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Rapid convergence of the Weinberg expansion of the deuteron stripping amplitude

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Transfer amplitude for (d, p)

T matrix can be expressed as $T_{dp} = \langle \chi_p^{(-)} \psi_n | V_{np} | \Psi^{(+)} \rangle$

χ_p is the scattering wave function of the outgoing proton

ψ_n is the overlap function which describes the relative motion of the neutron

Different expansion of the three body wave function lead to different models:

DWBA method: $\Psi^{\mathrm{DWBA}} = \phi_d \chi_o$

CDCC method: $\Psi^{\mathrm{CDCC}} = \phi_d \chi_o + \sum_i \phi_i^{\mathrm{bin}} \chi_i^{\mathrm{bin}}$

Weinberg state: $\Psi^{\mathrm{WSE}} = \sum_i \phi_i^{\mathrm{W}} \chi_i^{\mathrm{W}}$

Weinberg states expansion

Wave function has the expansion: $\Psi^{\mathrm{WSE}} = \sum_i \phi_i^{\mathrm{W}} \chi_i^{\mathrm{W}}$

Weinberg states are defined as the solution of the eigenvalue problem:

$$\left[-\epsilon_d - T_r - \alpha_i V_{np}\right] \phi_i(\mathbf{r}) = 0, \quad i = 1,2,\ldots$$

They satisfy a different orthogonal relation

$$\langle \phi_i^W | V_{\mathrm{np}} | \phi_j^W \rangle = -\delta_{ij}$$

The first Weinberg state is defined to be proportional to the ground state wave function:

Choose the first eigenvalue to be one: $\alpha_1 = 1$

Why Weinberg states?

T matrix $$T_{dp} = \langle \chi_p^{(-)} \psi_n | V_{np} | \Psi^{(+)} \rangle$$

Instead of the full three-body wave function, what we really need is the projected three body wave function $V_{np} | \Psi^{(+)} \rangle$

V_{np} is a short range interaction, only the inner part of the wave function is required.

$$\left[-\epsilon_d - T_r - \alpha_i V_{np} \right] \phi_i(\mathbf{r}) = 0, \quad i = 1,2,\ldots$$

Square integrable, discrete basis only for the short range.

Coupled equation for Weinberg states

The Schrödinger euation of the 3B wave function:

$$\left[E_d + i\epsilon - H_{np} - T_R - U_n\left(\boldsymbol{r}_n\right) - U_p\left(\boldsymbol{r}_p\right)\right]\Psi^{(+)}(\boldsymbol{r},\boldsymbol{R}) \quad = i\epsilon\phi_d(\boldsymbol{r})e^{i\boldsymbol{K}_d\cdot\boldsymbol{R}}$$

Carry out the projection, and we obtain the coupled equation:

$$\left[E_d + i\epsilon - T_R\right]\left|\chi_i^{(+)}\right\rangle = i\epsilon\delta_{i1}N_d\,|\,\mathbf{K}\rangle + \sum_{j=1}^{N} U_{ij}(\mathbf{R})\left|\chi_j^{(+)}\right\rangle$$

The coupling potential is much more complicated, because the Weinberg basis is not the eigenfunction of the n-p hamiltonian

$$U_{ij}(\mathbf{R}) = V_{ij}(\mathbf{R}) + C_{ij}$$

Coupling constant

$$V_{ij}(\mathbf{R}) = -\left\langle \phi_i \left| V_{np}U(\mathbf{r},\mathbf{R}) \right| \phi_j \right\rangle$$

$$C_{ij} = \beta_{ij}\left(\alpha_j - 1\right), \quad \beta_{ij} = \left\langle \phi_i \left| V_{np}^2 \right| \phi_j \right\rangle$$

Transformation between CDCC and WSE

WSE and CDCC is related with

$$\chi_i^W(\boldsymbol{R}) = C_{i0}\chi_0(\boldsymbol{R}) + \sum_{j=1} C_{ij}\chi_j^{\mathrm{bin}}(\boldsymbol{R})$$

Coefficient can be determined with the orthogonal relation:

$$\langle \phi_i^W | V_{\mathrm{np}} | \phi_j^W \rangle = -\delta_{ij}$$

$$C_{i0} = -\left\langle \phi_i^W \middle| V_{np} \middle| \phi_d \right\rangle \quad (=0, \quad i \neq 1)$$

$$C_{ij} = -\left\langle \phi_i^W \middle| V_{np} \middle| \phi_j^{\mathrm{bin}} \right\rangle \quad (i, j = 1,2,\ldots)$$

