z\Lambda]$)

Calculations: ^{31}Ne

- In every θ trajectory, there exists a point corresponding to the minimal rate of change of the resonance parameters with respect to θ.
- i.e. the optimal value of the resonance parameters

$$\frac{dE_\theta}{d\theta} \simeq 0 \tag{17}$$

Summary

Summary

- The CSM is extended to the resonances of deformed nuclei.
- The resonance and bound states remain almost unchanged with the varation of θ.
- The 3 degenerate resonances are separated into 12 resonances with the development of deformation.

Thank You!