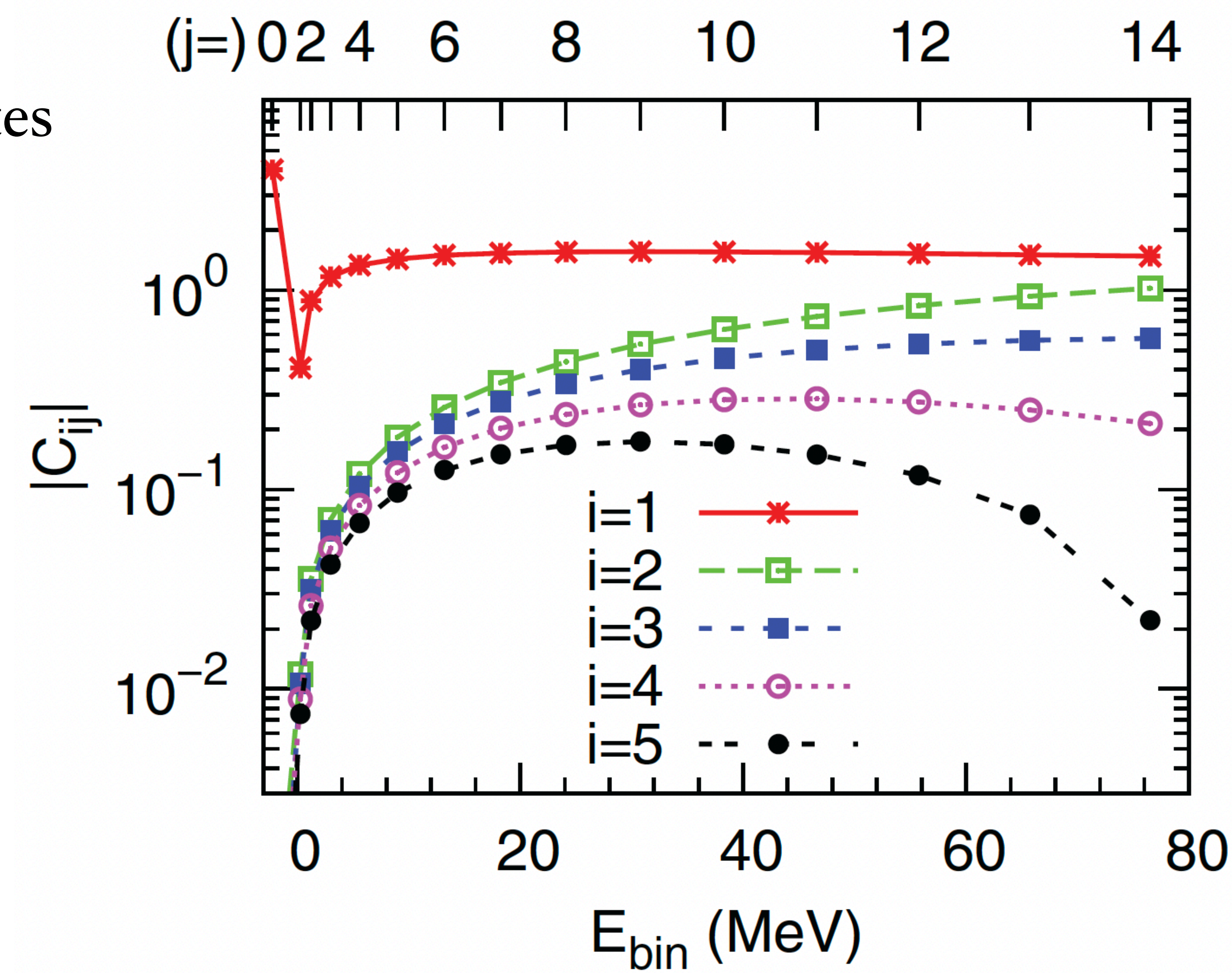
Coefficient determined by CDCC

Index for bin states

i: Index for Weinberg states

j: Index for bin states

Dominance for
the first Weinberg state



Energy region for bin states